EVALUATION FOR VARIOUS RESOURCE ALLOCATION METHODS FOR MULTIUSER-MIMO OFDMA SYSTEMS

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ABSTRACT

A simple algorithm to select user-pair and allocate frequency band is proposed for multiuser-MIMO OFDMA systems. For sake of comparison, various resource allocation algorithms, such as max-throughput, best-fit, first-fit, and random-fit algorithm, are examined with regard to the system throughput, computational complexity, and fairness among the users. Computer simulation results show that the first-fit algorithm can achieve a large reduction of computation for resource allocation as well as good fairness among users, with only a small reduction of throughput.

I. INTRODUCTION

Multiuser multiple-input multiple-output (MU-MIMO) techniques have been studied to increase the sum achievable rates in wireless communication. Among the existing MU-MIMO methods, linear processing techniques [1]–[4], such as zero-forcing (ZF) based joint-channel diagonalization (JCD) [1], [2] and minimum mean square error (MMSE)-based methods [3], [4], require simpler transceivers than those of nonliner processing. The ZF-based methods, which perfectly cancel inter-user-interferences, can obtain more throughput than MMSE-based methods when signal-to-noise ratio (SNR) is high. However, when SNR is low, MMSE-based methods that consider the noise level of the receiver can obtain more throughput than ZF-based methods [4]. This trade-off with respect to the SNR makes the operation of MU-MIMO systems complex. To avoid this trade-off problem a modified MMSE-based linear processing method has been proposed, which has merits of both the ZF- and MMSE-based methods [5]. To increase data rate for multimedia communications of next generation wireless communication devices, broadband communication techniques, such as orthogonal frequency division multiple access (OFDMA), have been integrated into the MU-MIMO method. As demonstrated by international standards, such as IEEE802.16 Air Interface for Fixed and Mobile Broadband Wireless Access Systems [6], IEEE802.20 [7], and Wireless World Initiative New Radio (WINNER) [8], it is inevitable that OFDMA and MIMO techniques will be combined.

In this paper, we examine the application of MU-MIMO OFDMA systems. For MU-MIMO, a simple ZF-based technique is employed and various resource allocation algorithms, such as max-throughput, best-fit, first-fit, and random-fit algorithms, are compared with respect to the system throughput, computational complexity, and fairness among the users. In this study, it is assumed that the supportable number of users in one frequency band is limited to two. Simulation results show that the first-fit algorithm can reduce computational complexity by 83.7% and can obtain 98.9% fairness among users with a sacrifice of an 7.5% rate reduction compared with the max-throughput algorithm.

II. MULTIUSER MIMO SYSTEM MODEL

The system configuration of a MU-MIMO downlink with T users, N_T transmit antennas, and N_R receive antennas, is shown in Fig. 1. According to the MU-MIMO techniques, up to N_T different users can be supported simultaneously by using the same frequency band. Then, the number of supportable users K is limited by $N_T B$, where B is the number of subbands for MU-MIMO. The MIMO channel for the jth user and bth subchannel band ($b \in \{1, \ldots, B\}$) is represented as a matrix $\mathbf{H}_j(b) \in \mathbb{C}^{N_R \times N_T}$, where the (m, n)th entry represents the complex gain from the nth transmit antenna to the mth receive antenna and is independent, identically distributed (i.i.d.) zero mean complex Gaussian random variables with a unit variance. It is assumed that the MIMO channel is static in the same frequency band, such as an adaptive modulation and coding (AMC) zone in IEEE802.16e [6] systems, and all $\{\mathbf{H}_j(b)\}$, $\forall j$ and $\forall b$, are known at the transmitter, while the jth receiver only knows the jth MIMO channel $\mathbf{H}_j(b)$, $\forall b$. Also, it is assumed that $\mathrm{E}[\mathbf{x}(k)\mathbf{x}(k)^H] = \mathbf{I}_{L_k}$, where $\mathbf{x}(k)$, $\mathrm{E}[\cdot]$ and $\mathbf{I}_{L_k}$ denote the transmitted symbol vector, the expectation, and a L_k-dimensional identity matrix. The spatial multiplexing is performed by forming a vector signal $\mathbf{x}(k)$ with L_k symbols, $\mathbf{x}(k) \in \mathbb{C}^{L_k \times 1}$, preprocessing each vector, and transmitting the elements of the resulting vector $\mathbf{T}_k(b)\mathbf{x}(k)$ through different antennas in the bth subchannel band, where $\mathbf{T}_k(b) \in \mathbb{C}^{N_T \times L_k}$ is the transmit processing matrix given by $\mathbf{T}_k(b) = \mathbf{W}_k(b)\bar{\mathbf{V}}_k(b)\mathbf{E}_k(b)$. Here, $\mathbf{W}_k(b) \in \mathbb{C}^{N_T \times (N_T - \sum_{j=1, j\neq k} L_j)}$ is the matrix for the inter-user-interference (IUI) suppression, $\bar{\mathbf{V}}_k(b) \in \mathbb{C}^{(N_T - \sum_{j=1, j\neq k} L_j) \times L_k}$ is the preprocessor for diagonalizing the kth user's MIMO channel $\mathbf{H}_k(b)$, and $\mathbf{E}_k(b) \in \mathbb{C}^{L_k \times L_k}$ is the diagonal matrix for power control. The preprocessed vectors for each user are combined to yield $\sum_{k=1}^{K} \mathbf{T}_k(b)\mathbf{x}_k(b)$ and then transmitted. At the kth receiver using the bth subchannel band, the received signal $\mathbf{H}_k(b) \sum_{j=1}^{K} \mathbf{T}_j(b)\mathbf{x}_j(b) + \mathbf{n}_k(b)$, where $\mathbf{n}_k(b) \in \mathbb{C}^{N_R \times 1}$ is a noise vector whose elements are i.i.d. with a zero mean and

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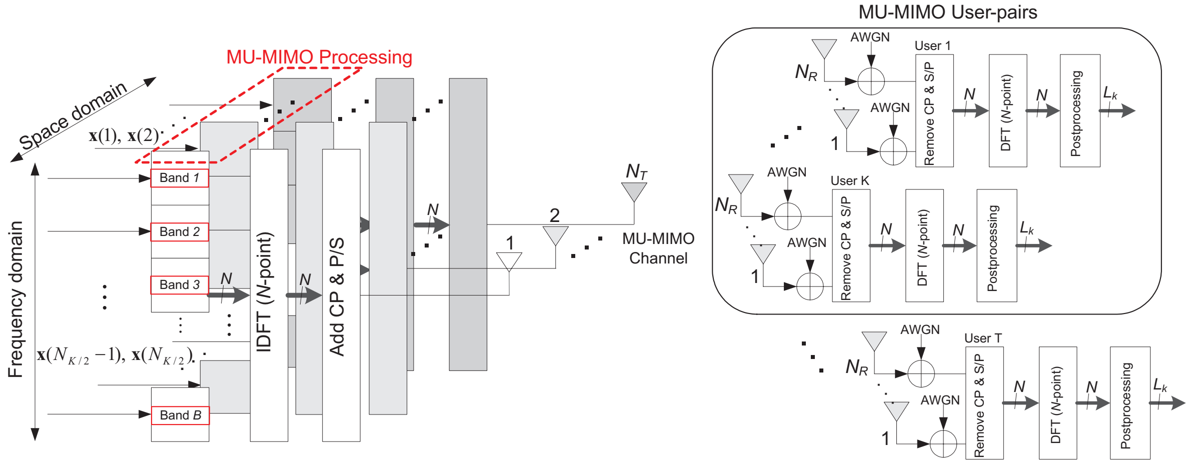


Figure 1: System configuration for multiuser MIMO-OFDMA downlink with K users among total T users.

variance σ^2, is postprocessed by $\mathbf{R}_k(b)^H \in \mathbb{C}^{L_k \times N_R}$ to yield

$$\tilde{\mathbf{x}}_k(b) = \mathbf{R}_k(b)^H \mathbf{H}_k(b) \sum_{j=1}^{K} \mathbf{T}_j(b)\mathbf{x}_j(b) + \mathbf{R}_k(b)^H \mathbf{n}_k(b) \quad (1)$$

where the superscript H denotes the Hermitian transpose. The receive processing matrix $\mathbf{R}_k(b)^H$ is given by $\bar{\mathbf{U}}_k^H(b)\mathbf{U}_k^H(b)$ where $\mathbf{U}_k^H(b) \in \mathbb{C}^{L_k \times N_R}$ is the matrix combining N_R received streams and $\bar{\mathbf{U}}_k^H(b) \in \mathbb{C}^{L_k \times L_k}$ is the postprocessor for diagonalizing the kth user's MIMO channel $\mathbf{H}_k(b)$. When BD followed by singular value decomposition (SVD)-based single-user MIMO processing is employed [1], [2], the IUI suppressing matrix becomes the nulling matrix as follows:

$$\mathbf{W}_k(b) = \text{null}\left(\mathbf{H}_{\bar{k}}^H(b)\mathbf{U}_{\bar{k}}^H(b)\right) \quad (2)$$

where null$\{\cdot\}$ denotes the span of an orthogonal basis for the kernel or nullspace of the input matrix; $\mathbf{U}_{\bar{k}}(b)$ consists of $L_{\bar{k}}$ dominant left singular vectors of $\mathbf{H}_{\bar{k}}(b)$ where $\bar{k} \neq k$; and $\bar{\mathbf{V}}_k(b)$ and $\bar{\mathbf{U}}_k(b)$ are the right and left singular matrices of $\mathbf{U}_k^H(b)\mathbf{H}_k(b)\mathbf{W}_k(b)$, respectively. In this case, the IUI can be cancelled out completely based on a ZF criterion, and thus $\tilde{\mathbf{x}}_k(b)$ in (1) becomes

$$\begin{aligned}\tilde{\mathbf{x}}_k(b) &= \mathbf{R}_k^H(b)\mathbf{H}_k(b)\mathbf{T}_k(b)\mathbf{x}_k(b) + \mathbf{R}_k^H(b)\mathbf{n}_k(b) \\ &= \bar{\mathbf{U}}_k^H(b)\mathbf{U}_k^H(b)\mathbf{H}_k(b)\mathbf{W}_k(b)\bar{\mathbf{V}}_k(b)\mathbf{E}_k(b)\mathbf{x}_k(b) \\ &\quad + \mathbf{R}_k^H(b)\mathbf{n}_k(b) \\ &= \bar{\mathbf{D}}_k(b)\mathbf{E}_k(b)\mathbf{x}_k(b) + \mathbf{R}_k^H(b)\mathbf{n}_k(b)\end{aligned} \quad (3)$$

where $\bar{\mathbf{D}}_k(b)$ is an $L_k \times L_k$ diagonal matrix consisting of the singular values of $\mathbf{U}_k^H(b)\mathbf{H}_k(b)\ \mathbf{W}_k(b)$, and the third equality comes from the fact that $\bar{\mathbf{U}}_k(b)$ and $\bar{\mathbf{V}}_k(b)$ are singular matrices of $\mathbf{U}_k^H(b)\mathbf{H}_k(b)\mathbf{W}_k(b)$.

III. Multiuser Pair and Band Allocation

The effective SNRs and the throughput[1] for each subchannel are determined by the MU-MIMO user-pairs, and the achievable rates for the k-th user can be represented by $\omega(\mathbf{A}_k) \triangleq \sum_{i=1}^{L_k} \log_2(1 + \lambda_i^2/\sigma^2)$, where λ_i is the ith largest singular value of an effective channel matrix $\mathbf{A}_k$. Then, we define the symmetric sum achievable rate matrix $\mathbf{M}(b)$; the (i, j)th element of $\mathbf{M}(b)$, $m_{i,j}(b)$, represents the sum achievable rates obtained by using the bth subchannel band when combining the ith and jth users and it can be written as follows:

$$m_{i,j}(b) = \begin{cases} \omega(\mathbf{H}_i(b)\mathbf{W}_j(b)) + \omega(\mathbf{H}_j(b)\mathbf{W}_i(b)), & i \neq j \\ \omega(\mathbf{H}_i(b)\mathbf{H}_j^H(b)), & i = j \end{cases} \quad (4)$$

Table 1: Number of Real Operations for Computing $m_{i,j}(b)$

$m_{i,j}(b)$	# of operations (addition plus multiplication)
$i \neq j$	$6L_k N_R N_T^2 + 24N_R L_k^2 - 8L_k^3$
$i = j$	$6(4N_R N_T^2 - 4N_T^3/3) + 1$

Here, the diagonal elements are achievable rates only when the ith user is allocated at the frequency band b, i.e., a single user MIMO transmission. The computational complexity for $\mathbf{W}_k(b)$ in (2) is $(24N_T N_R^2 + 48N_R^3)BT$. To count the number of operations, we consider an efficient eigenvalue decomposition method in [9], which is based on the bidiagonalization and singular value decomposition. In addition, we treat that every complex operation as multiplication. Table 1 summarizes the number of operations associated with finding the sum achievable rates with given $\mathbf{H}_k(b)$ and $\mathbf{W}_k(b)$.

A. *Max-throughput Algorithm*

To obtain the optimal MU-MIMO user-pair selection, sum achievable rate matrices $\{\mathbf{M}(b)\}$ are required for the transmitter. Then, the optimal MU-MIMO user-pair set in terms of maximizing total-throughput is defined as

$$\mathcal{S} \triangleq \underset{\{\mathcal{S}(b)\}}{\arg\text{maximize}} \sum_b m_{i,j}(b) \quad (5)$$

where MU-MIMO user-pair set $\mathcal{S} = \{\mathcal{S}(1), \cdots, \mathcal{S}(B)\}$ and $\mathcal{S}(b)$ represents the MU-MIMO user-pair (i, j) for subchannel band b. Even though the optimal user-pair set $\mathcal{S}$ satisfying (5) can yield the maximum total-throughput, fairness among

[1]For simplicity, we assumed that there is no power control, i.e., $\mathbf{E}_k(b)$ in (3) is a normalized identity matrix and up to two users are supportable simultaneously by using the same frequency band.

Table 2: Number of Computations for $m_{i,j}(b)$

Algorithm	$m_{i,i}(b)$	$m_{i,j}(b), i \neq j$
Max-throughput (best-fit)	TB	$B\sum_{i=1}^{T-1} i$
First-fit	$\left(\lceil \frac{K}{2} \rceil - 1\right) B$	$\sum_{i=1}^{\lceil T/2 \rceil - 1} (T - (2i-1))$
Random-fit	0	$\lceil K/2 \rceil$

users cannot be guaranteed at all. We call this method the max-throughput algorithm, and this algorithm is summarized as follows:

Max-throughput algorithm

Step 1. Initialization
$\mathcal{S}(b) = (0,0), \forall b \in \{1,2,\ldots,B\}$, $K > 2B$, and $k = 0$.

Step 2. Stage $k = k+1$

$b = k$

Select best user-pair in bth subchannel band:

$\{i,j\} = \arg\text{maximize}_{i,j}\{m_{i,j}(b)\}, \forall i, \ \forall j$

Update MU-MIMO user-pair:

$\mathcal{S}(b) = (i,j)$

Update subchannel band set:

$\Omega = \Omega + \{b\}$

Step 3. Stop and $\mathcal{S} = \{\mathcal{S}(1), \cdots, \mathcal{S}(B)\}$, if $k = B$. Otherwise, go to *Step 2*.

As we mentioned previously, the max-throughput algorithm cannot guarantee fairness among users, since the same user can be supported in the different scheduling stages termed k, i.e., some users multiply use the MU-MIMO bands. Here, if a certain scheduling algorithm, such as round robin or proportional fairness scheduling (see [10] and the references therein), is employed for selecting proper users, then fairness performance can be improved. However, the transmitter should know all $m_{i,j}(b)$ for all i, j, and b. This imposes a great burden of computational complexity for the transmitter. The total number of computations for $m_{i,j}(b)$ in (4) is summarized in Table 2, when T is larger than two. Furthermore, even though $\{\mathbf{M}(b)\}$ are known at the transmitter, it is a formidable task to find the optimal $\mathcal{S}$ in terms of both the throughput and fairness, since so many combinations, $(T(T+1)/2)^B$, should be compared to determine $\mathcal{S}$. Therefore, reducing the computational load for allocating resources is of practical importance.

B. Best-fit Algorithm

To avoid computational load due to the combinatorial search, the best-fit algorithm [11], which was proposed for a single carrier system, can be employed. The conventional best-fit algorithms utilize space-and-time resources for scheduling. On the other hand, the proposed best-fit algorithm utilizes space-and-frequency resources by using $\{\mathbf{M}(b)\}$. The transmitter using the best-fit algorithm selects the user-pair which yields the highest throughput among all subchannel bands in every scheduling stage. Here, note that the users and subchannels selected in the $(k-1)$th previous stage are discarded in the kth present stage for fairness among users, which is similar to round robin scheduling. The best-fit algorithm is summarized as follows:

Table 3: Simulation Parameters for MU-MIMO OFDMA System

Parameter	Values
$\{N_T, N_R\}$	$\{4, 2\}$
# of subcarriers N	1024
Carrier frequency	2.3 GHz
Sampling rates	10 MHz
Power profile (5 taps)	Pedestrian A in ITU
SNR	25 dB
# of OFDM symbols	1000
# of MU-MIMO subchannel band (B)	6 bands

Best-fit algorithm

Step 1. Initialization
$\mathcal{S}(b) = (0,0), \forall b \in \{1,2,\ldots,B\}$, $K > 2B$, $k = 0$, $i = 0$, $j = 0$, and $\Omega = \emptyset$

Step 2. Stage $k = k+1$

If i and j are not element of $\mathcal{S}(b)$

Select best user-pair and subchannel band:

$\{i,j,b\} = \arg\text{maximize}_{i,j,b}\{m_{i,j}(b)\}, \forall i, \forall j$, and $\forall b \not\subseteq \Omega$

Update MU-MIMO user-pair:

$\mathcal{S}(b) = (i,j)$

Update subchannel band set:

$\Omega = \Omega + \{b\}$

Otherwise go to *Step 3*.

Step 3. Stop and $\mathcal{S} = \{\mathcal{S}(1), \cdots, \mathcal{S}(B)\}$, if $k = B$. Otherwise, go to *Step 2*.

C. First-fit Algorithm

To further reduce computational complexity, the first-fit algorithm [12] can be employed in OFDMA systems. This algorithm can reduce computations by reducing the number of computations for $m_{i,j}(b)$ in (4) (see Table 2). The first-fit algorithm is described as follows: First, a transmitter decides the subchannel band for the 1st user. In this procedure, the transmitter compares $m_{1,1}(b)$'s for all b, and then selects the subchannel b', which has the largest sum achievable rates, i.e., $b' = \arg\max_b\{m_{1,1}(b)\}$. Secondly, in the allocated subchannel band b', all of $m_{a,j}(b')$ for all j are compared, and then the transmitter decides the 1st user's user-pair $(1, j')$, which has the largest sum achievable rates. Similarly, the transmitter can determine all user-pairs and their subchannels. Similarly to the best-fit algorithm, the previously selected users and allocated subchannels are discarded in the current resource allocation stage. The suboptimal first-fit user-pair selection algorithm

Table 4: Computational Complexity for MU-MIMO User-pair Selection When $N_T = 4$, $N_R = 2$, and $L_k = 2$

Algorithm	# of operations $(+, \times)$ when $T > 2$ and T is even
Max-throughput (best-fit)	$256BT^2 + 771BT$
First-fit	$128T^2 + 768BT + 259B - 771$
Random-fit	$768B$

is summarized as follows:

First-fit algorithm

Step 1. Initialization
$\mathcal{S}(b) = (0,0), \forall b \in \{1, 2, \ldots, B\}$, $T > 2$, and $\Omega = \emptyset$

Step 2. Stage $k = k + 1$

If k is not element of $\mathcal{S}(b)$

Select subchannel band:

$b' = \arg\text{maximize}_b\{m_{k,k}(b)\}, \forall b \notin \Omega$

Select MU-MIMO user-pair for kth user:

$i' = k$

$j' = \arg\text{maximize}_j\{m_{i',j}(b')\}, \forall j > i'$,

where j is not the element of $\mathcal{S}(b), \forall b$

Update MU-MIMO user-pair:

$\mathcal{S}(b') = (i', j')$

Update subchannel band set:

$\Omega = \Omega + \{b'\}$

Otherwise go to *Step 3*.

Step 3. Stop and $\mathcal{S} = \{\mathcal{S}(1), \cdots, \mathcal{S}(B)\}$, if $k = B$. Otherwise, go to *Step 2*.

D. Random-fit Algorithm

To minimize computational complexity, both the user-pairs and subchannels can be allocated randomly. This algorithm is termed as the random-fit algorithm. Though this algorithm is the simpler than those mentioned previously (see Table 2), the resulting throughput would be seriously degraded. The random-fit algorithm is summarized as follows:

Random-fit algorithm

Step 1. Initialization
$\mathcal{S}(b) = (0,0), \forall b \in \{1, 2, \ldots, B\}$, and $T > 2$

Step 2. $k = k + 1$

Select subchannel band:

$b = k$

Select MU-MIMO user-pair for kth user:

$i' = 2k - 1$ and $j' = 2k$,

Update MU-MIMO user-pair:

$\mathcal{S}(b) = (i', j')$

Step 3. Stop and $\mathcal{S} = \{\mathcal{S}(1), \cdots, \mathcal{S}(B)\}$, if $k = B$. Otherwise, go to *Step 2*.

IV. Comparison and Discussion

In this section, various resource allocation methods were examined in terms of the sum achievable rates, fairness among users, and computational complexity. The parameters for the simulation were summarized in Table 3. The MU-MIMO channel is obtained by generating independent Gaussian random variables with zero mean in the frequency domain, and the results shown below are the averages over 1 000 independent trials. In the simulation, Jain's fairness index [13] was used to compare fairness among users. Jain's index J is bounded between 0 and 1. The higher index represents higher fairness among users. When there are K users, Jain's index $J = \frac{\left(\sum_{k=1}^{K} r_k\right)^2}{K \sum_{k=1}^{K} r_k^2}$, where the normalized throughput $r_k = T_k/O_k$, T_k is the measured throughput, and O_k is the fair throughput, for the kth user. Here, we assumed that all O_k are identical for all k, i.e., traffic characteristics for each user are identical and required rates for each user are identical also.

By using Tables 1 and 2, the total numbers of operations to select MU-MIMO user-pairs are compared in Table 4 and illustrated in Fig. 2(a). Here, comparison complexities were not considered. Thus, the max-throughput and best-fit algorithms require the same computational complexities. The random-fit algorithm, which requires $\mathcal{O}(K)$ operations, provides great computational savings compared with the other algorithms. Both the best-fit and first-fit algorithms require $\mathcal{O}(T^2)$ operations in terms of T, yet the first-fit algorithm provides a savings in computation, since the order of T^2 operations for the first-fit algorithm is free from the B.

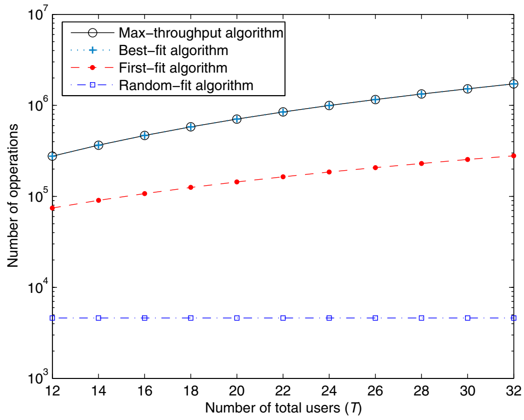
Figs. 2(b) and (c) show the average achievable rates and the Jain's fairness indices for four user-pair selection algorithms, respectively. It can be seen that the achievable rates, except for that of the random-fit algorithm, increase as the number of total users increases, i.e., only the random-fit algorithm cannot obtain multiuser diversity gain. Even though the max-throughput algorithm achieves the largest throughput among the various resource allocation algorithms, fairness among users is very poor. The fairness index of the max-throughput algorithm is 0.166, which means that only 16.6% of users are happy. The second largest throughput is obtained by the best-fit algorithm. Unfortunately, the average fairness index of the best-fit algorithm is only about 0.709. On the other hand, both the first- and random-fit algorithms provide about 98% fairness. Thus the first- and random-fit algorithms are worthwhile in terms of fairness.

In summary, when the total number of users is 32, the random-fit algorithm obtained computational complexity reduction of about 99.7% against the max-throughput (best-fit) algorithm. However, the achievable rate decreased about

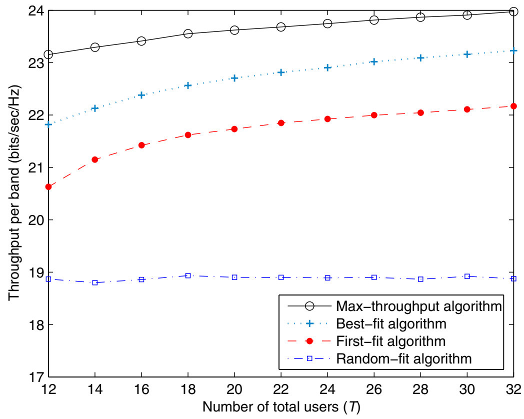
Table 5: Performance Comparison Among Resource Allocation Methods

Algorithms	MU diversity	Throughput	Complexity	Fairness
Max-throughput	○	Largest	Large	16.6%
Best-fit	○	−3.1%	Largest	70.9%
First-fit	○	−7.5%	Small	98.9%
Random-fit	×	−21.2%	Smallest	98.4%

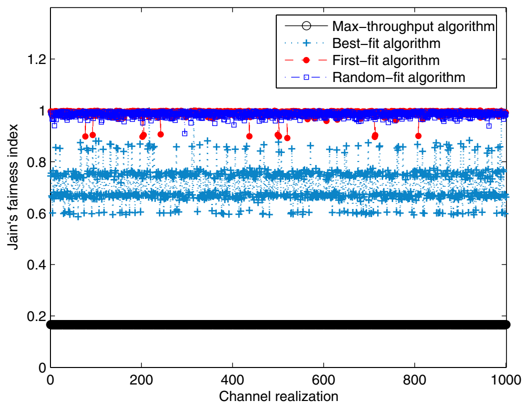
21.2% from that of the max-throughput algorithm. In the first-fit algorithm case, an 83.7% reduction of computational complexity is accompanied by an 7.5% rate reduction. According to the computing power of the transmitter and required throughput, a user-pair selection algorithm can be determined (see Table 5). Here, it can be surmised that the best-fit algorithm is attractive in terms of fairness, computational complexity, and total-throughput.



(a)



(b)



(c)

Figure 2: When $SNR = 25\,\text{dB}$, $B = 6$, $N_T = 4$, and $N_R = 2$. (a) computational complexity comparison. (b) Sum achievable rates comparison. (c) Fairness comparison.

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