A Consideration on Adaptive RAKE Receivers for Single-user MMSE Detection

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Abstract

The constrained MMSE (CMMSE) receiver, which has been developed for flat fading channels [6], is applied to each finger of a RAKE receiver in multipath fading environment. The optimal filter coefficients of the CMMSE receiver in steady state are derived. The results indicate that the performance of CMMSE receivers can be degraded by interpath interference (IPI) when the channel fade rate is slow. It is shown that the channel estimator employed by the CMMSE receiver can be suffered by some bias which is caused by IPI. Nevertheless, simulation results demonstrate that the proposed CMMSE RAKE receiver can perform well in multipath fading channels, and outperforms conventional MMSE RAKE receivers.

1. Introduction

Various adaptive MMSE receivers have been proposed for detection of DS-CDMA systems. For AWGN channels, adaptive MMSE receivers were developed based on the standard MSE cost function [1]-[3]. In the case of flat fading channels, a channel estimator was employed and, in an attempt to improve the tracking capability, the MSE cost was modified [4]-[6]. Furthermore, in [6] a constrained MMSE (CMMSE) receiver in which a constraint regarding filter coefficients was imposed on the MMSE problem has been proposed. For frequency selective fading channels, use of an adaptive filter for each resolvable transmission path has been suggested, and the receivers in [4]- [6] may be applied to each path. In this case, however, the receiver performance is degraded due to interpath interference (IPI).

In this paper, the CMMSE receiver in [6] is applied to each finger of a RAKE receiver in multipath fading environment, and its properties are analyzed. The results indicate that the performance of the CMMSE receiver can be degraded by IPI. The simulation results, however, demonstrate that the performance degradation caused by IPI is rather minor and that the proposed CMMSE RAKE receiver outperforms conventional MMSE RAKE receivers.

2. Signal model

Let us consider the desired user's channel impulse response for a multipath fading channel at time t as

$$c(t) = \sum_{l=1}^{L} c_l(t)\delta(t-\tau_l), \tag{1}$$

where $c_l(t)$ and τ_l are the time-varying fading factor and propagation delay of the l-th multipath, respectively, and $\delta(t)$ represents the Dirac-delta function. For the sake of simplicity, we assume that τ_l is integer multiple of T_c which is the chip duration, and that τ_l is sorted in ascending order. In addition, it is assumed that the fading factors $c_l(t)$ have zero mean and are invariant for a symbol duration; that is, $c_l(t) = c_l(n)$ for $nT \le t < (n+1)T$, where T is the symbol duration. Denote by $r(i)$ the matched filter output at the receiver. The normalized spreading code ($\|\mathbf{s}_k\|^2 = 1$) for the k-th user is denoted by $\mathbf{s}_k = [s_{k,1}\ s_{k,2}\ \cdots\ s_{k,N}]^T$, where $N = T/T_c$ is the processing gain. Without loss of generality, we assume the following: the desired user is the 1st user; the delay of the 1st user's first path $\tau_1 = 0$; adaptive filters are aligned to the first path of the 1st user. The input vector to the adaptive filter is written as

$$\begin{aligned}\mathbf{r}(n) &= [r(nN)\ r(nN+1)\ \cdots\ r(nN+N+M-1)]^T \\ &= \sum_{l=1}^{L} c_l(n) \underbrace{\begin{bmatrix} \mathbf{0}_{\tau_l} \\ \mathbf{s}_1 \\ \mathbf{0}_{M-\tau_l} \end{bmatrix}}_{\triangleq \mathbf{s}_{1l}} d_1(n) + \mathbf{u}(n), \end{aligned}\tag{2}$$

where $\mathbf{r}(n)$ is an $(N+M) \times 1$ vector, M is the maximum delay variation of the desired user's multipaths, $d_1(n)$ is the desired data symbol, $\mathbf{u}(n)$ represents the sum of MAI, ISI

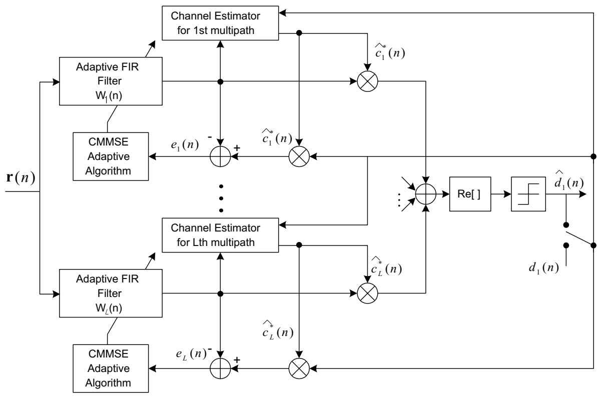

Figure 1. The block diagram of CMMSE RAKE receiver

and background noise; $\mathbf{0}_b$ is a $b \times 1$ vector consisting of all zeros. Let $\mathbf{S} = [\mathbf{s}_{11}\ \mathbf{s}_{12}\ \cdots\ \mathbf{s}_{1L}]$. Then, $\mathbf{r}(n)$ can be rewritten as

$$\mathbf{r}(n) = \mathbf{S}\mathbf{c}(n)d_1(n) + \mathbf{u}(n), \tag{3}$$

where $\mathbf{c}(n) = [c_1(n)\ c_2(n)\ \cdots\ c_L(n)]^T$. For flat fading channels, $\mathbf{r}(n)$ reduces to

$$\mathbf{r}(n) = c_1(n)d_1(n)\mathbf{s}_1(n) + \mathbf{u}(n). \tag{4}$$

3. CMMSE RAKE receiver

Figure. 1 illustrates the CMMSE RAKE receiver. It employs a CMMSE receiver, consisting of an adaptive filter and a channel estimator, for each resolvable path. The length of the adaptive filter is $N + M$, and its output at the l-th path is denoted by $\mathbf{w}_l^H\mathbf{r}(n)$, where $\mathbf{w}_l$ is the tap weight vector. Under the assumption that a pilot symbol is inserted at every Q symbol periods, the channel estimator for the l-th path is given by

$$\hat{c}_l = \frac{1}{N_p}\sum_{i=1}^{N_p} d_1^*(n - iQ)\mathbf{w}_l^H\mathbf{r}(n - iQ) \tag{5}$$

where $d_1(n)$ is the pilot and N_p is a positive integer. The output of the CMMSE RAKE receiver $\hat{d}_1(n)$ is obtained through maximal ratio combining. It is expressed as

$$\hat{d}_1(n) = \arg Re[\bar{d}_1(n)], \tag{6}$$

where

$$\bar{d}_1(n) = \sum_{l=1}^{L} \hat{c}_l{}^*(n)\mathbf{w}_l^H\mathbf{r}(n). \tag{7}$$

In what follows, we analyze the characteristics of the CMMSE RAKE receiver and derive its adaptation rule.

The l-th CMMSE receiver in Figure. 1 is based on the following optimization: the optimal weight $\mathbf{w}_l^o$ is given by

$$\begin{aligned} \mathbf{w}_l^o &= \arg\min_{\mathbf{w}_l} E[\|c_l(n)d_1(n) - \mathbf{w}_l^H\mathbf{r}(n)\|^2] \\ &\quad \text{subject to } \mathbf{w}_l = \mathbf{s}_{1l} + \mathbf{x}_l \text{ and } \mathbf{x}_l \perp \mathbf{s}_{1l} \end{aligned} \tag{8}$$

where the weight $\mathbf{w}_l$ is decomposed into the spreading code vector $\mathbf{s}_{1l}$ and a vector $\mathbf{x}_l$, which are orthogonal to each other. This optimization problem is solved for two types of channels. One is a fast fading channel for which channel parameters for different paths are uncorrelated; that is, $E[c_l(n)c_{l'}(n)] = 0$. The other is a static channel which models slow fading. For this channel, $c_l(n)$ is deterministic, and thus $E[c_l(n)c_{l'}(n)] = c_l(n)c_{l'}(n)$. The optimal weights are derived in the following properties.

Property 1 *When $E[c_l(n)c_{l'}(n)] = 0$, the optimal weight is given by*

$$\mathbf{w}_l^o = \frac{\mathbf{R}^{-1}\mathbf{s}_{1l}}{\mathbf{s}_{1l}^H\mathbf{R}^{-1}\mathbf{s}_{1l}} \tag{9}$$

where $\mathbf{R} = E[\mathbf{r}(n)\mathbf{r}^H(n)]$.

Proof: The optimum weight vector $\mathbf{w}_c$ can be solved by Lagrangian multiplier. The cost function can be rewritten as

$$\begin{aligned} &E[\|c_l d_1(n) - \mathbf{w}_l^H\mathbf{r}(n)\|^2] \\ &= \|c_l\|^2 - E[c_l^* d_1^*(n)\mathbf{w}_l^H\mathbf{r}] - E[c_l d_1(n)\mathbf{r}^H\mathbf{w}_l] \\ &+E[\|\mathbf{w}_l^H\mathbf{r}\|^2] \\ &= E[\|\mathbf{w}_l^H\mathbf{r}\|^2] - \|c_l\|^2 \end{aligned} \tag{10}$$

where the second equality comes from $E[c_l^* d_1^*(n)\mathbf{w}_l^H\mathbf{r}] = E\left[c_l^* d_1^*(n)\mathbf{w}_l^H\left(\sum_{i=1}^{L} c_i\mathbf{s}_{1l}d_1(n) + \mathbf{u}(n)\right)\right] = \|c_l\|^2 = E[c_l d_1(n)\mathbf{r}^H\mathbf{w}_l]$. Using the method of Lagrangian multipliers, the optimization in (8) is written as

$$\begin{aligned} J &= \sum_{k=0}^{N-1}\sum_{i=0}^{N-1} w_{lk}^* w_{li} r_R(i-k) - \|c_l\|^2 \\ &\quad +Re\left[\lambda^*\left(\sum_{k=0}^{N-1} w_{lk}^* s_{1l,k} - 1\right)\right] \end{aligned} \tag{11}$$

where $r_R(i-k) = E[r(n-k)r^*(n-i)]$. The kth element of the gradient of J is given by

$$\nabla_k J = 2\sum_{i=0}^{N-1} w_{li}^* r(i-k) + 2\lambda^* s_{1l,k} \tag{12}$$

By setting $\nabla_k J = 0$, we get

$$\sum_{i=0}^{N-1} w_{li}^{o*} r_R(i-k) = -\lambda^* s_{1l,k} \qquad k = 0, 1, \cdots, N-1 \tag{13}$$

where w_{li}^o is the i-th element of $\mathbf{w}_l^o$. From this equation,

$$\mathbf{w}_l^o = -\lambda \mathbf{R}^{-1}\mathbf{s}_{1l} \quad (14)$$

where $\mathbf{R}$ is an $N \times N$ matrix consisting of $r_R(i)$. If $\mathbf{s}_{1l}$ is post multiplied to the Hermitian transpose of (14), then $\mathbf{w}_l^{oH}\mathbf{s}_{1l} = -\lambda^* \mathbf{s}_{1l}^H \mathbf{R}^{-1}\mathbf{s}_{1l}$. Then, due to the fact that $\mathbf{w}_l^{oH}\mathbf{s}_{1l} = 1$ in (8)

$$\lambda = -\frac{1}{\mathbf{s}_{1l}^H \mathbf{R}^{-1}\mathbf{s}_{1l}} \quad (15)$$

Using (15) in (14), the result in (9) is obtained. ■

The optimal weight, say $\mathbf{w}^o$ for flat fading channels is essentially the same as the weight in (9): $\mathbf{w}^o$ is obtained by replacing $\mathbf{s}_{1l}$ in (9) with $\mathbf{s}_1$. This is because the cost function for flat fading is given by $E[\|\mathbf{w}^H\mathbf{r}\|^2] - \|c_1\|^2$, which is essentially identical to the cost in (10). The identity between $\mathbf{w}_l^o$ in (9) and $\mathbf{w}^o$ for flat fading indicates that the performance of the CMMSE receiver at each finger is *not* degraded by IPI in fast fading environment.

Property 2 *For static (or deterministic) channels, $\mathbf{w}_l^o$ becomes*

$$\mathbf{w}_l^o = \frac{\mathbf{R}^{-1}\mathbf{s}_{1l}}{\mathbf{s}_{1l}^H\mathbf{R}^{-1}\mathbf{s}_{1l}} + \sum_{i \neq l} \frac{c_l^* c_i (\mathbf{s}_{1l}^H\mathbf{R}^{-1}\mathbf{s}_{1l}\mathbf{R}^{-1}\mathbf{s}_{1i} - \mathbf{s}_{1l}^H\mathbf{R}^{-1}\mathbf{s}_{1i}\mathbf{R}^{-1}\mathbf{s}_{1l})}{\mathbf{s}_{1l}^H\mathbf{R}^{-1}\mathbf{s}_{1l}}. \quad (16)$$

Proof: Since $E[c_l(n)c_{l'}(n)] = c_l(n)c_{l'}(n)$,

$$\begin{aligned} & E[\|c_l d_1(n) - \mathbf{w}_l^H\mathbf{r}(n)\|^2] \\ & = \|c_l\|^2 - E[c_l^* d_1^*(n)\mathbf{w}_l^H\mathbf{r}(n)] - E[c_l d_1(n)\mathbf{r}^H(n)\mathbf{w}_l] \\ & + E[\|\mathbf{w}_l^H\mathbf{r}(n)\|^2] \\ & = \mathbf{w}_l^H\mathbf{R}\mathbf{w}_l - \mathbf{w}_l^H\mathbf{P} - \mathbf{P}^H\mathbf{w}_l - \|c_l\|^2 \end{aligned} \quad (17)$$

where $\mathbf{P} = E[c_l^* d_1^*(n)\mathbf{r}(n)]$. $\mathbf{w}_l$ in (16) can be obtained in a straightforward manner using the method of Lagrangian multipliers, as in the proof of Property 1, and thus the rest of the proof is omitted. ■

The 1st term in the right hand side of (16) is the same as (9), and the second term is caused by IPI in slow fading environment. This result indicates that the performance of the CMMSE RAKE can be gegraded by IPI. However, the performance degradation is usually minor – this will be shown in Section 4 through computer simulation. Next the adaptation rule for the CMMSE RAKE receiver is presented.

Property 3 *Tap adaptation rule for the l-th path of CMMSE RAKE receiver is*

$$\begin{aligned} \mathbf{x}_l(n+1) &= \mathbf{x}_l(n) + \mu[c_l d_1(n) - (\mathbf{s}_{1l} + \mathbf{x}_l(n))^H\mathbf{r}(n)]^* \\ & \cdot \left(\mathbf{r}(n) - \mathbf{s}_{1l}^H\mathbf{r}(n)\mathbf{s}_{1l}\right). \end{aligned} \quad (18)$$

Proof: Let $\mathbf{r}(n)$ be decomposed as

$$\mathbf{r}(n) = \mathbf{r}_{sl}(n) + \mathbf{r}_{xl}(n) \quad (19)$$

where $\mathbf{r}_{sl}(n) = \mathbf{s}_{1l}^H(n)\mathbf{r}(n)\mathbf{s}_{1l}$ and $\mathbf{r}_{xl}(n) = \mathbf{r}(n) - \mathbf{s}_{1l}^H(n)\mathbf{r}(n)\mathbf{s}_{1l}$.

Then the cost function can be represented as

$$\begin{aligned} J &= E\left[\|c_l d_1(n) - < \mathbf{r}_{sl}(n) + \mathbf{r}_{xl}(n), \mathbf{s}_{1l} + \mathbf{x}_l(n) > \|^2\right] \\ &= E\left[\|c_l d_1(n) - \mathbf{s}_{1l}^H\mathbf{r}_{sl}(n) - \mathbf{x}_l^H(n)\mathbf{r}_{xl}(n)\|^2\right]. \end{aligned} \quad (20)$$

Its gradient given by

$$\nabla J = -2E[c_l d_1(n) - \mathbf{s}_{1l}^H\mathbf{r}_{sl}(n) - \mathbf{x}_l^H(n)\mathbf{r}_{xl}(n)]^* \cdot \mathbf{r}_{xl}(n), \quad (21)$$

and the LMS algorithm corresponding to the cost in (21) is written as

$$\mathbf{x}_l(n+1) = \mathbf{x}_l(n) + \frac{1}{2}\mu[-\nabla\widehat{J}(n)] \quad (22)$$

where $\nabla\hat{J}(n)$ is the stochastic gradient which is obtained by removing the expectation $E[\cdot]$ in (21). The rule in (18) is derived by using (21) in (22). ■

The adaptation rule for flat fading channels in [6] is essentially the same as the one in (18). The adaptation requires channel information c_l, which is estimated using (5). In the following property, we examine the bias of the channel estimate.

Property 4 *Suppose that the adaptive filter is in the steady state and that $\mathbf{w}_l = \mathbf{w}_l^o$. In multipath fading channels, the estimate in (5) is biased.*

Proof: For fast fading channels ($E[c_l(n)c_{l'}(n)] = 0$), using the result in Property 1, $E[\hat{c}_l]$ is written as

$$\begin{aligned} E[\hat{c}_l] &= E\left[\frac{1}{N_p}\sum_{i=1}^{N_p} d_1^*(n-iQ)\mathbf{w}_l^{oH}\mathbf{r}(n-iQ)\right] \\ &= \frac{1}{N_p}\frac{\mathbf{s}_{1l}^H\mathbf{R}^{-1}}{\mathbf{s}_{1l}^H\mathbf{R}^{-1}\mathbf{s}_{1l}} \cdot \sum_{i=1}^{N_p} E\left[d_1^*(n-iQ)\mathbf{r}(n-iQ)\right] \\ &= \frac{\mathbf{s}_{1l}^H\mathbf{R}^{-1}}{\mathbf{s}_{1l}^H\mathbf{R}^{-1}\mathbf{s}_{1l}} \sum_{i=1}^{L} c_i\mathbf{s}_{1i} \\ &= c_l + \frac{\mathbf{s}_{1l}^H\mathbf{R}^{-1}}{\mathbf{s}_{1l}^H\mathbf{R}^{-1}\mathbf{s}_{1l}} \sum_{i\neq l} c_i\mathbf{s}_{1i} \end{aligned} \quad (23)$$

where the second equality comes from (9) and the third equality is originated from $E[d_1^*(n-iQ)\mathbf{r}(n-iQ)] = \sum_{i=1}^{L} c_i \mathbf{s}_{1i}$. Therefore, channel estimator in (5) is biased in fast fading channels. In static channels, the bias caused by the second term in the right hand side of (16), is added to the bias in (23). ■

In the case of flat fading, the estimate in (5) is *unbiased*, because $E[d_1^*(n)\mathbf{w}^{oH}\mathbf{r}(n)] = c_1$. Property 4 indicates that the channel estimators in the CMMSE RAKE receiver can be suffered by IPI. Developing a channel estimator which is robust to IPI would be useful for improving the receiver performance.

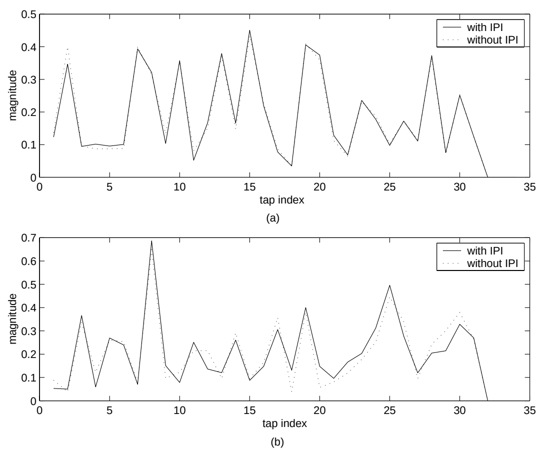

Figure 2. Optimal weights for the CMMSE RAKE receiver. (a) $f_D T = 10^{-2}$, (b) static channel.

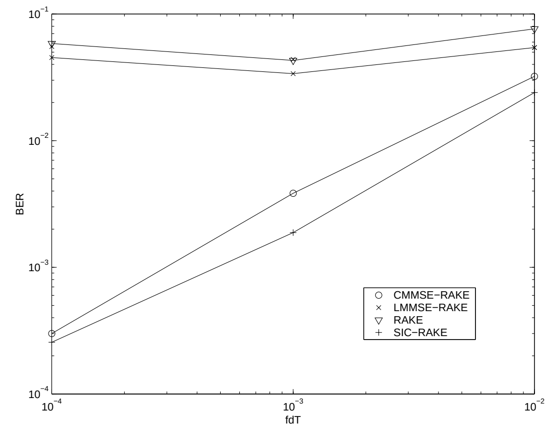

Figure 3. BER performances of adaptive MMSE receivers (K=10, channel estimation)

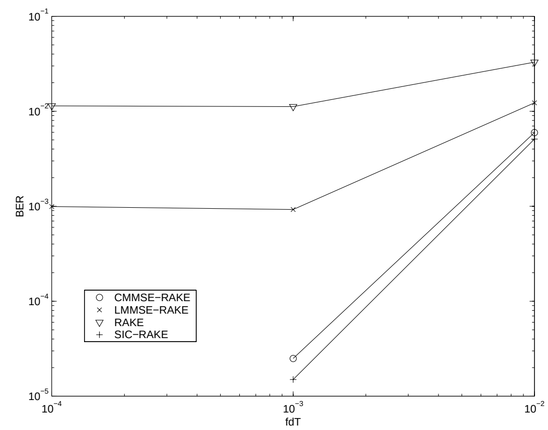

Figure 4. BER performances of adaptive MMSE receivers (K=5, channel estimation)

4. Simulation Results

To examine the influence of IPI on the optimal weights $\mathbf{w}_l^o$ of the CMMSE RAKE receiver, $\{\mathbf{w}_l^o\}$ were obtained through simulation for two types of channels; one was static and the other was a fast fading channels with $f_D T = 10^{-2}$, where $f_D T$ is the maximum normalized Doppler frequency. It was assumed that both channels have 3 paths ($L = 3$); E_b/N_o equals to 20dB, and that the filter length is 34 ($N + M = 34$). The system under consideration was system with BPSK modulation, which was the Gold code of length 31 for spreading ($N = 31$). The transmitted power of all active users were set to be equal and the number of users is 10 (K=10). During transmission, one pilot symbol was inserted for every eight data symbols. Figure 2 shows the optimal weight of the 1st path. For comparison, the optimal weights which were evaluated after *artificially* removing IPI are also shown. It is seen that the optimal weights with/without IPI are close. Therefore, the influence of IPI on filter coefficients was rather minor.

The performance of the CMMSE RAKE receiver was compared with the RAKE receiver employing the MMSE receiver in [5] which will be referred to as the LMMSE RAKE receiver and the conventional RAKE receiver that does not employ any MMSE receivers. The environment for the simulation was identical to the case of optimal weight comparison which is presented above. Figures 3 and 4 show the BER performances corresponding to 10 users and 5 users, respectively. In these figures, the performance bounds are also shown. These bounds are the BER of the CMMSE RAKE receiver which were obtained after *artificially* removing IPI. It is seen that the proposed CMMSE receiver outperformed the others. The LMMSE RAKE receiver behaved better than the conventional RAKE receiver, but its performance was considerably worse than that of the proposed. The performance of the CMMSE RAKE receiver was close to the bound. This indicates that the performance degradation due to IPI was rather minor.

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