Carrier Frequency Offset Compensation for Uplink of OFDM-FDMA Systems

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Abstract— **A frequency offset compensation method for OFDM-FDMA, that can correct offsets *after* the DFT via circular convolution, is proposed as an alternative to the direct method that corrects frequency offsets by multiplying the complex exponents of the offset estimates *before* the DFT. In contrast to the direct method whose complexity increases proportional to the number of users, the computational load of the proposed scheme decreases as the number of users increases. It is shown that the proposed method is simpler to implement than the direct method when the number of users is greater than two. Furthermore, the former can outperform the latter, because a frequency offset compensation for one user after the DFT does not affect the data of other users. Computer simulation results demonstrate the advantage of the proposed method.**

Keywords—**OFDM, OFDM-FDMA, Frequency offset correction, Carrier synchronization**

I. Introduction

Due to the fact that orthogonal frequency division multiplexing (OFDM) is based on using a number of subcarriers, the frequency division multiple access (FDMA) method that assigns clusters of subcarriers to different users has been recognized as a basic multiple access scheme for OFDM [1]–[6]. OFDM using such a FDMA method, herein referred to as OFDM-FDMA, is conceptually simple, however, its implementation needs some caution: in particular, OFDM-FDMA not only requires accurate receiver timing and a carrier frequency for each user, but also requires precise synchronization among all users in order to avoid any intercarrier interference (ICI). Synchronization between users is difficult in the uplink communication (from mobile stations to a base station). One approach to overcoming this difficulty is to use a downlink control channel which conveys the estimated synchronization information to each user [2], [3]. An alternative is to achieve synchronization via signal processing at the uplink receiver without the help of a control channel. This paper will be focused on the latter: in particular, an efficient carrier frequency compensation method at the uplink receiver will be developed.

Carrier frequency synchronization at the uplink receiver is achieved via frequency offset estimation and correction. Estimating the frequency offsets for OFDM-FDMA is rather straightforward: the method in [7], which was developed for a single user OFDM, can be applied, as shown in Section II below. In contrast, the direct correction of a frequency offset by multiplying the complex exponents of the estimated offsets before the discrete Fourier transform (DFT) tends to require heavy computation and may cause some performance degradation. Since the frequency offsets associated with different users are different to each other, an offset correction followed by the DFT should be performed separately for each user. Therefore, the direct frequency compensation for OFDM-FDMA with P users requires P DFT blocks. Performance degradation in the direct method is caused by the fact that a frequency compensation for one user before the DFT can increase the frequency offsets in the data of other users within the DFT block.

In this paper, an alternative signal processing method for frequency offset compensation is developed. This method corrects frequency offsets *after* the DFT using cir-

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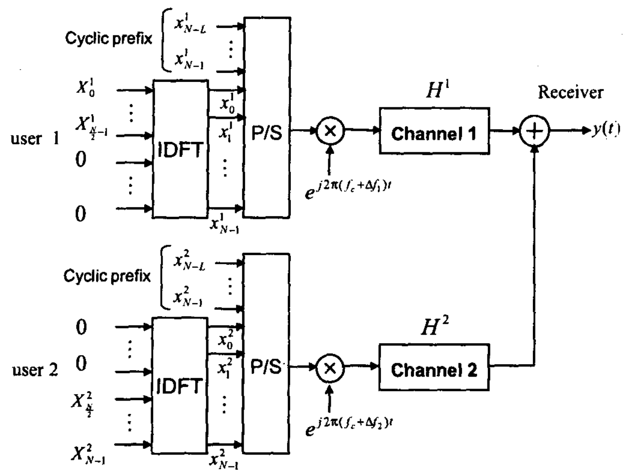

Fig. 1. Transmitters and channels in uplink of OFDM-FDMA with two users.

cular convolution. It is shown that the complexity of the proposed method decreases as the number of users P increases and that it is simpler to implement than the direct method when $P > 2$. Furthermore, the proposed method can outperform direct compensation, because a frequency offset compensation for one user after the DFT does not affect the data of the other users.

Throughout this paper, for the sake of simplicity, the number of subcarriers, denoted by N, and the number of users, P, are assumed to be a power of two. In addition, it is assumed that each user is assigned N/P subcarriers.

II. Frequency Offset Compensation in OFDM-FDMA Systems

In this section, for the sake of simplicity the frequency offset compensation algorithm will be explained for OFDM-FDMA with two users, however, its complexity will be analyzed for P users.

A. Estimation

Fig. 1 illustrates OFDM-FDMA with two users. Half of the inputs to the inverse DFT (IDFT) are set to zero, because each user occupies half of the subcarriers. As shown in 2, a training symbol at the transmitter is shared by the users and is repeated to assist frequency offset estimation. If the received symbols after the N-point DFT

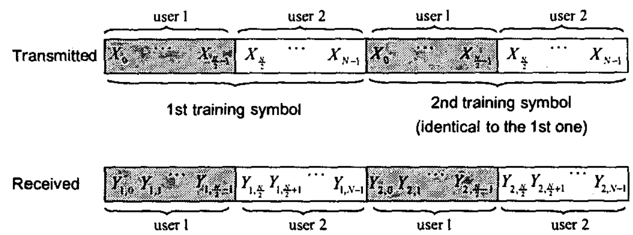

Fig. 2. Training symbols for OFDM-FDMA with two users.

(N subcarriers), corresponding to the 1st and 2nd training symbols are denoted by $\{Y_{1,k}|0 \le k \le N-1\}$ and $\{Y_{2,k}|0 \le k \le N-1\}$, respectively, then the maximum likelihood estimate of each user's frequency offset [7] can be given by

$$\begin{aligned} \hat{\epsilon}_1 &= \frac{1}{2\pi}\tan^{-1}\frac{\sum_{k=0}^{N/2-1}\mathrm{Im}(Y_{2,k}Y_{1,k}^*)}{\sum_{k=0}^{N/2-1}\mathrm{Re}(Y_{2,k}Y_{1,k}^*)}, \\ \hat{\epsilon}_2 &= \frac{1}{2\pi}\tan^{-1}\frac{\sum_{k=N/2}^{N-1}\mathrm{Im}(Y_{2,k}Y_{1,k}^*)}{\sum_{k=N/2}^{N-1}\mathrm{Re}(Y_{2,k}Y_{1,k}^*)}. \end{aligned} \tag{1}$$

where ϵ_1 and ϵ_2 are the frequency offsets normalized by the subcarrier spacing.

B. Correction

The frequency offsets can be corrected before the DFT, as shown in Fig. 3. In this structure, which will be referred to as the direct method, two N-point DFT blocks are employed and $\{e^{-j2\pi\hat{\epsilon}_i n/N}\}$ is pre-multiplied for the frequency offset correction of the received sequence $\{y_n\}$ for user i, $i = 1, 2$. After the DFT for each user, only the data recovered from the subcarriers associated with the user are retained and the rest are discarded. It should be noted that after the pre-multification of $\{e^{-j2\pi\hat{\epsilon}_i n/N}\}$, the frequency offset of the l-th user's data for $l = 1, 2$, becomes $\epsilon_l - \hat{\epsilon}_i$ which is close to zero when $l = i$; however, $|\epsilon_l - \hat{\epsilon}_i|$ may be larger than ϵ_l when $l \neq i$. This fact tends to cause some performance degradation. The number of complex multiplications for this compensator is given by $\frac{N}{2}\{P\log_2\frac{N}{P} + 3(P-1)\}$. Obviously, the complexity increases as the number of users grows.

Accordingly, in an attempt to reduce the computational load, frequency offset correction after the DFT is consid-

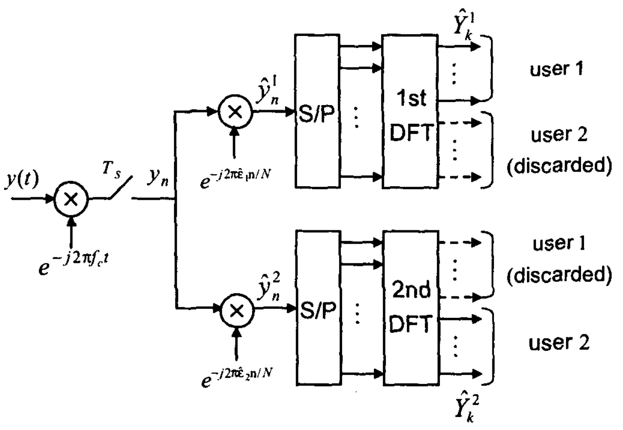

Fig. 3. Frequency offset compensation before DFT (direct method).

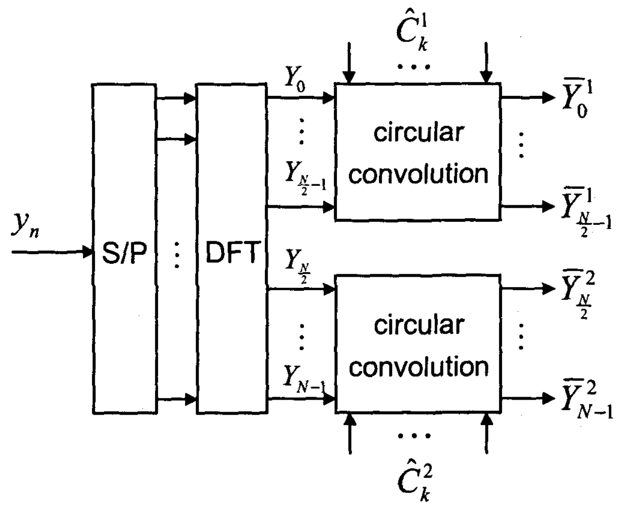

Fig. 4. Frequency offset compensation after DFT (proposed).

ered. Again referring to Fig. 3, if the output of the i-th DFT is denoted by $\{\hat{Y}_k^i, 0 \le k \le N-1\}$, then

$$\begin{aligned}\{\hat{Y}_k^i\} &= DFT_N\{y_n e^{-j2\pi\hat{\epsilon}_i n/N}\} \\ &= Y_k \otimes C_k^i \end{aligned} \quad (2)$$

where $DFT_N\{\cdot\}$ denotes the N-point DFT; $\{Y_k\} = DFT_N\{y_n\}$; $\otimes$ represents the N-point circular convolution; and $C_k^i = DFT_N\{e^{-j2\pi\hat{\epsilon}_i n/N}\}$, which can be rewritten as

$$C_k^i = N \cdot \frac{\sin\{\pi(k+\hat{\epsilon}_i)\}}{N \cdot \sin\{\pi(k+\hat{\epsilon}_i)/N\}} e^{-j\pi(N-1)(k+\hat{\epsilon}_i)/N}, \ \hat{\epsilon}_i \neq 0. \quad (3)$$

The calculation of $\{\hat{Y}_k^i\}$ using (2) is more complex than the calculation in Fig. 3 because a circular convolution generally requires a much heavier computation than the DFT. To reduce the computational load, $\{Y_k, 0 \le k \le N-1\}$ in (2) is replaced with $\{Y_k^1\} \equiv \{Y_0, ..., Y_{\frac{N}{2}-1}, 0, ..., 0\}$ for user 1, $\{Y_k^2\} \equiv \{0, ..., 0, Y_{\frac{N}{2}}, ..., Y_{N-1}\}$ for user 2. The use of $\{Y_k^1\}$ and $\{Y_k^2\}$ instead of $\{Y_k\}$ is justified by the fact that each user only occupies one half of the subcarriers (Fig. 1). Futhermore, $\{C_k^i\}$ can be approximated by

$$\{\hat{C}_k^i\} = \{C_0, ..., C_{\frac{M-1}{2}}, 0, ..., 0, C_{N-\frac{M-1}{2}}, ..., C_{N-1}\}, \quad (4)$$

where M is the number of nonzero elements in $\{\hat{C}_k^i\}$. It is assumed that M is an odd number. The use of $\{\hat{C}_k^i\}$ is justified because the values of $\{C_k^i\}$ in (3) are close to zero unless k is close to zero or $N-1$. Using $\{Y_k^i\}$ and $\{\hat{C}_k^i\}$, Fig. 3 is modified as in the structure in Fig. 4, which is the proposed method. In this method, the frequency offset compensation for one user does not influence the data of the other users. The number of complex multiplications required by the compensator can be given by

$$\begin{cases} f(N) + NM - P\{(\frac{M}{2})^2 - \frac{1}{4}\} & , 1 \le M < \frac{N}{P} \\ f(N) + \frac{N^2}{P} - P\{(\frac{N}{P} - \frac{M}{2})^2 - \frac{1}{4}\} & , \frac{N}{P} \le M < \frac{2N}{P} - 1 \\ f(N) + \frac{N^2}{P} & , \frac{2N}{P} - 1 \le M \le N, \end{cases}$$

where $f(N) = \frac{N}{2}\log_2 N$ represents the multiplications required by the DFT block (in case that the fast Fourier transform is used). It is interesting to note that for a given N the computational complexity of the proposed method decreases as the number of users P increases. This is because the number of zeros in $\{Y_k^i\}$ increases linearly with P.

Table I compares the complexity of the direct method and the proposed method for certain values of N, P, and M. For two users ($P = 2$), the direct method is the simplest, however, it becomes the most complex one when $P = 8$, whereas the complexity of the proposed method decreases as the number of users P increases. The proposed method with $M = 5$ is the simplest when $P \ge 4$.

III. Simulation Results

To compare the performances of the direct and the proposed method, computer simulations were performed for OFDM-FDMA. In the simulation, the following system parameters were assumed: DQPSK signaling with a rate of 25 Mbaud, N=64, cyclic prefix duration of 12, and 1/2 convolutional code with constraint length of 5. The number of users P was 2 and 4. The channel was a frequency selective Rayleigh fading model with an exponentially decaying delay profile. The root mean square delay spread was assumed to be 50 ns, and the maximum mobile speed was 5 km/h. Figs. 5 and 6 show the bit error rate (BER) of the compensators. The frequency offsets were: $\epsilon_1 = 0.10$ and $\epsilon_2 = -0.10$ in Fig. 5, and $\epsilon_1 = 0.10$, $\epsilon_2 = -0.10$, $\epsilon_3 = -0.05$, and $\epsilon_4 = 0.05$ in Fig. 6. As expected, the performance of the OFDM-FDMA system was significantly improved by employing the compensators. The direct and proposed compensators acted in a similar manner when $E_b/N_o < 15$ dB. However, for $E_b/N_o > 15$ dB the proposed method performed better than the direct method. The performances of the proposed method when $M = N$ and $M = 5$, were almost comparable. Therefore, the use of the proposed method is recommended with $M = 5$.

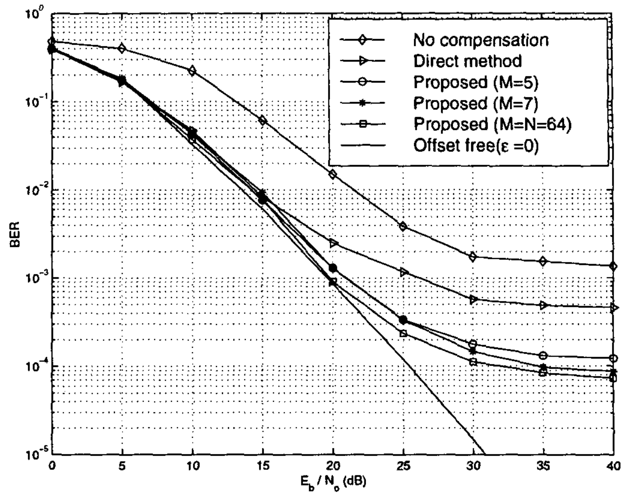

Fig. 5. Comparison of BER performances when N=64 and P=2 ($\epsilon_1 = 0.10$, $\epsilon_2 = -0.10$).

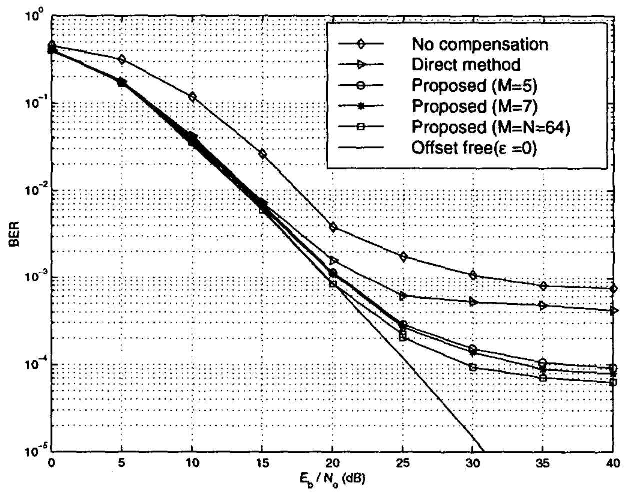

Fig. 6. Comparison of BER performances when N=64 and P=4 ($\epsilon_1 = 0.10$, $\epsilon_2 = -0.10$, $\epsilon_3 = -0.05$, $\epsilon_4 = 0.05$).

IV. Conclusion

A frequency offset compensation method for OFDM-FDMA was proposed. This method corrects frequency offsets after the DFT, and is able to outperform the direct method which compensates offsets before the DFT. It was shown that the computational complexity of the proposed method decreases as the number of users increases, plus the proposed method is simpler to implement than the direct method when the number of users is greater than two.

TABLE I

Number of complex multiplications required for frequency offset compensation.

Type		2 users	4 users	8 users
N=64	Direct method	416	800	1440
	Proposed (M=5)	500	488	464
	Proposed (M=7)	616	592	544
	Proposed (M=N)	2240	1216	704
N=128	Direct method	960	2112	4416
	Proposed (M=5)	1076	1064	1040
	Proposed (M=7)	1320	1296	1248
	Proposed (M=N)	8640	4544	2496

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