Measurement of Vibrational Motions using a Three-Facet Mirror

W. S. Park [a], H. S. Cho [b], and Y. K. Byun [c]

[a] Central R&D Center, Mando Corporation
343-1, Manho-ri, Poseung-myun, Pyungtaek 451-821, Korea

[b] Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology
373-1, Kusong-dong, Yusong-gu, Taejeon 305-701, Korea

[c] Samsung Advanced Institute of Technology
P.O. Box 111, Suwon 440-600, Korea

ABSTRACT

A new measurement method to measure vibrational motions of objects is presented. The original principle is similar to the previous work that utilized a 3-facet mirror to obtain three dimensional positions and orientations of rigid bodies. While the previous work was presented for only stationary objects, in this paper, we newly investigate the feasibility of this method for dynamic applications. The 3-facet mirror that looks like a triangular pyramid having an equilateral cross-sectional shape. The mirror has three lateral reflective surfaces inclined 45 degrees to its bottom surface, and is mounted on the object whose motion is to be measured. As optical components, a He-Ne laser source and three position-sensitive detectors(PSD) are used. The laser beam is emitted from the He-Ne laser source located at the upright position and vertically incident to the top of the 3-facet mirror. The laser beam is reflected from the 3-facet mirror and splits into three sub-beams, each of which is reflected from the three facets and finally arrives at three PSDs, respectively. Since each PSD is a 2-dimensional sensor, we can acquire the information on the three dimensional position and orientation of the 3-facet mirror. From this principle, we can get the motion of any object simply by mounting the 3-facet mirror on the object. In this paper, the measurement principle and a series of experiments are presented. The experiments include measurements of vibrational motions of a piezoelectric actuator that moves the 3-facet mirror in a single axis. The experimental results are compared with those of a laser doppler vibrometer. Through the experiments, the proposed sensor is proven to be an effective means for measuring dynamic motions of objects.

Keywords: 3-dimensional pose, 3-facet mirror, position-sensitive detector, vibration analysis

1. INTRODUCTION

A rigid body which is not constrained to any kinematic condition has six degrees of kinematic freedom in space. Therefore, it needs six spatial parameters such as x, y, z in translation and roll, pitch, yaw in rotation to fully describe the 3-dimensional position and orientation of a rigid body. In order to simultaneously measure the 3-dimensional position and orientation of rigid bodies, several approaches have been proposed. Ivarsson and Sanderson[1] have proposed a 6-dof vibration sensor consisting of six accelerometers. This approach is relatively easy to implement and thus has been widely used. But it should be carefully designed so as to weigh less to minimize loading effect. Furthermore, to obtain translational and rotational displacement, it needs integration, which might cause error accumulation and drift.

Ratcliffe and Lieven[2], Briggs and Talke[3-4], and Jeong and Bogy[5] used laser doppler vibrometers(LDV) to obtain vibrations in both translations and rotations. This approach has no loading effect but cause error accumulation when one needs displacement. Lee et. al.[6] proposed a new measurement method that can measure 6-dof displacements simultaneously. They used a laser, four position sensitive detectors(PSD), and several beam splitters. This method needs no integration, but it should load several beam splitters on the object of interest. Thus, it is not suitable for high speed motions or vibrational motions since it cause much loading effect.

Bokelberg and Sommer[7-8] have proposed an optical system that consists of three laser sources, three PSDs, and a tetrahedral mirror. This method mounts a tetrahedral mirror on a structure to be analyzed. Three laser beams meet the three mirror surfaces, respectively. And three PSDs detects the reflected beams. This method is effective to get 6-dof simultaneously with minimized loading and without error accumulation. But it is quite difficult to implement since they should align three lasers and three PSDs accurately. This indicates it is probable to be errorneous.

Park et. al.[9] proposed to adopt a 3-facet mirror to measure 6-DOF displacement of objects in the precision order of a few micrometers in translation and a few micro radians in rotation. This method also adopts a tetrahedral mirror as Bokelberg and Sommer[7-8] method, but this method uses only a single laser. Thus, this method is much easier to implement and yields improved accuracy.

In this paper, a new measurement method is adopted to measure vibrational motion of objects. The method has been originally proposed by Park et. al.[9] to measure three dimensional positions and orientations of objects. However, they applied the method only to stationary objects. In this paper, the potential ability of the method to measure dynamic motion of objects is investigated. In section 2, The principle of the measurement method is presented including its mathematical model and some experiments for stationary objects. In section 3, experiments for simple translational vibration of an object excited by a piezoelectric actuator is presented. The data, in both aspects of time series and frequency domain, comparing the measurement results with those measured by a reference instrument, laser doppler vibrometer, are also presented, and discussed in detail.

2. MEASUREMENT OF THREE-DIMENSIONAL POSITION AND ORIENTATION

Sensing principle

Figure 1 shows the overall configuration of the sensor system that we propose to measure vibrational motions of objects. As stated previously, it has been originally proposed to obtain six degrees of freedom of objects[9]. As shown in the figure, the sensor system is composed of a mirror of pyramidal shape, a He-Ne laser source, three position-sensitive detectors(PSD). As shown in the figure, the laser beam is emitted from the He-Ne laser source located at the upright position and vertically incident on the top of a mirror of pyramid shape. The mirror is specially fabricated, which has an equilateral triangular cross-section as shown in Figure 2. The mirror is called 3-facet mirror in this paper, since the mirror has three lateral reflective surfaces inclined 45° to its bottom surface, and it reflects and splits the laser beam into three sub-beams.

As shown in the figure, three PSDs are located at three corner points of a triangular formation, which is an equilateral triangular formation lying parallel to the reference plane. The sensitive areas of three PSDs are oriented toward the center point of the triangular formation. The object whose position and orientation are to be measured is situated at the center with the 3-facet mirror on its top surface. Each reflective facet of the 3-facet mirror faces toward each PSD and the three laser beams reflected at the 3-facet mirror arrive at three PSDs, respectively. From the outputs of three PSDs, one can acquire the information on the 6-DOF pose of the 3-facet mirror. Furthermore, one can easily acquire 6-DOF pose of objects simply by mounting the 3-facet mirror on them.

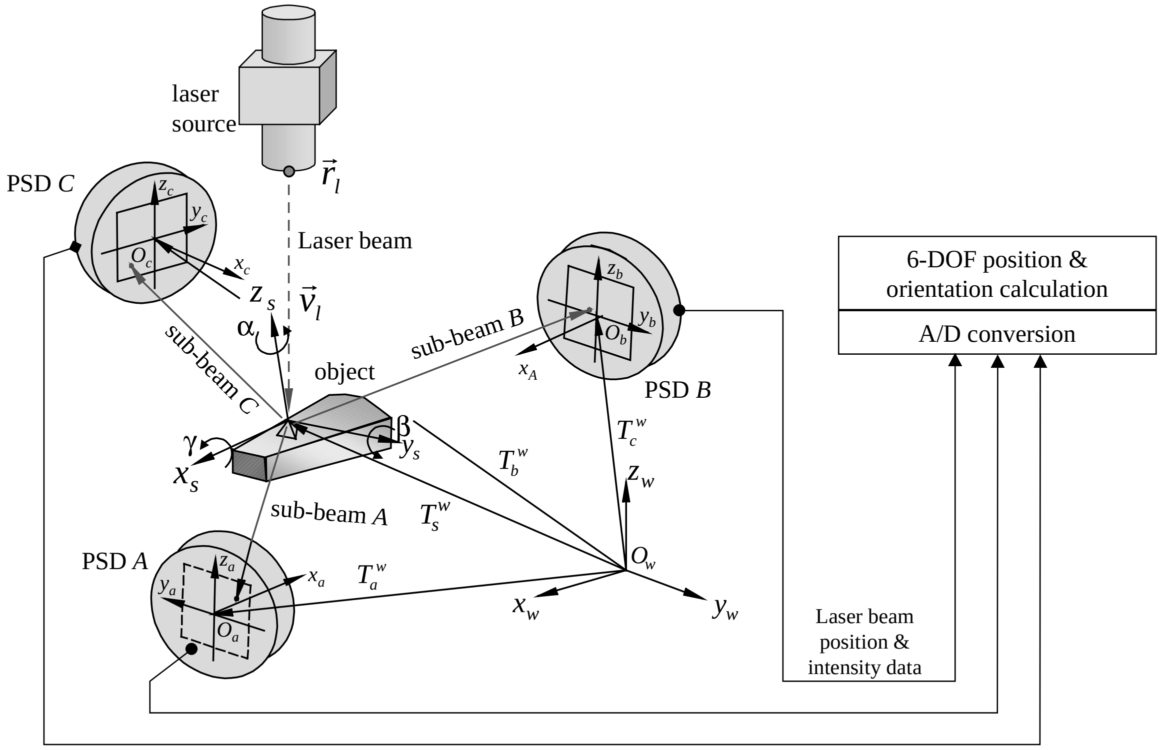


Figure 1 Schematic of the measurement system

Since each PSD provides the 2-dimensional position of laser beam spot and its irradiant power, three PSDs provide totally nine outputs related with the positions of the laser beam spots and power data. Park et. al.[9] presented the mathematical relationship between the 6-DOF pose, x, y, z-translation and roll, pitch, yaw rotation, and the nine outputs from the PSDs. The mathematical model is to be briefly reviewed.

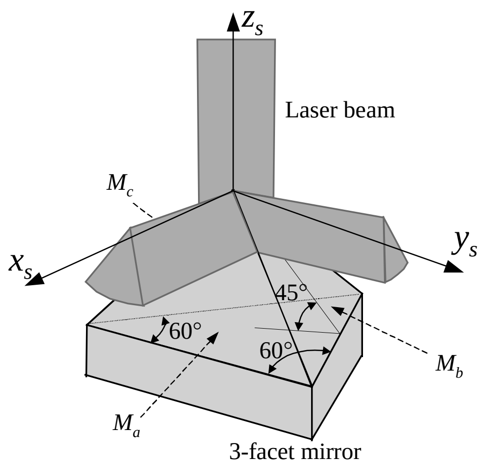


Figure 2 3-facet mirror reflecting a laser beam into three beams

Sensor model

The mathematical model relating 3-dimensional pose of the 3-facet mirror and the PSDs' outputs can be given in a form of following equation:

$$[\psi_a \;\; \zeta_a \;\; \Phi_a^* \;\; \psi_b \;\; \zeta_b \;\; \Phi_b^* \;\; \psi_c \;\; \zeta_c \;\; \Phi_c^*]^T = \mathbf{G}_f^*(t_x,\; t_y,\; t_z,\; \gamma,\; \beta,\; \alpha) \tag{1}$$

Here, $\mathbf{G}_f^*$ represents a vector function that yields the PSDs' outputs from given 3-dimensional pose of the 3-facet mirror. (ψ_a, ζ_a), (ψ_b, ζ_b), (ψ_c, ζ_c), Φ_a^*, Φ_b^*, and Φ_c^* denote three couples of 2-dimensional positions of laser spots on three PSDs and their irradiant powers, respectively. $\vec{t}^{\,w} = [t_x^w \quad t_y^w \quad t_z^w]^T$ and $\vec{\omega}^w = [\gamma \quad \beta \quad \alpha]^T$ represent 3-dimensional position and orientation of 3-facet mirror with respect to the reference coordinate system O_w, respectively.

Figure 3 shows that a sub-beam reflected from a mirror surface M_a goes incident on *PSD A* to form an image looking like a piece of pie, say P_a. In the figure, the dotted ellipse L_a is the imaginary cross-section of laser beam as if the laser beam is not split by the 3-facet mirror but totally reflected by a planar mirror whose pose is the same as that of M_a. S_a (ψ_a^o, ζ_a^o) is the center of the dotted ellipse L_a and also the projection of the center of laser beam. Two vertices of P_a, l_{ab}^a and l_{ca}^a, are the projections of the vertices of the 3-facet mirror, l_{ab} and l_{ca}. L_a including the arc of P_a is presented as the projection of the arc of laser beam defined by beam diameter ϕ_l. And the intersection of l_{ab}^a and l_{ca}^a, Q_a, is the point projected from the top of the mirror, as shown in the figure. If the center of laser beam coincides with the mirror top, then Q_a and S_a are located at same position. And, R_a (ψ_a, ζ_a) is the light centroid of P_a weighted with intensity distribution, which is the 2-dimensional position output of *PSD A*. And laser power Φ_a^* is total power incident on *PSD A* that can be calculated through integration of intensity distribution over P_a. Thus, (ψ_a, ζ_a) and Φ_a^* can be calculated through following equations:

$$\psi_a = \frac{\iint\limits_{P_a} y_a I_a(r_a)\,dy_a dz_a}{\iint\limits_{P_a} I_a(r_a)\,dy_a dz_a}, \qquad \zeta_a = \frac{\iint\limits_{P_a} z_a I_a(r_a)\,dy_a dz_a}{\iint\limits_{P_a} I_a(r_a)\,dy_a dz_a} \tag{2}$$

$$\Phi_a^* = \iint\limits_{P_a} I_a(r_a)\,dy_a dz_a \tag{3}$$

Here, $I_a(r_a)$ represents irradiant power distributions of the laser spot P_a on *PSD A*, and calculated from the intensity distribution of original laser beam $I(r)$ as follows.

$$I_a(r_a) = \frac{\vec{p}\cdot\hat{x}_a}{\|\vec{p}\|} I(r_a) \tag{4}$$

where

$$r_a = \sqrt{(y_a - \psi_a^o)^2 \frac{p_x^2}{p_x^2 + p_y^2} + (z_a - \zeta_a^o)^2 \frac{p_x^2}{p_x^2 + p_z^2}}\;. \tag{5}$$

and

$$\vec{p} = [\, p_x \quad p_y \quad p_z \,]^T \; . \tag{6}$$

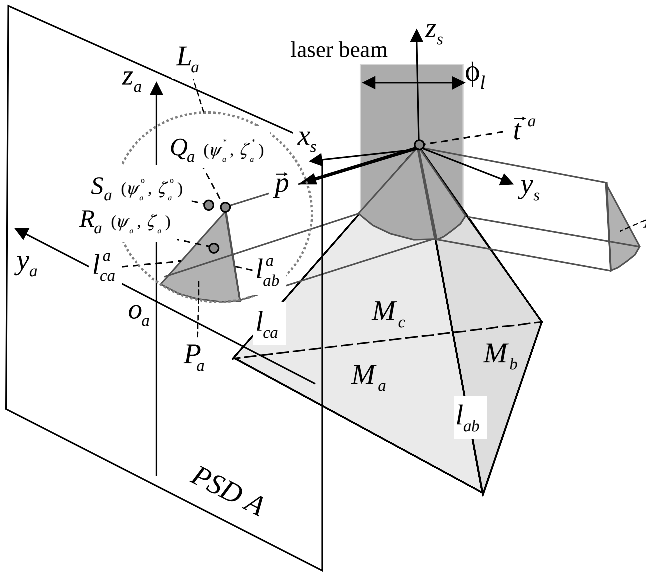


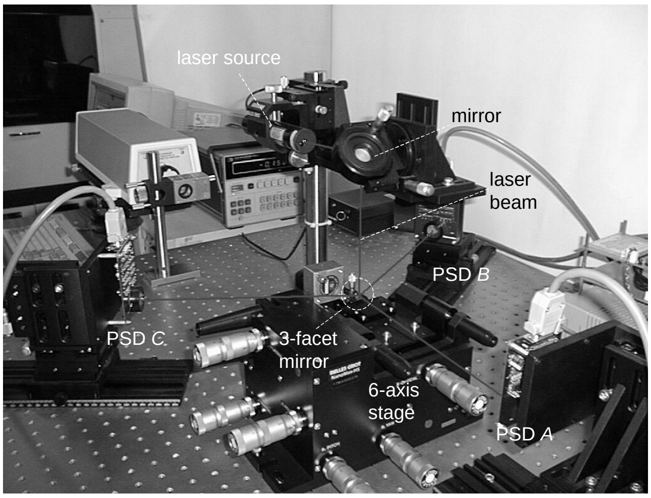
Figure 3 3-facet mirror reflecting a laser beam into *PSD A*

As for *PSD B* and *C*, similar procedures can be taken.

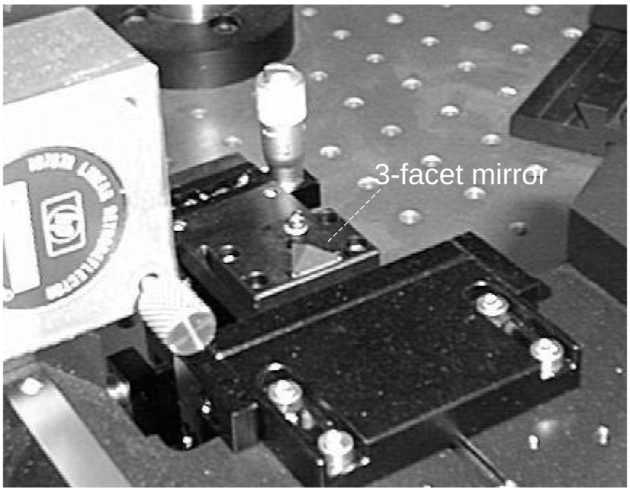
So far, we have modeled the relationship between the 3-dimensional pose, i.e. $t_x,\ t_y,\ t_z,\ \gamma,\ \beta,$ and α, and the three 3-tuples of PSD outputs, $(\psi_a, \zeta_a, \Phi_a)$, $(\psi_b, \zeta_b, \Phi_b)$, and $(\psi_c, \zeta_c, \Phi_c)$. Through the procedures explained above, we can calculate $(\psi_a, \zeta_a, \Phi_a)$, $(\psi_b, \zeta_b, \Phi_b)$, and $(\psi_c, \zeta_c, \Phi_c)$ with given $t_x,\ t_y,\ t_z,\ \gamma,\ \beta,$ and α. It should be noted that the sensor model explained in this paper can not calculate $\vec{t}^{\,w}$ and $\vec{\omega}^{w}$ from (ψ_a, ζ_a), (ψ_b, ζ_b), (ψ_c, ζ_c), Φ^*_a, Φ^*_b, and Φ^*_c. When we perform actual measurement with the proposed sensor system, we have to perform the inversion of above sequences of calculation, i.e. calculation of $t_x,\ t_y,\ t_z,\ \gamma,\ \beta,$ and α with given $(\psi_a, \zeta_a, \Phi_a)$, $(\psi_b, \zeta_b, \Phi_b)$, and $(\psi_c, \zeta_c, \Phi_c)$. This can be performed through a numerical way, Newton's method[11], through which we have successfully got the solution, $t_x,\ t_y,\ t_z,\ \gamma,\ \beta,$ and α, with given $(\psi_a, \zeta_a, \Phi_a)$, $(\psi_b, \zeta_b, \Phi_b)$, and $(\psi_c, \zeta_c, \Phi_c)$ in the experiments to be presented in this paper.

Measurement for three dimensional position and orientation of stationary objects

In the previous work of Park et. al.[9], a series of experiments to validate the proposed method through obtaining three dimensional position and orientation of a 3-facet mirror have been presented. In the experiments, the 3-facet mirror has been stepped at an even spacing in one of six axes while its position was measured using the model described above. And, similar measurement experiments were repeated in other axes. Through the experiments, the measurement resolutions for x_w, y_w, and z_w-translations are found to be 2.6μm, 1.3μm, and 2.8μm, respectively. And, those of roll, pitch, and yaw-rotations are found to be 9.3μrad, 8.7μrad, and 13.0μrad.



(a)

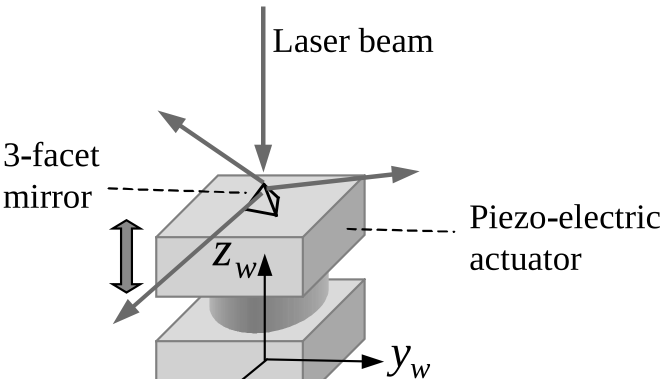


(b)

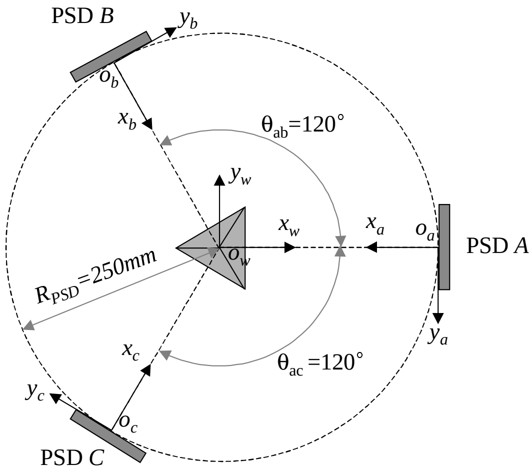
Figure 4 Photograph of experimental system: (a) Whole system; (b) 3-facet mirror

3. MEASUREMENT OF VIBRATORY MOTION

After reviewing that the proposed method can measure three dimensional pose of stationary objects, the feasibility on dynamic objects is to be investigated in this section through some experiments. The experiments include the measurement of translational vibrations of an object excited by a piezoelectric actuator. The 3-facet mirror is mounted on the moving part of a piezoelectric actuator, while the stationary part is fixed on a table as shown in Figure 5. The actuator works in z_w-axis so that the 3-facet mirror moves in same direction.



(a)



(b)

Figure 5 Measurement of translational vibration of a piezoelectric actuator
(a) 3-facet mirror mounted on piezoelectric actuator; (b) Layout of PSDs

In the experiments, the actuator moves the 3-facet mirror in such motions as sinusoidal wave, triangular wave, and rectangular wave at the exciting frequency of 10 Hz. Then, the motions of the 3-facet mirror are detected and at the same time, the motions are also measured with a reference instrument, laser dopper vibrometer(LDV). Figure 6 through Figure 8 present the experimental results of the motions. Figure 6 shows the time series data of the sinusoidal wave motion measured by the instruments, the proposed method and the LDV, and the amplitude spectra in frequency domain. In the figure, the displacement data of the proposed method are plotted in a solid line while those of LDV in a dotted line. We can see the proposed method estimates the motion of 3-facet mirror well. For quantitative evaluation, errors between the results of both instruments are also plotted. In the error plot, the deviation ranges from –0.01 mm to +0.01 mm, which is 5% of the peak-to-peak value of the wave read by the LDV. Spectra from both instruments indicate the principal frequency of the motion as 10 Hz. Both spectra look very close to each other.

Figure 7 shows the experimental results obtained from triangular wave motion. In the error plot, the amplitude of error is similar to that of the sinusoidal wave motion, which is 5% of the amplitude of displacement. The spectra of the data of both instruments are close to each other, in which they indicate the principal frequency as 10 Hz.

As for the rectangular wave motion, the error is rather larger than those of sinusoidal or triangular wave motions. As shown in Figure 8, the error abruptly rises at every instance that the actuator steps. This is because the 3-facet mirror was not fixed stiff enough. The spectra from both instruments are close to each other as those of sinusoidal or triangular wave motions. Both spectra read 10 Hz as the principal frequency of the motion.

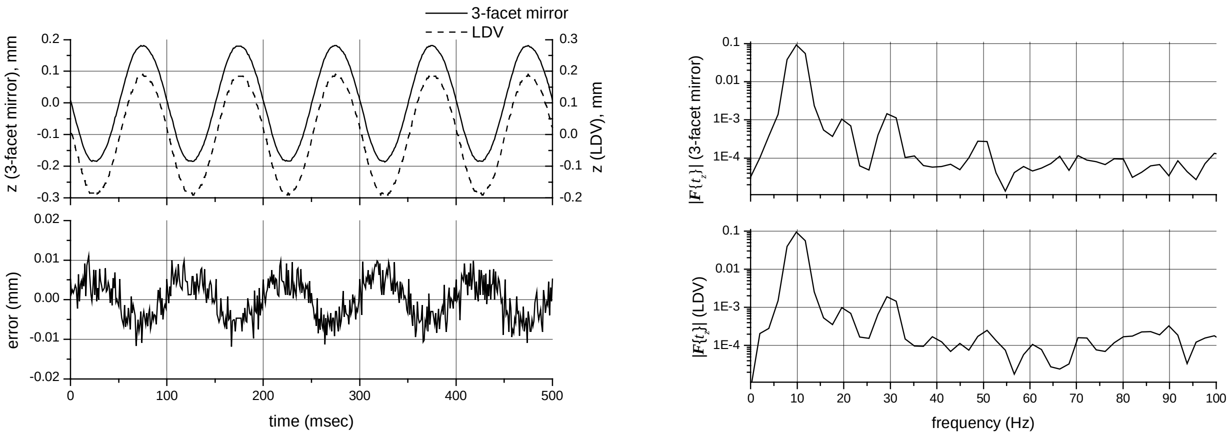


Figure 6   10 Hz sinusoidal wave motion of the piezoelectric actuator

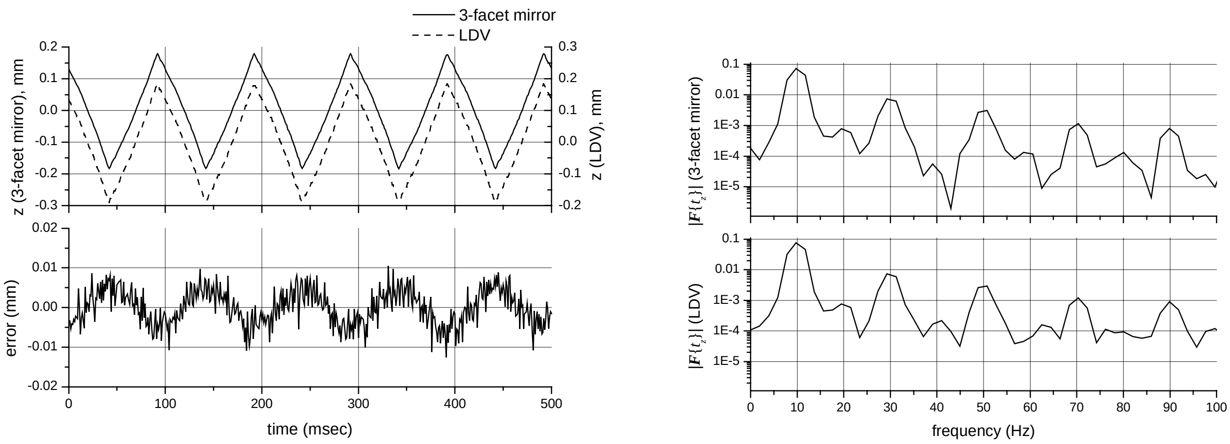


Figure 7   10 Hz triangular wave motion of the piezoelectric actuator

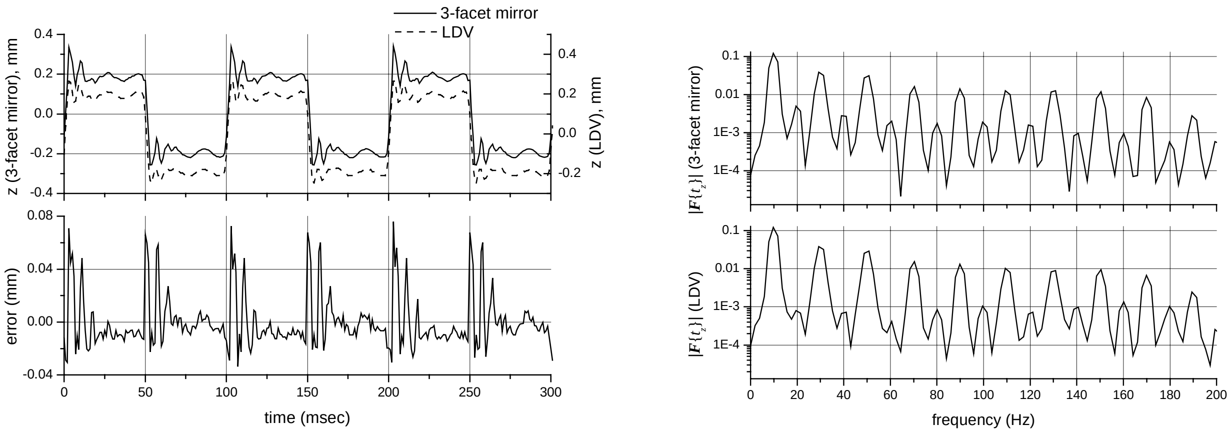
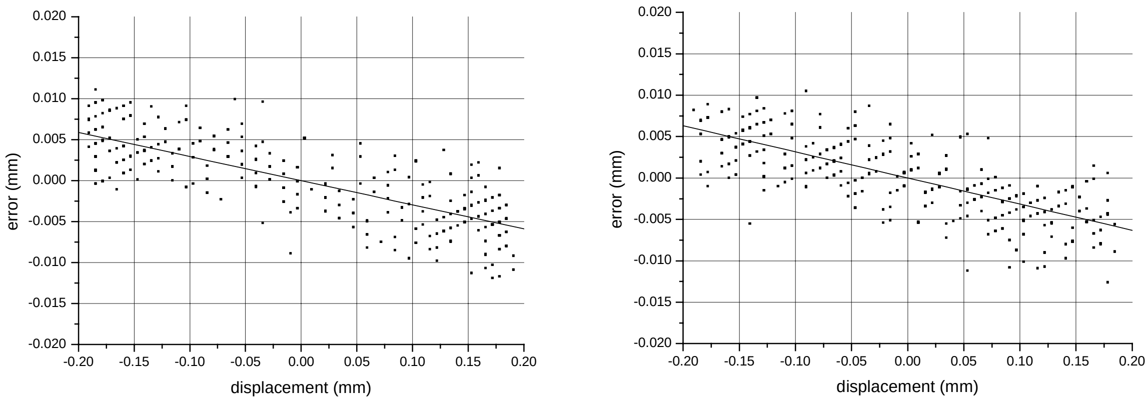


Figure 8 10 Hz rectangular wave motion of piezoelectric actuator

As shown in Figure 6 through Figure 8, the errors oscillate at 10 Hz, thus they look synchronized with the displacements. And they cross the zero value at the every instance that the displacement plots cross the zero value. From this, one can expect some correlation between errors and displacements. Figure 9 shows the errors with respect to displacements in both of sinusoidal wave motion and triangular motion. In the figure, the errors of both plots are negatively proportional to the displacements, in which the slopes of linear regression are the same as -0.03 as indicated in a solid line. This means it is possible to reduce errors to 2% of the span of vibration.



(a) (b)

Figure 9 Error distribution with respect to displacement: (a) Sinusoidal wave motion; (b) Triangular wave motion

4. CONCLUSION

A new measurement method has been adopted to measure vibrational motion of objects. In this work, the potential ability of the method to measure dynamic motions of objects is investigated. The proposed sensor system is based on laser optics which adopts a special mirror, 3-facet mirror, a He-Ne laser, and three PSDs. The sensor system simultaneously measures the 3-dimensional position and orientation of the 3-facet mirror mounted on objects in motion.

In this paper, the principle of the measurement method is reviewed including its mathematical model and some experiments for stationary objects. Experiments for simple translational vibration of an object excited by a piezoelectric actuator are presented. The data, in both aspects of time series and frequency domain, comparing the measurement results with a reference instrument, laser doppler vibrometer, are also presented. Through the experiments, we could verify that the measurement principle is valid and the proposed sensor system can measure the vibrational motion of rigid bodies within 5% uncertainty. This uncertainty is expected to be improved up to 2% through simple error compensation technique.

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