A Novel Full-Diversity Full-Rate Transmit Beamforming Systems Combined with V-BLAST over Correlated Fading Channels

Jung-Eun Kim, Hyoungsuk Jeon, Wooram Shin, and Hyuckjae Lee
School of Engineering
Information and Communications University
119 Munjiro Yuseong-gu Daejeon, Korea
Email: {keiswel, hschun, smoothy85, hjlee}@icu.ac.kr

***Abstract*— In this paper, we propose a novel transmit beamforming algorithm combined with vertical Bell laboratories layered space-time (V-BLAST) over correlated fading channels. Using the antenna groups at the transmitter, not only spatial multiplexing gain but also beamforming gain can be achieved. We encode the transmit signals in order to enhance the diversity gain compared with conventional V-BLAST system combined with beamforming without changing the number of antennas. Simulation result shows that our proposed algorithm provides a significant reduction in the BER compared with the conventional V-BLAST and transmit beamforming combined with V-BLAST without encoding transmit data.**

***Keywords*—Space-time code (STC), Linear dispersion code (LDC), Golden code, Beamforming, Spatial multiplexing, Full-rate full-diversity gain**

I. INTRODUCTION

In the rich scattering environments, the multiple-input multiple-output (MIMO) antenna systems are well known to promise higher data rates and quality of service (QoS). High data rate can be achieved through the spatial multiplexing. The vertical Bell laboratories layered space-time (V-BLAST) is well known for achieving the spatial multiplexing gain by setting the antenna spacing is ten times of the carrier wave length in order to make the independent data paths. On the other hands, beamforming systems require to obtaining spatial directivity. The critical beamforming pattern can be achieved by spacing antenna elements as half times to the carrier wave length. Because of this inconsistent demands at antenna spacing, the researches about combining with beamforming and V-BLAST are regarded as difficult problems. Although the contradictive requirements for antenna spacing, in order to improve the system performance employing STC or V-BLAST, new beamforming structures combined with STC or V-BLAST to achieve diversity gain or spatial multiplexing (SM) gain as well as beamforming gain have been researched in [4], [5]. When the MIMO channel is given, diversity gain and SM gain can be achieved simultaneously, but there is a fundamental tradeoff between how much of each type of gain can extract.

In this paper, we propose a novel transmit beamforming system that can be achieved the MIMO diversity-multiplexing frontier by encoding transmit symbols. The MIMO antenna structure is equivalent to [4] and the Golden code [6] is used at the transmitter for achieving not only full-diversity but also full-rate in the 2-Tx, 2-Rx V-BLAST system. Moreover, beamforming weight vector is optimized for maximum mutual information in the given channels.

This paper is organized as follow. The system model of a novel transmit beamforming combined with V-BLAST is presented in section 2. Section 3 describes how to achieve diversity-multiplexing frontier as well as beamforming gain. Then, the simulation result with analysis is followed in Section 4. Finally the paper is concluded in section 5.

II. SYSTEM MODEL

Fig. 1. shows the proposed transmit beamforming combined with V-BLAST system. It divides $N_G N_t$ antennas into N_G groups to form virtual $N_G \times N_r$ V-BLAST system with the N_r received antennas. We assume that antenna correlations among N_G groups at the transmitter and among N_r receive antennas are negligible but N_t antennas of each group are fully correlated. The N_t correlated antennas of each group send same symbols to make directional beam. Then, we can model the received signal as follows:

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\tilde{\mathbf{x}} + \mathbf{n} \\ &= \begin{bmatrix} \mathbf{H}_1 & \dots & \mathbf{H}_{N_G} \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 x_1 \\ \vdots \\ \mathbf{w}_{N_G} x_{N_G} \end{bmatrix} + \mathbf{n} \\ &= \sum_{i=1}^{N_G} \mathbf{H}_i \mathbf{w}_i x_i + \mathbf{n} \end{aligned} \tag{1}$$

where $\mathbf{H} \in \mathcal{C}^{N_r \times (N_t N_G)}$ is the channel fading matrix which consists of sub-channel matrix $[\mathbf{H}_1 \dots \mathbf{H}_{N_G}]$. The sub-channel matrix $\mathbf{H}_i \in \mathcal{C}^{N_r \times N_t}$ is a channel of i-th group and modeled as $\mathbf{H}_i = \mathbf{R}_r^{1/2} \mathbf{H}_\omega \mathbf{R}_t^{1/2}$ where $\mathbf{R}_t$ and $\mathbf{R}_r$ are covariance matrices of the transmit and receive antennas respectively. The receive covariance matrix is replaced by $\mathbf{I}_{N_r \times N_r}$ according to our assumption, and the transmit covariance matrix follows the widely used one in [5]. $N_r \times N_t$ random matrix $\mathbf{H}_\omega$ is independent and identically distributed (i.i.d) circular symmetric Gaussian matrix. The transmit symbol vector $\tilde{\mathbf{x}}$ consists of beamforming weight vector $\mathbf{w}_i \in \mathcal{C}^{N_t}$ ($\|\mathbf{w}_i\|_2^2 = N_t$), $i = 1, 2, ..., N_G$ in the i-th group and modulated data symbol vector $\mathbf{x}$. The random noise vector $\mathbf{n} \in \mathcal{C}^{N_r}$ is complex Gaussian with zero-mean and noise variance $\frac{N_0}{2}\mathbf{I}_{N_r \times N_r}$.

III. A NOVEL FULL-DIVERSITY FULL-RATE TRANSMIT BEAMFORMING SYSTEMS

In this section, we show how to obtain the diversity-multiplexing frontier by encoding the transmit symbols and how to determine the beamforming weight vector which is optimized to maximize the capacity.

A. Transmit Data Encoding

General families of 2-Tx, 2-Rx full-diversity full-rate linear dispersion codes (LDC) achieve the optimum diversity-multiplexing tradeoff. The Golden code, which is a kind of LDC, outperforms all previous LDC because the non-vanishing determinants of the Golden code make the maximum coding gain. Therefore, we adopt the Golden code to encoding the transmit symbols.

Coding matrix of the Golden code is

$$\mathbf{x} = \Phi\mathbf{s} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha\left(s_1 + \theta s_2\right) & \alpha\left(s_3 + \theta s_4\right) \\ i\overline{\alpha}\left(s_3 + \overline{\theta} s_4\right) & \overline{\alpha}\left(s_1 + \overline{\theta} s_2\right) \end{bmatrix} \tag{2}$$

where $\mathbf{s} = [s_1, s_2, s_3, s_4]$ are the modulated data symbols and $\theta = \frac{1+\sqrt{5}}{2}$, $\overline{\theta} = \frac{1-\sqrt{5}}{2}$, $\alpha = 1 + i - i\theta = 1 + i\overline{\theta}$ and $\overline{\alpha} = 1 + i - i\overline{\theta} = 1 + i\theta$. For capacity lossless design, dispersion matrix Φ should be $\Phi\Phi^H = \mathbf{I}$ and the factor $\frac{1}{\sqrt{5}}$ is necessary.

B. Beamforming Weight Vector Calculation

In the proposed scheme, the beamforming criterion is based on maximizing the mutual information and the problem is solved by Eigen decomposition. In this part, we briefly derive the beamforming weight vector based on mutual information maximization criterion. The mutual information is given by

$$C = \ln\det\left(\mathbf{I}_{N_r \times N_r} + \frac{SNR}{N_G N_t} \sum_{i=1}^{N_G} \mathbf{H}_i \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_i^H\right) \tag{3}$$

which depends on the number of group and beamforming weight vectors. Because the natural logarithm is monotonically increasing, $\det(\cdot)$ can be used instead of $\ln[\det(\cdot)]$. For any symmetric positive semi-definite matrices $\mathbf{A}$ and $\mathbf{B}$, $\det(\mathbf{I}_{n\times n} + \mathbf{A}) \leq \det(\mathbf{I}_{n\times n} + \mathbf{A} + \mathbf{B})$ is satisfied. Therefore, the mutual information is lower bounded

$$\det\left(\mathbf{I}_{Nr \times Nr} + \frac{SNR}{N_t}\mathbf{H}_j \mathbf{w}_j \mathbf{w}_j^H \mathbf{H}_j^H\right) \leq C \begin{matrix} \mathbf{w}_1, & \ldots & , \mathbf{w}_{N_r} \\ n_1, & \ldots & , n_{N_r} \end{matrix} \tag{4}$$

Using Jacobi's formula, $\det(\mathbf{I} + \mathbf{X}) \cong 1 + \mathrm{tr}(\mathbf{X})$, the approximated mutual information can be represented like this,

$$\begin{aligned} &\det\left(\mathbf{I}_{N_r \times N_r} + \tfrac{SNR}{N_G N_t} \sum_{i=1}^{N_G} \mathbf{H}_i \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_i^H\right) \\ &\cong 1 + \mathrm{tr}\left(\tfrac{SNR}{N_G N_t} \sum_{i=1}^{N_G} \mathbf{H}_i \mathbf{w}_i \mathbf{w}_i^H \mathbf{H}_i^H\right) \\ &= 1 + \tfrac{SNR}{N_G N_t} \sum_{i=1}^{N_G} \mathbf{w}_i^H \mathbf{H}_i^H \mathbf{H}_i \mathbf{w}_i \end{aligned} \tag{5}$$

The maximum mutual information is achieved by choosing $\mathbf{w}_i$ as the eigenvector of the maximum eigenvalue of the $\mathbf{H}_i^H \mathbf{H}_i$.

IV. SIMULATION RESULT

The performance of the beamforming system combined with V-BLAST with golden code is demonstrated by the Monte-Carlo simulation. We consider the downlink transmission system with 2 groups and 2 receive antennas. Each group consists of 2-element ULA. The channel is assumed quasi-static Rayleigh flat fading and the correlation coefficient ρ is set to 0.8 as the high spatially correlated case respectively. We compare our proposed scheme(BF+V-BLAST GC) with the Golden code(GC), 2-Tx, 2-Rx conventional V-BLAST(2x2 V-BLAST), transmit beamforming combined with V-BLAST without transmit data encoding (BF+V-BLAST w/o GC) in terms of the bit error rate(BER) vs. SNR.

Fig. 2 shows that BF+V-BLAST w/o GC achieves SNR gain compared with 2-Tx, 2-Rx conventional V-BLAST about 3dB at a BER of 10^{-3}. On the other hands, the Golden code achieves diversity gain at 2-Tx, 2-Rx antenna structures because the slope is significantly changed. The proposed scheme, therefore, attains full-diversity gain and SNR gain while sending full rate of transmit data.

V. CONCLUSION

In this paper, we have developed a novel transmit beamforming structure combined with V-BLAST in the correlated fading channel. The Golden code is a kind of the linear dispersion code which achieves full-diversity and full-rate only to MIMO antenna structure.

Comparing with the MIMO antenna structure combining transmit beamforming and spatial multiplexing, the proposed scheme has lower bit error rate due to diversity gain and coding gain.

REFERENCES

[1] F. Rashid-Farrokhi, K. J. R. Liu, L. Tassiulas, "Transmit Beamforming and Power Control for Cellular Wireless Systems," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1437-1450, Oct. 1999.

[2] P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel," in *Proc. URSI Int. Symp. Signals, Systems, and Electronics*, Pisa, Italy, pp. 295-300, 29 Sept.-2 Oct. 1998.

[3] V. Tarokh, N. Seshadri, A. R. Calderbank, "Space time codes for high data rate wireless communication: Performance analysis and code construction," *IEEE Trans. Info. Theory*, vol. 44, no.2, pp. 744-765

[4] I. Kim, K. Lee, J. Chun, "A MIMO Antenna Structure that Combines Transmit Beamforming and Spatial Multiplexing," *IEEE Trans. Wireless Commun.*, vol. 6, no. 3, Mar. 2007.

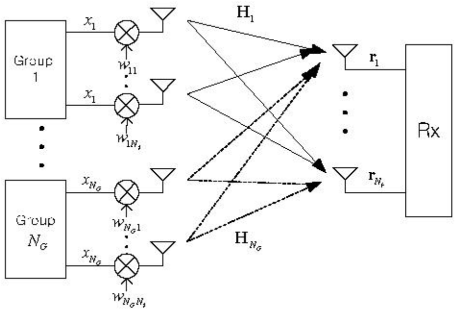

Fig. 1. The block diagram for the combined scheme of the beamforming and V-BLAST

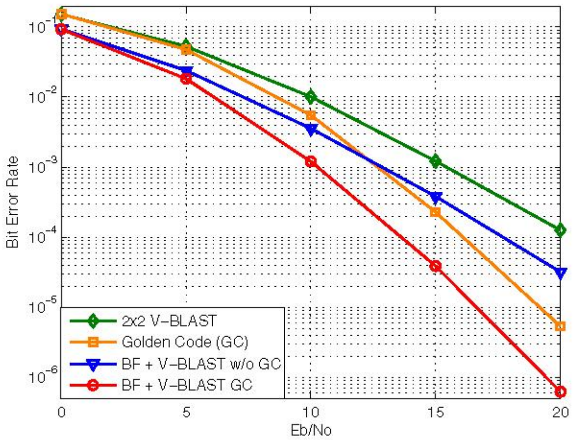

Fig. 2. BER performance comparison of proposed schemes vs. conventional V-BLAST for QPSK with ML decoding in the high correlation ($N_G = N_t = N_r = 2$, $\eta = 4\text{bps/Hz}$)

[5] Z. Lei, C. F. P. S., Y. C. Liang, "Combined Beamforming with Space-Time Block Coding for Wireless Downlink Transmission," *IEEE 56th, Conf on Vehicular Technology Proceedings*, vol. 4, Sept. 2002.

[6] L. Zheng, D.N.C. Tse, "Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels", *IEEE Trans. Info. Theory*, vol. 49, Issue 5, May 2003

[7] J.-C. Belfiore, G. Rekaya, E. Viterbo, "The Golden Code: A 2X2 Full-Rate Space-Time Code with Non-Vanishing Determinants," *IEEE Trans. Info. Theory*, vol. 51, no. 4, pp. 1432-1436, Apr. 2005.