Multihop Performance of Geographic Random Forwarding (GeRaF) under Lognormal-Shadowed Rician Channels

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***Abstract*— The performance of GeRaF is recently evaluated on Rayleigh fading channels. However, in practical systems, line-of-sight paths might exist between sensor nodes. We evaluate the performance of GeRaF under a more realistic channel model. Specifically, multihop performance of GeRaF is studied under lognormal-shadowed Rician fading channels in this paper. Both analytical and simulation results are presented and yield further insight to the performance of GeRaF.**

I. INTRODUCTION

Recently, research on wireless sensor network (WSN) has attracted enormous interest from the wireless communications community. Motivated by technology advancement in several areas such as micro-electro-mechanical systems (MEMs), tiny sensors and actuators, low-power and low-cost radio transmitters/receivers, WSN is speculated to be a major impact in various new applications e.g., medical, animal observation, environment monitoring, agriculture, product tracking, etc. [1]. In order to reach this target, numerous design challenges need to be addressed at all design layers and even cross-layer optimization is mandatory [2]. The main issue of WSN is its energy efficiency since sensor nodes are energy-limited and the replacement of their batteries is difficult. Therefore, most research in WSN aims to reduce energy consumption or to increase the energy efficiency of the network.

One way to significantly save the energy is to put some sensor nodes in the network to sleep. A node wakes up when there is an event triggering or it can wake up regularly according to the sleeping schedule. When a node is sleeping, it cannot transmit or receive signal. Since communications in WSN is usually done by multihop relaying in which a sensor node has a capability to relay signal for other nodes, if there are too many nodes sleeping, the network will lose its connectivity or data transmission will experience a long delay.

Several multiple access (MAC) and routing schemes have been proposed especially for WSN. For example, SPAN [5] proposed the division of nodes into several sets. As long as one of these sets is active, the network remains connected and the rest nodes can be put to sleep. GAF [6] is similar to SPAN but it divides the coverage area into grids. One of the nodes in each grid is scheduled to wake up in order to provide network connectivity. STEM [7] is a scheme in which a node periodically wakes up. When a node has a packet to transmit, it sends a beacon to a specific node until that node becomes active and transmission can take place. Recently, GeRaF [8], [9] has been introduced as a multiple access/routing scheme that is robust to the node sleeping schemes. Nodes need not to be synchronized and this drastically reduces the cost and complexity of WSN. GeRaF exploits location information for routing a packet to the destination. Other MAC and routing schemes for WSN can be vastly found in the literature.

Previous performance evaluation of GeRaF assumes no fading scenario [8] or Rayleigh fading with path loss [10]. It was concluded that GeRaF is robust to Rayleigh fading channels. The assumption of Rayleigh fading channels means that multipath scattering is rich between any pairs of nodes and therefore signal arriving at a node is mainly diffused with its amplitude being Rayleigh. However, in practice, several sensor nodes might be more likely to have line-of-sight paths. In this case, Rician channel model is a more accurate model.

In this paper, we extend the performance analysis and evaluation of GeRaF in [10] to a Rician fading channel. We also include the effect of lognormal shadowing in the evaluation. Both analytical and simulation results are included and discussed. The contribution of this paper can be considered twofold. On one hand this paper addresses the performance of GeRaF under lognormal-shadowed Rician fading channels. On the other hand, the paper reinforces the importance of the issue on the effect of realistic physical channel to the network layer performance. It has been admitted recently that the deterministic "disk model" usually assumed in order to evaluate the performance of a network protocol in wireless channels is not accurate and might lead to different conclusion when realistic channel model is incorporated [11], [12].

II. SYSTEM MODEL

We are considering a fixed wireless network of nodes that are randomly placed in the area of interest according to a uniform Poisson process. All the nodes share the same frequency resource. A parameter M describes the average number of neighbour nodes per unit circle. Note that M already excludes those sleeping nodes which might occur in the sleeping scheme of an energy-limited WSN. Among these

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This work was supported by Brain Korea 21 Project, The school of information technology, KAIST in 2005.

M nodes, some may be able to serve as a relay and some may not depending on the channel characteristic and location at the time of transmission request.

In this paper, we are considering a Rician fading channel which includes a Rayleigh fading channel as a special case. Suppose y is a Rician r.v. with its pdf

$$p(y) = \begin{cases} 2y(1+K)\exp\left(-K - y^2(1+K)\right) \\ \qquad \cdot I_0\left(2y\sqrt{K(1+K)}\right) \quad ,y \geq 0 \\ 0, \text{otherwise}, \end{cases} \tag{1}$$

where K is a Rician factor which is a ratio of the line-of-sight component to the diffused components, $I_0(.)$ is a zeroth order modified Bessel function. When $K = 0$, y reduces to a Rayleigh r.v. The transmission signal is affected only from small scale Rician/Rayleigh fading and large scale fading by a path loss and possible shadowing. More specifically, the received signal power at distance r from the transmitter encounters an overall gain $\alpha\gamma r^{-\eta}$ where η is a pathloss exponent (typically valued 2-4), $\alpha = y^2$ is a squared Rician r.v. with pdf

$$p(\alpha) = \begin{cases} (1+K)\exp\left(-K - \alpha(1+K)\right) I_0\left(2\sqrt{\alpha K(K+1)}\right) \\ \qquad\qquad ,\alpha \geq 0 \\ 0, \text{otherwise}, \end{cases} \tag{2}$$

and γ represents lognormal shadowing which can be written as $\gamma = 10^{x_0/10}$ where x_0 is a zero mean Gaussian r.v. with a standard deviation σ which is usually referred to as shadowing spread. We assume the network load is light and there is no cochannel interference. We also assume that the Rician fading gain α (as well as γ) changes slowly within a transmission frame and can be treated as a constant during a frame.

A node is able to act as a relay when its received signal power is greater than some chosen threshold. Whenever a node can serve as a relay, the transmission is assumed to be successful. Similar to the approach in [10], the probability of a successful transmission across a distance r is written as

$$\begin{aligned} P_s(r) &= P\left[\alpha\gamma r^{-\eta} > b\right] = P\left[\alpha > b\gamma^{-1} r^{\eta}\right] \\ &= P\left[\alpha > b10^{-x_0/10} r^{\eta}\right] \end{aligned} \tag{3}$$

where b is a chosen threshold. We also normalize all distances to the coverage radius of for the nonfading case [8], then $b = 1$. The probability in (3) can be interpreted as a probability of no outage for a link to a node at distance r away from the transmitter. If that link is not in outage, the node can be a relay and the transmission is successful. Note that for the Rayleigh fading without shadowing case, the probability in (3) is in an exponential form while for the Rician fading case, it has no closed-form solution.

III. MULTIHOP PERFORMANCE OF GERAF

The detailed operation of GeRaF is referred to [8], [9]. Next, we will only briefly explain the main idea. GeRaF assumes that each node knows its own location and its distance to the destination (not necessary the direction). A unique feature of GeRaF is the fact that a relay node is not known in advance. A relay node will be determined once a source node has a packet to send. When a node has a packet to send, two main steps are performed for a successful transmission.

1) The source node sends a request for transmission indicating its address and the intended destination. Each active neighbour node which hears the request determines its own priority in serving as a relay. Its priority is determined from its distance to the destination.
2) The highest priority node informs the source that it will serve as a relay for this transmission and the packet transmission starts while other neighbour nodes can be put to sleep.

In this manner, the packet can be forwarded to the destination within some finite time with high probability if there are enough available neighbour nodes [8]. Note that in Step 2), it is possible that more than one node inform the source about its readiness for relaying. In this case, collision occurs at the source node and some form of collision resolution is needed. And if there is no neighbour nodes available, the source node has to send a request again possibly after some waiting time. In this case, the number of hops is increased by one without packet transmission. The above issues are discussed thoroughly in [9].

In this paper, we assume that GeRaF can always choose the best node (the active neighbour node closest to the destination) as a relay and then there is no collision. In addition, once there is an available relay, we do not take into account the transmissions in Step 1) in the performance evaluation.

Following the approach in [10], the analytical performance can be evaluated as follows. Consider a source node at distance D from the destination. Packet transmission can take place only when there is an available node closer to the destination than the source node. In each transmission (hop), we are interested in the advancement towards the destination. From Fig. 1, the probability of advancement ζ less than a corresponds to the probability of no available relays in the circular area of radius $D - a$ centered at the destination. The considered area can be found by spanning the shaded differential area in Fig. 1 with the radius r from a to $2(D-a) + a = 2D - a$. From the law of cosine, the angle θ can be found as a function of r, D, a as [10]

$$\theta = \arccos\left[\frac{D^2 + r^2 - (D-a)^2}{2rD}\right]. \tag{4}$$

One may find another approach in evaluating the considered area by considering the differential area at distance r and angle θ from the destination (not shown) and spanning the r' from 0 to $D - a$ and θ' from 0 to 2π. However, doing that will result in double integration which is more difficult to evaluate numerically. Next, the average number of available relays in the considered area is [10]

$$M_r = \int_a^{2D-a} M P_s(r) 2\theta r dr \tag{5}$$

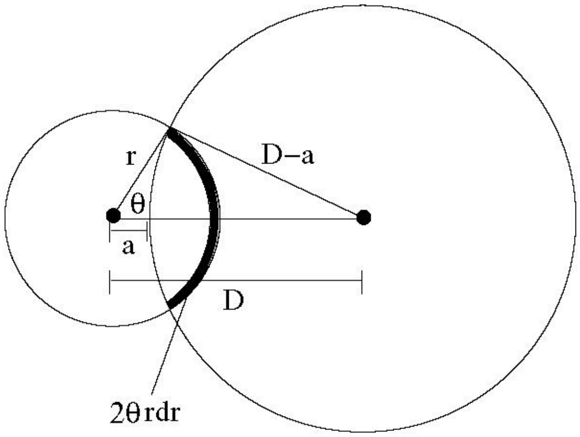

Fig. 1. circle intersection for the analysis

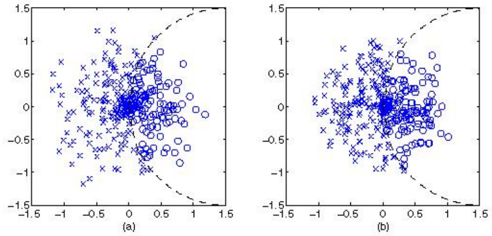

Fig. 2. Available node cloud for the source at (0,0) and destination at (1.5,0). × represents available nodes but not used for relaying, o represents available nodes possibly used for relaying, in a channel with (a) Rayleigh fading (b) Rician fading with $K = 10$. A node inside the dash arc can be used as a relay.

where $P_s(r)$ is the probability of no outage given in (3) which has to be evaluated numerically. Therefore, from (5), we can see that the number of neighbour nodes M have been modulated by the probability of no outage which depends on the distance r.

A. No Shadowing

For the Rician channel without shadowing, we let $\gamma = 1$ in (3). Therefore,

$$\begin{aligned} P_s(r) &= P\left[\alpha > r^\eta\right] \\ &= 1 - \int_0^{r^\eta} (1+K)\exp\left(-K - \alpha(1+K)\right) \\ &\quad \cdot I_0\left(2\sqrt{\alpha K(K+1)}\right) d\alpha. \end{aligned} \tag{6}$$

Let $\beta = (K+1)\alpha$, (6) becomes

$$\begin{aligned} P_s(r) &= 1 - \exp(-K) \int_0^{(K+1)r^\eta} \exp(-\beta) \\ &\quad \cdot I_0\left(2\sqrt{K\beta}\right) d\beta. \end{aligned} \tag{7}$$

Using (7) in (5), we obtain the average number of relays in the considered area for the Rician channel without shadowing.

Fig. 2 shows the available node cloud under a Rayleigh fading channel and a Rician fading channel with $K = 10$. Roughly, in both cases, we can see that the available nodes are more concentrated around the source and are more sparse at further distance away from the source. Also, we can see that the available nodes are more sparse in the Rayleigh channel than in the Rician channel. The results in more available nodes existing at further distance from the source in the Rayleigh channel. This characteristic affects the routing performance which will be explored in Section IV.

B. With Shadowing

For Rician channel with shadowing, we first evaluate the probability of no outage given a fixed x_0 in (3) then we average it over the Gaussian statistics. Therefore,

$$\begin{aligned} P_s(r) &= \int_{x_0} P\left[\alpha > 10^{-x_0/10} r^\eta | x_0\right] p(x_0) dx_0 \\ &= 1 - \exp(-K) \int_{-\infty}^{\infty} \int_0^{(K+1)10^{-x_0/10} r^\eta} \exp(-\beta) \\ &\quad \cdot I_0\left(2\sqrt{K\beta}\right) d\beta \cdot p(x_0) dx_0 \end{aligned} \tag{8}$$

where $p(x_0) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x_0^2}{2\sigma^2}\right)$.

Using (8) in (5), we obtain the average number of relays in the considered area for the Rician channel with shadowing.

Note that the average number of available relays in the considered area depends on a, D and we also need to average it over a to evaluate the performance measure as follows. Since the number of relays in the considered area is a Poisson r.v., the probability that the advancement $\zeta \le a$ is given by [10]

$$P[\zeta \le a] = e^{-Mr}. \tag{9}$$

Therefore, the average advancement (in one hop) towards a destination is found as [10]

$$E[\zeta(D)] = \int_0^D (1 - P[\zeta \le a]) da = D - \int_0^D P[\zeta \le a] da. \tag{10}$$

The performance measure in this paper is the average number of hops to reach the destination at distance D. It can be approximated by $D/E[\zeta(D)]$ and this approximation will be used in the performance evaluation. One can resort to a more precise bound of the average number of hops to reach the destination at distance D by [8]

$$\frac{D-1}{E[\zeta(D)]} + 1 \le \text{average no. of hops} < \frac{D}{E[\zeta(1)]} + 1. \tag{11}$$

When $E[\zeta(D)] \approx 1$, the approximation $D/E[\zeta(D)]$ is close to the lower bound. However, when $E[\zeta(D)] > 1$ (possibly occurring in the fading case), the approximation $D/E[\zeta(D)]$ is even smaller than the lower bound and it may not be a good approximation in this case.

IV. RESULTS AND DISCUSSION

In this section, we plot the performance measure $D/E[\zeta(D)]$ as a function of an average number of neighbour relays in unit circle centered at the source. The Rician factor varies from zero, which corresponds to Rayleigh fading, to ten, which corresponds to a relatively strong line-of-sight channel. We evaluate the multihop performance of GeRaF when $\eta = 4$ in all cases. In the performance measure plots, simulation results are shown as markers and analytical results by numerical integration are shown as solid lines.

A. No Shadowing

Fig. 3 illustrates the probability of no outage as a function of a node distance from the source. If the link is not outage, the node can be served as a relay and transmission is successful in one hop. As the node distance is greater than some value, the probability of no outage starts to decrease. At the node distance less than one, the channel with higher K has greater probability of no outage. That means strong line-of-sight is beneficial for relaying at distance less than one, i.e., Rician channel has better chance for relaying than Rayleigh channel. However, this behavior is reversed when the node distance is greater than one. The probability of no outage in the cases of high K's drop sharply. As a result, Rayleigh channel tends to have better chance for relaying at node distance greater than one. That means Rayleigh channel actually has better chance to reach a node at further distance than the Rician channel does. The above situation is analog to the higher outage probability of Rician channel at low SNR [3]. The high node distance corresponds to the low SNR channel in which the Rayleigh channel has lower outage probability than the Rician channel does.

Figs. 4 and 5 show the average number of hops for $D = 5$ and $D = 10$, respectively. The lowest solid line corresponds to the analytical result when $K = 0$ and the lines above it correspond to $K = 1, 2, 5, 10$, respectively. When $M > 5$, it is clear that Rayleigh channel performs best, i.e., Rayleigh channel has the lowest average number of hops although the differences from other cases are not much. This result follows from the previous discussion that Rayleigh channel has better chance to reach a node at far distance. Since GeRaF always chooses the best node for relaying, a smaller number of hops is needed for the Rayleigh channel than the Rician channel when there is sufficient node density. At a very low node density, e.g., $M = 1$, there are not many (far distance) nodes available. Relaying occurs mostly at short distance in which the strong line-of-sight channel has better advantage. Therefore, the performance under Rician channel with strong line-of-sight is better than those with weak line-of-sight and Rayleigh channels when node density is very low. Note that the average number of hops at high M can be less than D for both Rician and Rayleigh channels since it is possible for the node to reach a next node further from its nominal transmission range. This point has been pointed out clearly in [10] for the Rayleigh channel case.

Note that the analytical results show the same behavior as the simulation results although they seem to underestimate the performance measure. This is because the analytical results are only the approximated ones and they are not truly lower bounds. Nevertheless, the analytical results match closer to the simulation results when D is higher.

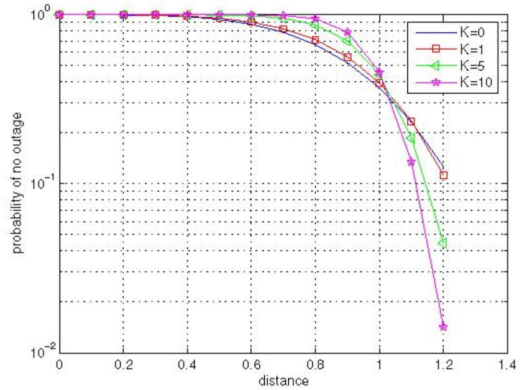

Fig. 3. Probability of no outage vs node distance with various K's

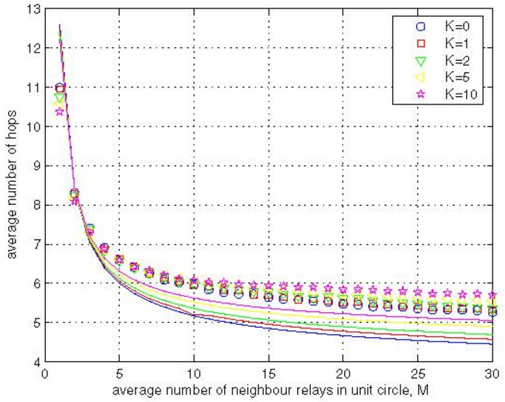

Fig. 4. Average number of hops vs average number of neighbour nodes in unit circle, M, when $D = 5$

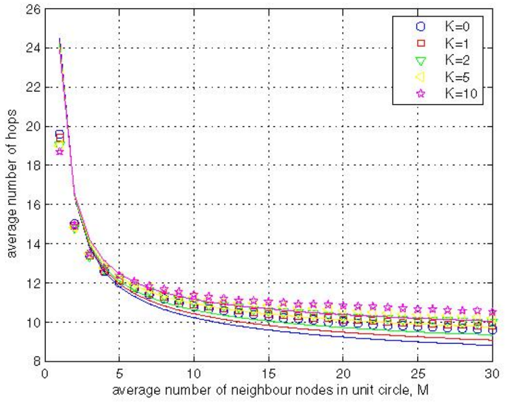

Fig. 5. Average number of hops vs average number of neighbour nodes in unit circle, M, when $D = 10$

B. With Shadowing

Fig. 6 illustrates the probability of no outage as a function of a node distance from the source in a Rican channel with $K = 5$ with/without shadowing effect. Similar discussion to that for Fig. 6 can be applied. For a node distance less than one, a heavy shadowed Rican channel ($\sigma^2 = 100$) has poorer probability of no outage while the situation is reversed when a node distance is greater than one. That means a node in a heavy shadowed Rician channel has better chance to reach a next node at far distance than a node in light shadowed or no shadowing Rician channel. This may be counterintuitive at beginning but it should not be surprising when one realizes that a heavy shadowed Rician channel means a large shadowing spread which would result in the possibility of reaching a node at further distance. This situation is analog to the case when the outage probability is lower for the heavy shadowed Rician channel than that for the light shadowed Rician channel at low SNR [4]. A large distance corresponds to a low SNR as discussed previously.

Fig. 7 shows the average number of hops for $D = 10$ with shadowing spread $\sigma^2 = 10, 100$. We can see clearly that heavy shadowed Rician channel $\sigma^2 = 100$ has lower average number of hops than light shadowed channel $\sigma^2 = 10$. Compared with the no shadowing case in Fig. 5, both shadowing cases have better multihop performance since GeRaF always chooses the best node for relaying. With shadowing effect, the performance difference with different K is negligible. Note that the (approximated) analytical results too much underestimate the performance measure with heavy shadowed channel case. Nevertheless, the analytical results match closely with the simulation results for light shadowed channel case.

Note that these results should not be interpreted that one should put more obstacles in the network in practice in order to have heavy shadowed channel since doing that would in fact attenuate the transmission signal energy. Here we just show that large shadowing spread has more possibility to reach nodes at further distance which results in lower average number of hops to reach the destination.

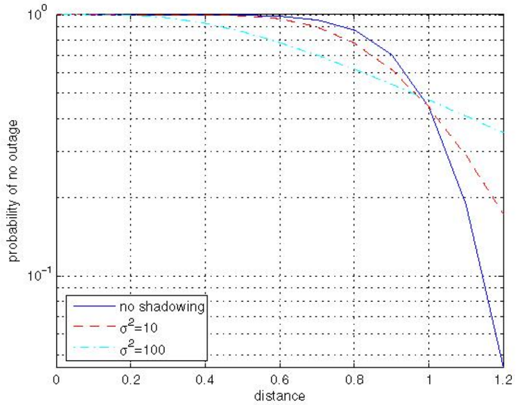

Fig. 6. Probability of no outage vs node distance in Rician channel with $K = 5$ with/without shadowing effect

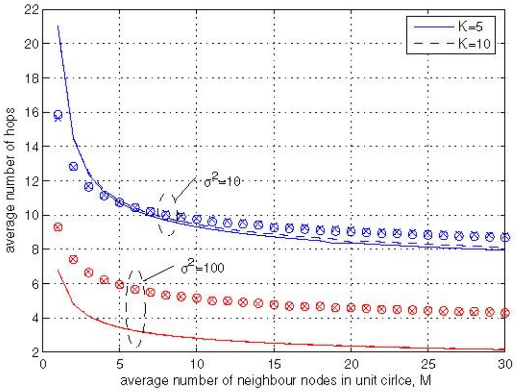

Fig. 7. Average number of hops vs average number of neighbour nodes in unit circle under shodowed Rician channel, M, when $D = 10$. o, × are simulation results for $K = 5, 10$, respectively

V. CONCLUSIONS

We extend the performance evaluation of GeRaF to a more realistic scenario in which a lognormal-shadowed Rician fading channel model is used. It is found that GeRaF has lower average number of hops in a Rayleigh fading channel than in a Rician fading channel. The paper also presented the beneficial effect of shadow spread to the GeRaF performance. Further extension might include cochannel interference and a more practical relay selection in the performance evaluation.

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