PERFORMANCE OF LDPC CODE BASED D-BLAST SYSTEM

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Abstract **In this paper, we derive a closed form of symbol error probability of low density parity check (LDPC) code based diagonal Bell laboratories layered space-time (D-BLAST) system over independent fading channel, especially for low SNR case. To derive the expression, we use Craig's Q-function and moment generating function, which result in simple expression. This resulting expression is a function of weight distribution of LDPC code. Our approach is also applicable to any block coded space-time code with long blocklength.**

I. INTRODUCTION

Recently, many researches have focused on multiple-input multiple-output (MIMO) system to achieve large capacity and to combat fading environment. It is shown that if the fading coefficients between all pairs of transmit and receive antenna are statistically independent and known to the receiver, the theoretical capacity over such MIMO Rayleigh fading channels increases linearly with the smaller number of transmit and receiver antennas [1].

Motivated by these results, many transmission schemes have been proposed. Among those, Bell Labs Layered Space-Time (BLAST) is one of the transmit-receive architecture using spatial multiplexing and sub-optimal processing to detect transmitted signal from each transmit antenna. It gives a reasonable tradeoff between complexity and performance. One of the BLAST system is Diagonal BLAST (D-BLAST) which spreads each layer in space and time, and relies on layer's encoding to achieve transmit diversity gain [2].

In [3], the performance of D-BLAST over fading channel is derived using Chernoff bound. In [4], the Gaussian approximation is used to extend the results of [3]. In [5], the pairwise error probability for general space-time code using the Gauss-Chebyshev quadrature is derived. In [6], Craig develops another form of Q-function which has exponential dependence on the argument. In [7], the performance of modulation over fading channel is derived using Craig's Q-function.

LDPC codes were proposed by Gallager [8] and their performance is very close to the Shannon limit with practical decoding complexity like Turbo codes. In fading channel, LDPC codes can significantly reduce the error floor with a modest computational complexity [9]. Especially, it is shown that LDPC code has excellent performance in the low SNR environment. In [10], LDPC codes are applied to Space-time coded LDPC system. In [11][12], the asymptotic distance distributions of LDPC code are derived.

In the paper, we derive symbol error probability for LDPC codes based D-BLAST architecture using the Craig's Q-function and moment generating function.

This paper is organized as follows. In section II, we describe the system model of proposed scheme, In section III, we derive the expression for error probability for low SNR case, and In section IV, we show the distance spectrum for regular LDPC, and simulation results over independent fading channel when both transmitter and receiver antennas are equal to four.

II. SYSTEM MODEL

The system model we consider is shown in Figure 1. We denote a MIMO system with n_T transmit antenna and n_R receive antenna as (n_T, n_R). It is assumed uncorrelated Rayleigh fading, and channel transfer function is known at the receiver. Also total transmitter power P is equally divided at each transmitting antenna. Therefore the received signal can be represented by

$$\mathbf{r} = \sqrt{\frac{P}{n_T}}\mathbf{H}\mathbf{x} + \mathbf{n}, \quad (1)$$

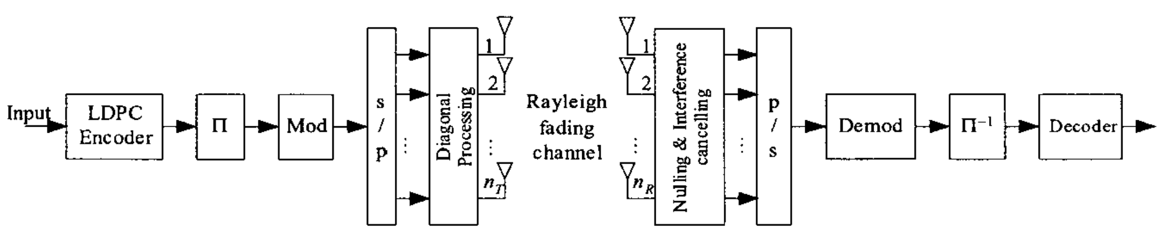

Figure 1. (n_T, n_R) LDPC codes based D-BLAST

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where $\mathbf{H}$ is a $n_R \times n_T$ matrix having independent identically distributed, complex, zero-mean, unit-variance entries and $\mathbf{n}$ is a complex n_R dimension Gaussian with zero mean, variance of N_0. Here we assume $n_T = n_R$.

At the transmitter, a primitive data stream is de-multiplexed into n_T data streams of equal rate, and each data stream is encoded using LDPC code encoder and interleaved. Each interleaved data streams are dispersed along diagonals in space-time. Each diagonal layer constitutes a complete codeword so detection is performed layer by layer. Assuming codeword length of N, the length of transmitted data at each antenna is $\frac{N}{n_T}$. Therefore, transmitted codeword over one diagonal can be represented $\mathbf{x} = [\mathbf{x}^0, \mathbf{x}^1, \ldots, \mathbf{x}^{n_T-1}]$ where each subvector $\mathbf{x}^n$ with blocklength $\frac{N}{n_T}$ represents the signal to be transmitted by antenna $n+1$. For example, $\mathbf{x}^0 = (x_0^0, x_1^0, \ldots, x_{\frac{N}{n_T}-1}^0)$ means transmitted signal from first transmit antenna. The coded substreams fill the space-time codeword in a block-diagonal form as shown below:

$$\begin{bmatrix} x_0^0 x_1^0 x_2^0 \cdots x_{\frac{N}{n_T}-1}^0 & \cdots & & \\ \mathbf{0} & x_0^1 x_1^1 x_2^1 \cdots x_{\frac{N}{n_T}-1}^1 & & \\ \mathbf{0} & \ddots & & \vdots \\ \mathbf{0} & \mathbf{0} & & x_0^{n_T-1} x_1^{n_T-1} x_2^{n_T-1} \cdots x_{\frac{N}{n_T}-1}^{n_T-1} \end{bmatrix}$$

D-BLAST receiver processing consists of canceling and nulling. Interferences that are not detected from upper diagonals are nulled out and already detected signals are subtracted out using QR decomposition. By multiplying Hermitian matrix of $\mathbf{Q}$ by received signal (1), nulled signal can be represented by

$$\mathbf{y} = \mathbf{Q}^* \mathbf{r} = \mathbf{R}\mathbf{x} + \mathbf{v}, \tag{2}$$

where $\mathbf{Q}$ is a unitary matrix, * represents conjugate transpose operation, and $\mathbf{R}$ is a upper triangular matrix.

$$\mathbf{R} = \begin{bmatrix} R^{1,1} & R^{1,2} & \cdots & R^{1,n_T} \\ 0 & R^{2,2} & & R^{2,n_T} \\ 0 & & R^{3,3} & R^{3,n_T} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & R^{n_T,n_T} \end{bmatrix}.$$

Since $\mathbf{Q}$ is a unitary matrix, the statistical properties of the noise term $\mathbf{v} = \mathbf{Q}^* \mathbf{n}$ is unchanged and nth subvector of $\mathbf{y}$ having length $\frac{N}{n_T}$ becomes

$$\mathbf{y}^n = R^{n,n}\mathbf{x}^n + \mathbf{v}^n + \sum_{i=n+1}^{n_T-1} R^{n,i}\mathbf{x}^i, \tag{3}$$

where $R^{n,i}$ is $(n,i)^{\text{th}}$ element of matrix $\mathbf{R}$. This received substream consists of the transmitted signal plus noise and interferences from other antennas.

III. PERFORMANCE ANALYSIS

Assuming $\mathbf{x}$ is transmitted, the pair-wise error probability that the receiver chooses $\hat{\mathbf{x}}$ instead of $\mathbf{x}$ conditioned on channel matrix $\mathbf{H}$ is [3]:

$$\begin{aligned} &P(\mathbf{x} \to \hat{\mathbf{x}} \mid \mathbf{H}) \\ &= P\left(\sum_{n=0}^{n_T-1} \left\{ \left\| \mathbf{y}^n - R^{n,n}\hat{\mathbf{x}}^n \right\|^2 - \left\| \mathbf{y}^n - R^{n,n}\mathbf{x}^n \right\|^2 \right\} < 0 \right) \\ &= Q\left(\sqrt{\frac{E}{2N_0} \sum_{n=0}^{n_T-1} \left\| R^{n,n}\left(\mathbf{x}^n - \hat{\mathbf{x}}^n\right) \right\|^2} \right) \end{aligned} \tag{4}$$

Since $\left\| R^{n,n} \right\|^2$ is a sum of squares of $2(n_T - n)$ zero-mean Gaussian distributed random variables, each with a variance 1/2 [4], the moment generating function of $\xi = \frac{E}{2N_0} \sum_{n=0}^{n_T-1} \left\| R^{n,n}\left(\mathbf{x}^n - \hat{\mathbf{x}}^n\right) \right\|^2$ is [5]:

$$M_\xi(s) = \prod_{n=0}^{n_T-1} \left(1 - \frac{sE\left\|\mathbf{x}^n - \hat{\mathbf{x}}^n\right\|^2}{2N_0} \right)^{-(n_T-n)}. \tag{5}$$

Using the approximation $(1+mx)^{-1} \approx (1+x)^{-m}$ for small mx [3], (5) can be approximated by

$$M_\xi(s) \approx \left(1 - \frac{sE}{2N_0} \right)^{-\sum_{n=0}^{n_T-1} \left\|\mathbf{x}^n - \hat{\mathbf{x}}^n\right\|^2 (n_T-n)}. \tag{6}$$

This is valid assumption since LDPC works well at low SNR regime. Average pairwise-error probability is obtained by averaging the conditional error probability.

$$P(\mathbf{x} \to \hat{\mathbf{x}}) = \int Q(\sqrt{\xi}) p(\xi) d\xi. \tag{7}$$

Craig [6] defined a new form of Q-function as $Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta$. This form has advantages of finite integration limits independent of the argument of the function x and of having an Gaussian form with respect to x [7]. Now using the definition of Craig's Q-function, we get average pairwise error probability as:

$$\begin{aligned} P(\mathbf{x} \to \hat{\mathbf{x}}) &= \int_0^\infty \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{\xi}{2\sin^2\theta}\right) p(\xi) d\xi \\ &= \frac{1}{\pi} \int_0^{\pi/2} M_\xi\left(-\frac{1}{2\sin^2\theta}\right) d\xi \end{aligned} \tag{8}$$

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From (6) and (8), we get [7]:

$$
\begin{aligned}
P(\mathbf{x} \to \hat{\mathbf{x}}) &= \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{\sin^2 \theta}{\sin^2 \theta + \frac{E}{4N_0}} \right)^p d\theta \\
&= \frac{1}{2} \left[1 - u \sum_{k=0}^{p-1} \binom{2k}{k} \left(\frac{1-u^2}{4} \right)^k \right]
\end{aligned}, \tag{9}
$$

where $u = \sqrt{\frac{E/4N_0}{1+E/4N_0}}$ and $p = \sum_{n=0}^{n_T-1} \left\| \mathbf{x}^n - \hat{\mathbf{x}}^n \right\|^2 (n_T - n)$.

In (9), p represents weighted distance between two codeword $\mathbf{x}$ and $\hat{\mathbf{x}}$ in DBLAST. For large block length like LDPC code, we can assume that using D-BLAST, each antenna has the similar weight distribution such as $\left\| \mathbf{x}^n - \hat{\mathbf{x}}^n \right\|^2 \approx \frac{d}{n_T}$, where d is a Hamming distance between two codewords $\mathbf{x}$ and $\hat{\mathbf{x}}$. Therefore p is simplified as:

$$
p \approx \frac{d(n_T+1)}{2}. \tag{10}
$$

Given pair-wise error probability, the symbol error probability is represented by:

$$
P(e) = \sum_{\mathbf{x} \neq \hat{\mathbf{x}}} q(\mathbf{x} \to \hat{\mathbf{x}}) P(\mathbf{x} \to \hat{\mathbf{x}}), \tag{11}
$$

where $q(\mathbf{x} \to \hat{\mathbf{x}})$ is the number of errors associated with each pairwise error probability.

For LDPC codeword which has a long blocklength N, the pairwise error probability $P(\mathbf{x} \to \hat{\mathbf{x}})$ is a function of a Hamming distance between two codewords $\mathbf{x}$ and $\hat{\mathbf{x}}$. Without loss of generality, we can assume the transmitted $\mathbf{x}$ be all-zero codeword. Then $P(\mathbf{x} \to \hat{\mathbf{x}})$ is a function of Hamming weights of $\hat{\mathbf{x}}$ and $q(\mathbf{x} \to \hat{\mathbf{x}})$ is reduced to be number of codewords of weight of $\hat{\mathbf{x}}$. Therefore, using the union bound, the symbol error probability (11) can be upper-bounded as:

$$
\begin{aligned}
P(e) &\leq \sum_{d=d_{\min}} A_d P(\mathbf{x} \to \hat{\mathbf{x}}) \\
&= \frac{1}{2} \sum_{d=d_{\min}} A_d \left[1 - u \sum_{k=0}^{\frac{d(n_T+1)}{2}-1} \binom{2k}{k} \left(\frac{1-u^2}{4} \right)^k \right]
\end{aligned}, \tag{12}
$$

where A_d is the number of codewords of Hamming weight d and where $u = \sqrt{\frac{E/4N_0}{1+E/4N_0}}$.

IV. EXAMPLES

In this paper, we use regular LDPC codes proposed by Gallager for D-BLAST architecture. In [8][11][12], the distance spectrums of (n, j, k) regular LDPC codes with codeword length N, uniform column weight j ($j \geq 3$), and uniform row weight k ($k > j$) are described. Over an ensemble of LDPC codes, the minimum distance is a random variable. But, as block length N increases, the probability distribution function of minimum distance is upperbounded by a unit step function at $\theta \in (0,1)$. Thus, for large N, the minimum distance of LDPC codes in the ensemble have at least $N\theta$. The average distance distributions for code rate R and the weight fraction θ of N are (when k is even) [12];

$$
b_0 = H(\theta) + p_0^R, \tag{13}
$$

where p_0^R is

$$
p_0^R = R \ln \left(\frac{(1+t)^k + (1-t)^k}{2t^{\theta k}} \right) - RkH(\theta), \tag{14}
$$

and t is the only positive root of

$$
\frac{(1+t)^{k-1} + (1-t)^{k-1}}{(1+t)^k + (1-t)^k} = 1 - \theta. \tag{15}
$$

Using (13), (14), (15), the average distance distributions of Gallager's ensemble of $(j,k) = (3, 6)$ regular LDPC codes are evaluated as shown in Figure 3. In this case, the minimum distance ratio is $\theta_{3,6} = 0.023$ which is first zero crossing and the minimum distance increase linearly with the code word length N.

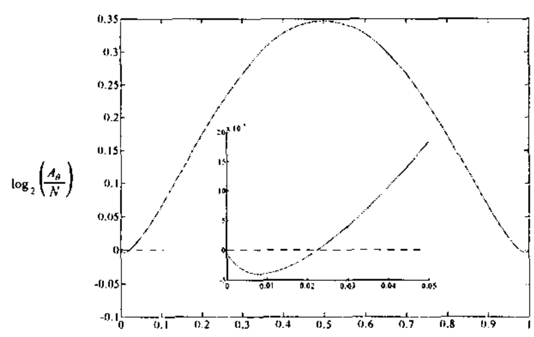

Figure 3. Average normalized distance distribution for $(j,k) = (3,6)$ [12]

Since Figure 3 shows asymptotic normalized distance distribution of regular LDPC code, we calculate for distance distribution for blocklength N = 1008. Since

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LDPC codes have even weights, the minimum Hamming distance for (6, 3) code is 24. The next distance spectrum (d, A_d) are (26, 4), (28, 14), (30, 45), (32, 156), (34, 559), ... In Figure 4, we show the distance distributions up to 30 for regular (6, 3) LDPC code.

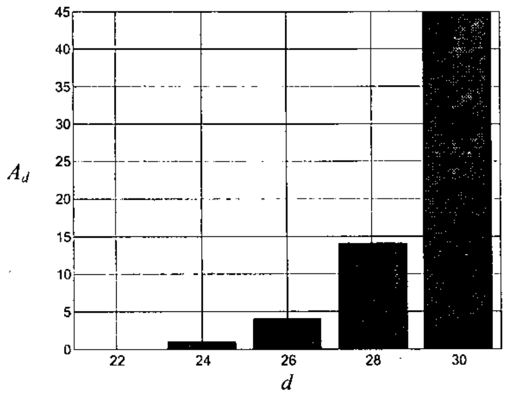

Figure 4: Weight distribution of (6, 3) regular LDPC codes up to 30 when blocklength is 1008.

Figure 5, we show the performance of LDPC coded (4, 4) D-BLAST system and (1, 1) LDPC coded systems on both AWGN and Rayleigh fading channel. The length of codeword of LDPC is 1008 and the maximum iteration number for LDPC decoding is 50. The spectral efficiency of (4, 4) system is four times than that of (1, 1) system and coding gain is about 1.3dB when bit error rate is equal to 10^{-4}. This shows that in diagonally dispersed transmission none of the individual sub-stream is hostage to the worst of the multiple paths

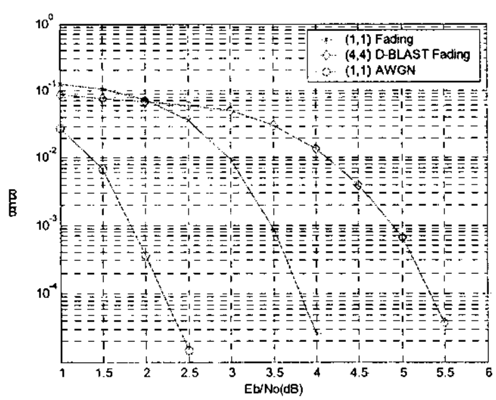

Figure 5. Performance of (6, 3) LDPC code based (4, 4) D-BLAST system over independent fading channel.

V. CONCLUSIONS

We derived the pairwise error probability for LDPC coded D-BLAST using Craig's Q-function and moment generating function. Since LDPC code has a large blocklength, we assume the weight distribution for each antenna is approximately same. This results in simple closed expression for symbol error probability which is a function of weight distribution of LDPC code. Simulation result shows that the performance of proposed scheme on flat fading channel using four transmit and four receive antenna is only 1.5dB away from that of LDPC code on AWGN when bit error rate is equal to 10^{-4}.

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