Efficient Modified Fano Detection with Reduced Branches for DSTTD System

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***Abstract*— A sub-optimal but computationally efficient modified Fano detection (MFD) algorithm for DSTTD system is presented. The proposed algorithm utilizes the sequential detection scheme based on tree searching in order to find the optimal symbol sequence. For more reliable signal detection and complexity reduction, the decoder is designed to move backward for the specified value at the end of the tree and to compute the reduced branch metrics. Simulation results show that the performance of MFD is comparable to that of ML detector while reducing the computational effort of ML method significantly.**

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) techniques are known to be efficient to achieve the capacity increase of wireless channel over rich scattering environment [1]. The multiple antenna systems can be used either for diversity gain or for spatial multiplexing gain. For spatial multiplexing, the V-BLAST (Vertical Bell Labs Layered Space-Time) architecture was proposed in [2]. On the other hand, Space-Time Transmit Diversity (STTD) [3] as well as Alamouti's Space-Time Block Codes (STBC) [4] was introduced to increase transmit diversity gain. To exploit these two advantages at the same time, Double Space-Time Transmit Diversity (DSTTD) with four transmit antennas was suggested in [5]. In this system, two STTD encoders are used at the transmitter and interference cancellation based detector is employed at the receiver.

DSTTD system can be considered as a special case of the multiuser scenario introduced in [7] (e.g., two synchronous co-channel terminals where each user uses Alamouti's space-time block codes and communicates with the same base station equipped with two antennas for reception). Thus, the received signals in DSTTD system can be detected using a combined interference cancellation and ML decoding scheme (ICML). Though ICML detection scheme provides reasonable performance, the performance may not be sufficient enough when higher-order modulation like 16QAM or higher is used. The Maximum Likelihood detector for MIMO systems is known to be optimal with perfect Channel State Information (CSI) at the receiver. However, when a large number of antennas are used together with higher modulation, ML detection is no longer feasible for real-time implementations. Inspired by this problem, many approaches related to sphere detection (SD), one of very actively studied areas, have been proposed in [9]-[13] to avoid the exponential complexity of ML detector after the concept of SD was presented in [8]. It provides the exact ML performance but it still has a crucial issue largely influencing the complexity of any SD in compliance with the selection of the search radius of the sphere.

In this paper, we propose a sub-optimal but computationally efficient modified Fano detection algorithm for DSTTD system. The proposed algorithm yields the performance close to that of ML with significantly less complexity. Sequential detection scheme applied to MIMO system may not find the optimal symbol sequence due to the insufficient depth of tree. Thus, the proposed MFD is designed to move backward for the specified value at the end of the tree for reliable signal detection and to compute the reduced branch metrics for complexity reduction.

This paper is organized as follows. In Section II, the system model is described. Proposed DSTTD detection scheme using modified Fano algorithm with reduced branches is introduced in Section III. We present the simulation results in terms of the average BER performance and computational complexity in Section IV and finally make conclusions in Section V.

Notation: Throughout this paper, bold symbols denote matrices or vectors. $(.)^T$, $(.)^H$, $(.)^*$, $\mathbf{I}_{n_t}$, and $\mathbf{0}_{n_t,1}$ denote matrix transpose, Hermitian, conjugate, $n_t \times n_t$ identity matrix, and $n_t \times 1$ vector with all zero elements, respectively. L_i denotes the average number of real operations at the i-th tree level and n_r represents $2\bar{n}_r$.

II. SYSTEM DESCRIPTION

We consider DSTTD system with four transmit ($n_t = 4$) and two receive antennas ($\bar{n}_r = 2$), in general the receiver can have $\bar{n}_r \geq 2$ receive antennas, as shown in Fig. 1. At

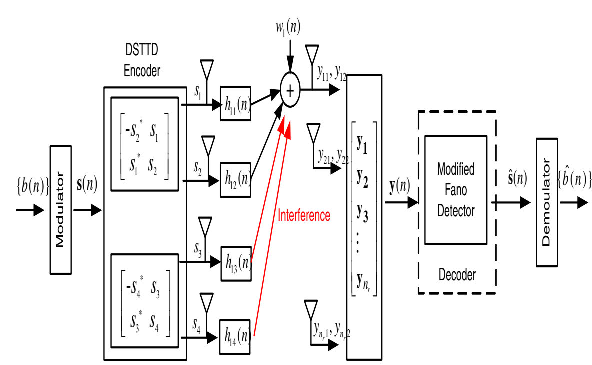

Fig. 1. System model for DSTTD system with $(4, n_r)$ transceiver

the transmitter, input data stream $\{b(n)\}$ at time index n is demultiplexed into four data substreams. These substreams are mapped into $M-$PSK or $M-$QAM symbols $\mathbf{s}(n)$ and divided into groups of two symbols each (i.e., $\{s_1, s_2\}$ and $\{s_3, s_4\}$). During two consecutive symbols periods, space-time block coded symbols are transmitted over the four antennas at the same time respectively.

We assume ideal timing and symbol-synchronous receiver sampling, thus omit the time index for convenience. Let h_{ij} denote the fading channel property between the i-th receive and j-th transmit antennas, $i = 1, 2, j = 1, 2, 3, 4$. At the receiver, the received signals over two consecutive symbols periods at the first receive antenna, y_{11} and y_{12} can be written as

$$y_{11} = h_{11}s_1 + h_{12}s_2 + h_{13}s_3 + h_{14}s_4 + w_1 \tag{1}$$

$$y_{12} = -h_{11}s_2^* + h_{12}s_1^* - h_{13}s_4^* + h_{14}s_3^* + w_2. \tag{2}$$

Similarly, the received signals at the second receive antenna y_{21} and y_{22} can be written as

$$y_{21} = h_{21}s_1 + h_{22}s_2 + h_{23}s_3 + h_{24}s_4 + w_3 \tag{3}$$

$$y_{22} = -h_{21}s_2^* + h_{22}s_1^* - h_{23}s_4^* + h_{24}s_3^* + w_4. \tag{4}$$

Then, we rewrite from Eq. (1) to Eq. (4) in a matrix form as

$$\begin{bmatrix} y_{11} \\ y_{12}^* \\ y_{21} \\ y_{22}^* \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{12}^* & -h_{11}^* & h_{14}^* & -h_{13}^* \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{22}^* & -h_{21}^* & h_{24}^* & -h_{23}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2^* \\ w_3 \\ w_4^* \end{bmatrix} \tag{5}$$

or equivalently

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w} \tag{6}$$

where the vector $\mathbf{w}$ is a complex Gaussian random vector with zero mean and covariance $\sigma_w \mathbf{I}_{2\bar{n}_r}$ while the average transmit power of each antenna is normalized to one and the channel matrix $\mathbf{H}$ contains uncorrelated complex Gaussian fading gains with unit variance and varies independently from frame to frame. We assume that the independent fading gains are perfectly known by the receiver.

To diminish the effect of noise enhancement and error propagation simultaneously, we introduce MMSE-SQRD of the extended channel matrix

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_w \mathbf{I}_{n_t} \end{bmatrix} = \underline{\mathbf{Q}}\mathbf{R} = \begin{bmatrix} \mathbf{Q} \\ \mathbf{Q}_1 \end{bmatrix} \mathbf{R} \tag{7}$$

where $\underline{\mathbf{Q}}$ is the $(2\bar{n}_r + n_t) \times n_t$ matrix with orthonormal columns and $\mathbf{R}$ is the $n_t \times n_t$ upper triangular matrix. Multiplying the extended received signal $\underline{\mathbf{y}}$ with $\underline{\mathbf{Q}}^H$, the modified received signal vector can be represented as

$$\widetilde{\mathbf{y}} = \underline{\mathbf{Q}}^H \underline{\mathbf{y}} = \mathbf{R}\mathbf{s} + \mathbf{w} \tag{8}$$

where $\underline{\mathbf{y}} = [\mathbf{y}\ \mathbf{0}_{n_t,1}]^T$ and $\mathbf{w} = \mathbf{Q}^H \mathbf{n}$, the statistical properties of which remain unchanged due to the $2\bar{n}_r \times n_t$ unitary matrix $\mathbf{Q}$. Since $\mathbf{R}$ has the upper triangular structure, the k-th component of $\widetilde{\mathbf{y}}$ is given by

$$\widetilde{y}_k = r_{k,k}s_k + \sum_{i=k+1}^{n_t} r_{k,i}s_i + w_k \tag{9}$$

where $r_{k,i}$ represents the (k, i) element of $\mathbf{R}$.

III. PROPOSED MFD DETECTION ALGORITHM

This section develops the computationally efficient modified Fano detection with reduced branches for DSTTD system. The Fano algorithm [14], [15] was originally introduced as a sequential decoding scheme for convolutional codes to avoid the exponential complexity of a Viterbi algorithm (VA) caused by large constraint lengths. It achieves asymptotically the same error probability as ML decoding without searching all possible states. The decoder is based on generating hypotheses about the transmitted codeword sequence and computing a metric between these hypotheses and the received signal. It goes forward or backward repeatedly in tree searching, so that it finds the most likely path. In the following subsections, we present the details of the proposed detector.

A. Fano-Like Metric Bias

In order to employ the Fano algorithm in MIMO systems, it is necessary to define a metric on symbol based operation taking the lengths of the different paths compared. This metric throughout this paper is called a Fano-like metric bias [9]. Like the Fano metric for the different path length in sequential sequence detection, it provides a measure that compares the unequal length symbol sequences. Hence, the branch metrics in higher levels have a larger bias than those in lower levels. It reflects that they are much closer to the end of the tree and thus more likely to be the part of the best path. The best path implies the one with the smallest Fano-like metric.

From Eq. (6), the average value of the smallest path is given by

$$E\{\|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2\} = E\{\|\mathbf{w}\|^2\} = 2\bar{n}_r\sigma_w^2. \tag{10}$$

Therefore, it is reasonable to choose $\alpha\sigma_w^2$ as the Fano-like metric bias for computing the branch metric at each tree level, where $0 \leqslant \alpha \leqslant 1$. The value of α is simply set to be 0.5 in our simulations, though it can be adjusted empirically by simulations. Moreover, the squared distance, shown in Eq. (9), is proportional to $r_{k,k}$ for the level k. The Fano-like metric bias for the tree level k can be thus defined as follows

$$F_k = F_{k-1} + \alpha\sigma_w^2 r_{k-1,k-1} \ \ (k = 2, \cdots, n_t) \tag{11}$$

where F_1 is defined as zero. Using the Fano-like metric bias in Eq. (11), the i-th biased branch metric at the tree level k is computed as

$$B_{ik} = |\tilde{s}_k - c_i|^2 + F_k \ (k = 1, \cdots, n_t, i = 1, \cdots, M) \tag{12}$$

where $|\cdot|$ denotes the Euclidean distance and c_i represents the i-th element of the signal constellation.

B. Modified Fano Algorithm Description

Now we consider a M-ary tree structure for the modified Fano detection with reduced branches for DSTTD system. Fig. 2 illustrates a simple binary tree structure as an example. To improve the decoding performance, we introduce two parameters, N_{bm} and N_{rnmm}, which indicate the number of backward movements at the level $n_t + 1$ and the repeated number of same accumulated minimum metrics at the level n_t, respectively. The decoding process of the modified version begins at the root node and terminates if a parameter N_{rnmm} reaches to the specified value during the pre-determined value N_{bm} while the original Fano decoding process terminates only when the decoder reaches at the level n_t of the tree which corresponds to the number of the transmit antennas.

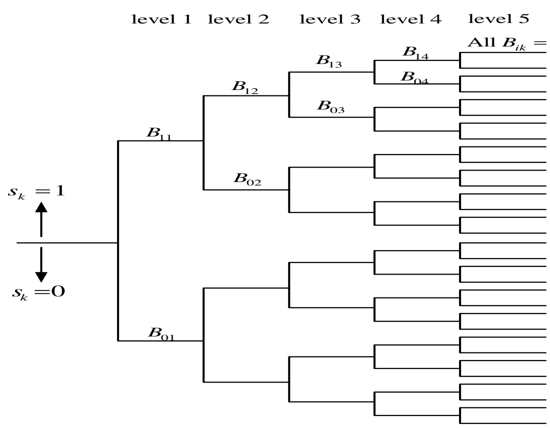

Fig. 2. A binary tree structure for signal detection when $n_t = 4$

From the root node with a threshold $T = 0$ and a metric value $MV = 0$, the decoder looks forward to find the best of the only N succeeding nodes at each level, where N denotes the number of reduced branches. The best node is determined by the biased cumulative path metric which is based on the Euclidean distance as defined in Eq. (12). Once the decoder moves forward and looks forward, it computes the M branch metrics at each level. Then the desired number of branches N is chosen in the order with the smallest branch metric to reduce the computational complexity in advance. It is different from the QRD-M algorithm applied in [6], where the QRD-M algorithm chooses the M branches of all M^k branches. On the other hand, MFD chooses the N branches among the M branches. Especially we assume that all the branch metrics are zeros if the decoder reaches at the level $n_t + 1$.

The flowchart of the proposed modified Fano algorithm for DSTTD system is shown in Fig. 3. Let M_F and M_B represent the metrics of the forward and backward node being examined, respectively [15]. If $M_F \leq T$, the decoder moves forward to the best of the N succeeding nodes. Furthermore, if this node is visited for the first time and the parameter N_{rnmm} does not reach to the specified value, a threshold tightening is performed. On the other hand, if $M_F > T$, the decoder looks backward to its ancestor node at the tree level $k - 1$ and checks whether M_B is less than or equal to T. If it is

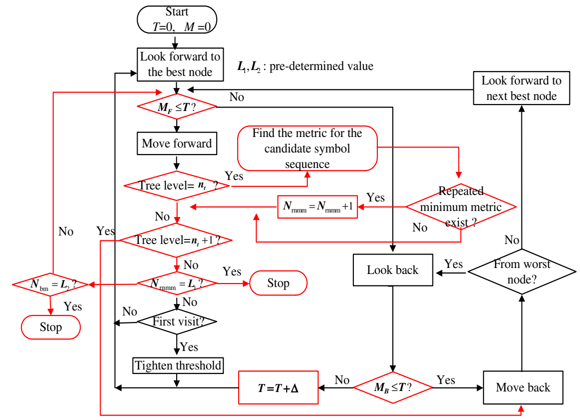

Fig. 3. Flowchart of the modified Fano algorithm: red line - modification

greater than T, then T is increased by $\triangle$ and the look forward to the best node step is repeated. If $M_B \leq T$, the decoder moves back to the preceding node. For convenience, we call this node P. If this backward movement was from the worst of the N nodes succeeding node P, the decoder again looks back to the ancestor node. If not, the decoder looks forward to the next best of the N nodes succeeding node P and checks if $M_F \leq T$. Especially, in case that the decoder ever looks backward from the origin node, we assume that the metric value of the preceding node is ∞.

Through this decoding process, if the decoder reaches to the tree level $k = n_t$, it computes the following metric for the limited N_{bm} (i.e., 200 or 400 in our case)

$$J(\mathbf{s}) = \|\widetilde{\mathbf{y}} - \mathbf{R}\mathbf{s}\|^2. \tag{13}$$

Among the metrics computed, the metric with same minimum value would be repeated. It means that MFD finds the same symbol sequences. Finally, the decoder finds the optimal symbol sequence which has the minimum metric calculated in Eq. (13) among the candidate symbol sequences.

IV. SIMULATION RESULTS

We investigate the average bit error (BER) performance of our proposed algorithm with reduced branches $N = 6$ for DSTTD system with 4 transmit and 2 receive antennas employing uncoded 16QAM modulation in Rayleigh fading channels. We consider two cases of N_{bm} =200, N_{rnmm}=2 and N_{bm} =400, N_{rnmm}=4 to obtain the reasonable BER performance and the corresponding complexity. These parameters can be adjusted depending on the desired performance and complexity. The average bit energy to noise power ratio (E_b/N_o) is defined as the SNR at the receiver normalized by the number of bits per symbol, thus $E_b/N_o = n_t/(\log_2(M)\sigma_w^2)$ is used.

The proposed DSTTD detector is compared with ML and Interference Cancellation and Maximum Likelihood (ICML [7]) detectors using the Monte Carlo simulations with respect to the average BER performance and the corresponding computational complexity. We evaluate the computational efforts

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of each algorithm by counting the real operations including multiplication, addition, and division. We also consider all the steps required to detect the transmitted signal at the receiver for each algorithm respectively. For this evaluation, each complex operation is converted into real operation to get a clear idea of complexity. For instance, one complex multiplication is equivalent to three real multiplications and five real additions.

In Fig. 4, we observe that the BER performance of proposed algorithm approaches that of ML detection as the repeated number of minimum metric (N_{rnmm}) increases. However, MFD with $N_{\text{rnmm}} = 4$ is still inferior to ML by approximate 0.5dB, whereas MFD outperforms ICML by 3dB at the target BER of 10^{-3} even in the case of $N_{\text{rnmm}} = 2$.

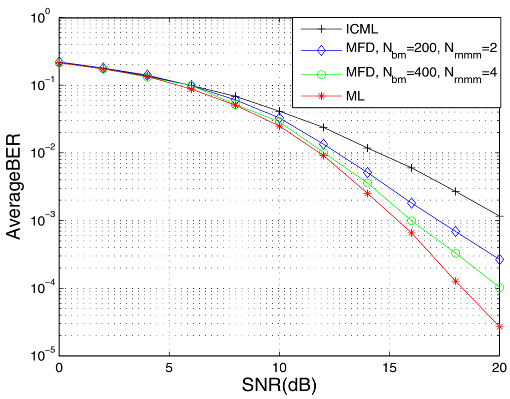

Fig. 4. Performance comparison between MFD and other schemes with 4 transmit and 2 receive antennas

TABLE I presents the complexity evaluation of the proposed algorithm with ML and ICML. Moreover, the average computational complexity for MFD and the DSTTD detectors is plotted in Fig. 5 to show the tendency of computational efforts of our scheme according to various N_{rnmm} values together with other schemes. Even though the SNR at the receiver is 0dB and the parameter N_{rnmm} is 4, the computational complexity of the proposed MFD for DSTTD system is about 1.9×10^3 times less than that of ML detector achieving near ML performance. Furthermore, the computational gap becomes even more increased about 1.0×10^4 times at higher SNR region. From this result, it is seen that the computational efforts of MFD is much less than those of ML, but MFD still has more complexity than ICML. This is because MFD finds the same candidate symbol sequences and increases its complexity.

TABLE I

COMPLEXITY EVALUATION OF EACH ALGORITHM [16]

ML	$(30n_t^2 n_r + 8n_t^2 + 6n_r n_t - 3n_t)M^{n_t}$
ICML	$\frac{25}{3}n_r^3 + 20n_r^2 n_t + 9n_r^2 + 8n_t n_r - \frac{17}{3}n_r - 2n_t + 5Mn_t$
MFD	$\frac{5}{2}n_t^3 + 9n_t^2 n_r + \frac{7}{2}n_t^2 + 7n_t n_r - 3n_t + \sum_{i=1}^{n_t} L_i$

From the simulation results, we observe the trade-off between the BER performance and the decoder complexity. For more reliable signal detection, we introduce two parameters,

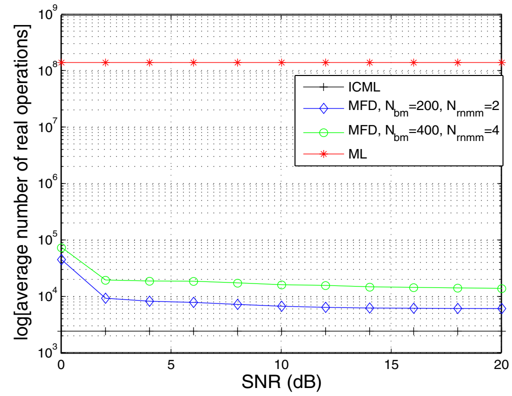

Fig. 5. Comparison of computational complexity between MFD and other schemes with 4 transmit and 2 receive antennas

N_{bm} and N_{rnmm}. As N_{bm} and N_{rnmm} increase, the BER performance of MFD is closer to that of ML. However, the improvement is achieved at the cost of the increasing computational efforts. Nevertheless, from Fig. 5, it is clear that the proposed MFD has significantly less computational complexity compared to the ML detection with highly favorable performance.

V. CONCLUSIONS

We presented a sub-optimal, reduced complexity MFD algorithm well-suited to DSTTD system. The proposed MFD is based on sequential sequence detection scheme using tree searching with backward movement. From the simulation results, although there is a tradeoff between performance and complexity, the performance of MFD detection algorithm is shown to be quite comparable to that of ML with a fraction of complexity. Hence, in many cases of interest, the proposed algorithm can be a good candidate detection scheme for DSTTD system. We also expect that our scheme can be applied to other types of MIMO schemes in high traffic demanding environments.

VI. ACKNOWLEDGEMENT

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