Efficient Transmit Strategy Using a Fraction of Feedback for QRD based V-BLAST Systems

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***Abstract*— In this paper, we propose an efficient transmit strategy utilizing the partial channel information feedback for the QRD based V-BLAST systems. In the proposed scheme, after QR decomposition (QRD) of the channel matrix, positive real-valued diagonal elements of $\mathbf{R}$ are forwarded to the transmitter through the feedback channel. Based on the feedback information, the transmitter allocates the optimum transmit power to minimize the bit error rate (BER) of the system. With small feedback overhead, the proposed scheme shows significantly improved BER performance compare to the conventional sorted QR decomposition (SQRD) based V-BLAST systems. Moreover, we can reduce the receiver complexity by removing the unnecessary sorting process at the receiver.**

I. Introduction

Multiple-input multiple-output (MIMO) systems can provide enormous capacity in rich-scattering environment by exploiting the spatial dimension [1]. Especially, the V-BLAST (Vertical Bell Labs Layered Space-Time) system has attracted much attention due to its simple architecture and high throughput performance [2]. At the receiving end of the system, the detection algorithm using ordered successive interference cancelation (OSIC) has been proposed simultaneously as an original work. Although OSIC detection provides outstanding system error performance, it requires a plenty of receiver complexity for recursive calculation on pseudo-inverse of the channel matrix. To reduce such receiver complexity, the detection algorithm utilizing sorted QR decomposition of the channel matrix is proposed that provides close detection performance to that of OSIC detection with far less receiver complexity [3].

In such open-loop systems, enormous increase of the receiver complexity is necessary to improve the system performance. As a solution for such an tradeoff between receiver complexity and system error performance, closed-loop systems are getting momentum recently. Corresponding to these research movements, there have been some studies concerned in performance improvement of the V-BLAST systems in closed-loop environments [4]-[7]. In closed-loop systems, channel information is forwarded to the transmitter with feedback mechanism, and the transmitter can adapt efficient transmission strategies according to the channel information. Although the closed-loop systems generally outperform the conventional open-loop systems, feedback overhead is a major concern for the implementation of systems. In this regard, power control approaches are noticeable for their relative simplicity and small feedback overhead [4]-[5]. As a previous work, a transmit power allocation specialized to the OSIC detection based V-BLAST system has been proposed to improve the system error performance [4]. However, relatively little attention was paid to the closed-loop SQRD based V-BLAST systems in spite of various advantages over the systems based on the OSIC detection.

In this paper, we propose an efficient transmit strategy for the QRD based V-BLAST systems using a fraction of channel information feedback. In the proposed system, after QR decomposition of the channel matrix, the diagonal elements of $\mathbf{R}$ matrix are forwarded to the transmitter. Using the feedback information, the transmitter allocates the optimum transmit power to minimize bit error rate (BER) of the system. Previous conventional power control approaches calculate the transmit power at the receiver since they requires full channel information for the power allocation. However, the proposed scheme can calculate the transmit power at the transmitter with fraction of channel feedback because it needs only partial channel information. Therefore, the proposed scheme has a great benefit on system implementation aspects while simple calculation of the transmit power only need extremely small complexity increase at the transmitter. By employing the proposed transmit strategy, error performance of the systems can be significantly improved compare to the conventional QRD based V-BLAST systems. Moreover, the employing of the proposed optimal transmit power allocation eliminates the need of sorting process at the receiver. Thus, it can also reduce the receiver complexity in comparison with the conventional SQRD based V-BLAST systems. It means that the proposed transmit strategy not only improves the system performance but also reduces the receiver complexity.

The rest of this paper is organized as follows. Section II provides brief descriptions about the system model compared to the conventional SQRD based V-BLAST systems. In Section III, the proposed transmit strategy to minimize BER of the system is numerically derived with Lagrange optimization. The system performance of the proposed transmit strategy is verified with the computer simulation, and some discussions are given in Section IV. Finally, we make a conclusion in Section V.

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II. SYSTEM MODEL

A. SQRD based V-BLAST System

The V-BLAST system is composed of M transmit antennas and N ($\geq M$) receive antennas. At the transmitter, independent data streams are transmitted simultaneously over the antenna array. In rich-scattering environment, the received signal vector $\mathbf{x} = [x_1 x_2 \cdots x_N]^T$ corresponding to the transmitted signal vector $\mathbf{c} = [c_1 c_2 \cdots c_M]^T$ is given by

$$\mathbf{x} = \mathbf{Hc} + \mathbf{n}, \tag{1}$$

where $\mathbf{n} = [n_1 n_n \cdots n_N]^T$ is the additive white Gaussian noise with zero mean and variance of σ_n^2. The $N \times M$ channel matrix $\mathbf{H}$ is composed of independent and identically distributed (i.i.d.) complex Gaussian channel coefficients.

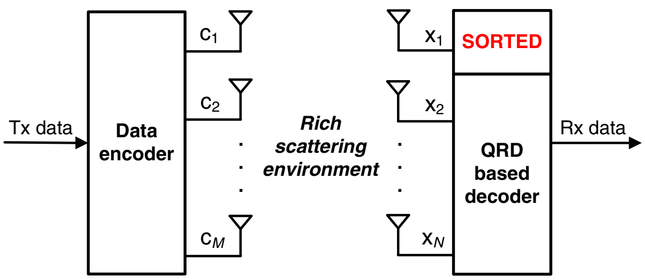

Fig. 1. Block Diagram of SQRD based V-BLAST System

The system block diagram of the SQRD based V-BLAST system is shown in Fig. 1. For the signal detection, the SQRD based V-BLAST system employs a simple detection algorithm utilizing sorted QR decomposition of the channel matrix. The importance of detection ordering in V-BLAST system has been discussed in [2]. To construct the best ordering, column permuting is performed with QR decomposition of the channel matrix in the QRD based detector. The QRD based detector with such a permutation process is called SQRD based detector, and it provides better performance but need more complex receiver structure.

At the SQRD based detector, using the modified Gram-Schmidts algorithm, the channel matrix $\mathbf{H}$ is decomposed into $N \times M$ unitary matrix $\mathbf{Q}$ and $M \times M$ upper triangular matrix $\mathbf{R}$ as

$$\mathbf{H} = \mathbf{QR}, \tag{2}$$

where the ordering information is ignored for simplicity. To detect the signal, by multiplying the Hermitian matrix of $\mathbf{Q}$, the received signal vector can be modified as

$$\mathbf{y} = \mathbf{Q}^H \mathbf{x} = \mathbf{Rc} + \mathbf{w}, \tag{3}$$

where the statistics of the noise components $\mathbf{w} = \mathbf{Q^H n}$ are unchanged due to the unitary property of $\mathbf{Q}$ matrix. With this kind of modification, the k-th received signal is represented by

$$y_k = r_{k,k} \cdot c_k + \sum_{i=k+1}^{M} r_{k,i} \cdot c_i + w_k, \tag{4}$$

where $r_{k,i}$ is the k-th row and i-th column element of $\mathbf{R}$ matrix. This special structure of the modified received signal makes possible to utilize the back substitution detection mechanism.

With the SQRD based V-BLAST system including sorting process at the receiver, a reasonable system error performance can be achieved. However, in the conventional SQRD based V-BLAST systems with equal transmit power, the occurrence of error caused by deep fading channel are stochastically unavoidable. This drawback limits the overall error performance of the system. Moreover, the problem of complexity increase at the receiver for sorting process is not negligible for the system implementation.

B. Proposed QRD based V-BLAST System

To overcome the problem of the conventional SQRD based V-BLAST systems, we propose an efficient transmit strategy using a fraction of feedback for the QRD based V-BLAST systems.

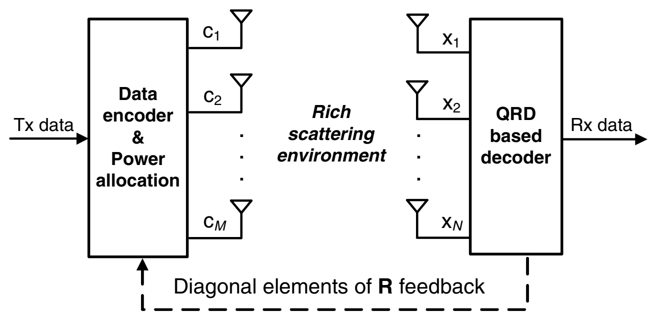

Fig. 2. Block Diagram of Proposed QRD based V-BLAST System

Fig. 2 shows the system architecture of the proposed QRD based V-BLAST system. At the receiver, after QR decomposition of the channel matrix, the positive real-valued diagonal elements of $\mathbf{R}$ matrix are forwarded to the transmitter. Since they are only real-valued vector with M elements, feedback overhead is relatively small. With the assumption of quasi-static channel environment, the transmitter can access current channel information through the feedback process. According to the feedback information, the transmitter allocates the optimum transmit power for each antenna to minimize BER of the system. The computation of transmit power is straightforward, so it only needs extremely small additional complexity at the transmitter. The details of the transmit power allocation algorithm will be dealt in Section III.

Applying the transmit power allocation to the system yields received signal as

$$\underline{\mathbf{x}} = \mathbf{HPc} + \mathbf{n}, \tag{5}$$

where $\mathbf{P} = \mathrm{diag}(\sqrt{P_1}, \sqrt{P_2}, \ldots, \sqrt{P_M})$ is the diagonal power allocation matrix whose element P_k represents the amount of power allocated to the k-th transmit antenna. For signal detection in the proposed system, modified QR decomposition is employed.

TABLE I

QR DECOMPOSITION ALGORITHM FOR PROPOSED SYSTEM

(1)	$\mathbf{R}=0, \mathbf{Q}=\mathbf{H}\cdot\mathbf{P}$
(2)	for $i=1,\dots,M$
(3)	$\quad r_{i,i}=\lvert\mathbf{q}_i\rvert$
(4)	$\quad \mathbf{q}_i=\mathbf{q}_i/r_{i,i}$
(5)	$\quad$ for $j=i+1,\dots M$
(6)	$\qquad r_{i,j}=\mathbf{q}_i^H\cdot\mathbf{q}_j$
(7)	$\qquad \mathbf{q}_j=\mathbf{q}_j-r_{i,j}\cdot\mathbf{q}_i$
(8)	$\quad$ end
(9)	end

TABLE I shows the modified zero-forcing (ZF) QR decomposition algorithm for the proposed system where $\mathbf{q}_k$ denotes the k-th column of $\mathbf{Q}$ matrix. Based on the QR decomposition algorithm in [3], the proposed algorithm eliminates the unnecessary sorting process. In addition, the channel matrix $\mathbf{H}$ is replaced to a multiplication of the channel matrix and the power allocation matrix, i.e. $\mathbf{H}\cdot\mathbf{P}$. With the same principle, extension to the modified QR decomposition for minimum mean square error (MMSE) criterion can be simply applicable according to the algorithm in [8].

The channel matrix multiplied with power allocation matrix is decomposed into the unitary $\mathbf{Q}$ matrix and the upper triangular $\mathbf{R}$ matrix. Applying the channel decomposition yields the received signal as

$$\underline{\mathbf{x}}=\mathbf{QRPc}+\mathbf{n}. \tag{6}$$

Utilizing the unitary property of the $\mathbf{Q}$ matrix, multiplying Hermitian matrix of $\mathbf{Q}$ provides the modified received signal represented by

$$\underline{\mathbf{y}}=\mathbf{Q}^H\underline{\mathbf{x}}=\mathbf{RPc}+\mathbf{w}. \tag{7}$$

Correspond to equation (7), the modified k-th received signal is represented as

$$\underline{y}_k=r_{k,k}\cdot P_k\cdot c_k+\sum_{i=k+1}^{M} r_{k,i}\cdot P_i\cdot c_i+w_k. \tag{8}$$

On the other hand, as will be explained in Section III, employing of the proposed transmit strategy eliminates necessity of sorting at the receiver. It directly related to the complexity reduction at the receiver that makes the system more outstanding in point of practical implementation.

III. PROPOSED TRANSMIT STRATEGY

In this section, we derive the optimum transmit power allocation for the proposed QRD based V-BLAST system. The system BER function in the proposed QRD based V-BLAST system is modeled first, and the proposed transmit strategy is derived using the Lagrange multiplier method.

When $\Pr(x_k^e)$ represents the erroneous bit detection probability at the k-th stage, the bit error probabilities for each detection stages U_k $(k=M,M-1,...,1)$ for the QRD based V-BLAST system can be approximated to

$$\begin{aligned}
U_M &= \Pr\left(x_M^e\right)\\
U_{M-1} &= \Pr\left(x_{M-1}^e|x_M^c\right)(1-U_M)+\Pr\left(x_{M-1}^e|x_M^e\right)U_M\\
&\cong \Pr\left(x_{M-1}^e|x_M^c\right)\\
U_{M-2} &= \Pr\left(x_{M-2}^e|x_{M-1}^c,x_M^c\right)(1-U_{M-1})(1-U_M)\\
&\quad+\Pr\left(x_{M-2}^e|x_{M-1}^c,x_M^e\right)(1-U_{M-1})U_M\\
&\quad+\Pr\left(x_{M-2}^e|x_{M-1}^e,x_M^c\right)U_{M-1}(1-U_M)\\
&\quad+\Pr\left(x_{M-2}^e|x_{M-1}^e,x_M^e\right)U_{M-1}U_M\\
&\cong \Pr\left(x_{M-2}^e|x_{M-1}^c,x_M^c\right)\\
&\ \vdots\\
U_1 &\cong \Pr\left(x_1^e|x_2^c,\,\dots,x_{M-1}^c,x_M^c\right),
\end{aligned} \tag{9}$$

where $\Pr\left(x_k^e|x_j^c\right)$ and $\Pr\left(x_k^e|x_j^e\right)$ are bit error probabilities at the k-th stage conditioned on the events of correct and erroneous detection for the j-th signal, respectively. The approximation is justified with a sufficiently high SNR environment which makes possible to neglect the terms of U_k. Based on (9), we can generally approximate bit error probability for the k-th detection stage to

$$U_k\cong\Pr\left(x_k^e|x_{k+1}^c,\,\dots,x_{M-1}^c,x_M^c\right). \tag{10}$$

The QRD based detector, unlike the OSIC detection scheme, has distinct characteristic due to the special form of $\mathbf{R}$ matrix in (4). Since $\mathbf{R}$ is a upper triangular matrix, the first detection stage $(k=M)$ is free of interference. The next detection stage $(k=M-1)$ is performed with interference of the M-th signal. However, if we assume that the detection of M-th signal is correct, the interference from M-th signal can be completely removed. With this in mind, for every detection stage, if previous detections are perfect, current detection can be peformed without interference. In this regard, we can conclude that the probability of bit error is determined by the diagonal elements of $\mathbf{R}$ matrix. Then the bit error probability for the k-th detection stage is expressed in terms of received SNR (e.g., see [10] for more detail) as

$$U_k=\Pr\left(x_k^e|x_{k+1}^c,x_{k+2}^c,\dots,x_M^c\right)\cong f\left(\frac{P_k\cdot r_{k,k}^2}{\sigma_n^2}\right), \tag{11}$$

where $f(\cdot)$ is an arbitrary BER function defined by a specific modulation employed in the system. From equation (11), the system bit error probability can be represented as a sum of bit error probability for each stage as

$$S_b=f\left(\tfrac{P_1\cdot r_{1,1}^2}{\sigma_n^2}\right)+f\left(\tfrac{P_2\cdot r_{2,2}^2}{\sigma_n^2}\right)+\cdots+f\left(\tfrac{P_M\cdot r_{M,M}^2}{\sigma_n^2}\right). \tag{12}$$

To calculate the optimum transmit power for system BER performance, we employ the Lagrange multiplier method. The constraint function for the optimization is the total transmit power constraint given by

$$\sum_{i=1}^{M} P_i = M. \tag{13}$$

The cost function with the objective function (12) and constraint function (13) is

$$C(P_1, P_2, \dots P_M) = S_b + \lambda \left(\sum_{i=1}^{M} P_i - M \right), \tag{14}$$

where λ is the Lagrange multiplier. In accordance with Lagrangian method, we can find the optimal transmit power by solving the equation (13) and M equations $\partial C / \partial P_k = 0$ simultaneously. Then the optimal transmit power for the proposed system is

$$P_k = \frac{M}{r_{k,k}^2 \left(1/r_{1,1}^2 + 1/r_{2.2}^2 + \cdots + 1/r_{M,M}^2 \right)}, \tag{15}$$

where simple form of the derived transmit power allocation only requires a fraction of complexity increase at the transmitter.

When employing the proposed transmit power allocation to the QRD based V-BLAST systems, we can find a interesting result. Applying the proposed transmit strategy results in same received signal power for every layer as

$$P_1 \cdot r_{1,1}^2 = P_2 \cdot r_{2,2}^2 = \cdots = P_M \cdot r_{M,M}^2. \tag{16}$$

In this regard, we can conclude that the BER performance of the QRD based V-BLAST systems are optimized when the received signal powers for all layers are equal. In other words, the system bit error probability is minimized when every detection stage shows the same bit error probability as

$$f\left(\frac{P_1 \cdot r_{1,1}^2}{\sigma_n^2} \right) = f\left(\frac{P_2 \cdot r_{2,2}^2}{\sigma_n^2} \right) = \cdots = f\left(\frac{P_M \cdot r_{M,M}^2}{\sigma_n^2} \right). \tag{17}$$

It implies that every detection ordering gives the same system BER performance. Taking the result into account, it is justifiable to conclude that the performance of the proposed system is independent on detection ordering. Therefore, as has been noted above, the proposed scheme can reduce the system complexity by removing sorting process at the receiver. Furthermore, the proposed transmit strategy can be applied to the MMSE criterion based on [8], and applicable to any modulation schemes generally.

IV. Simulation Results

In this section, the performance of the proposed transmit strategy is evaluated with Monte Carlo simulation. For the simulation, transmit power normalized QPSK modulation is employed, and QRD based detection algorithm with ZF and MMSE criterion are used. The channel environment is assumed as Rayleigh flat fading channel, and they are uncorrelated each other. Also, the availability of perfect channel information at the receiver and error-free feedback channel is assumed.

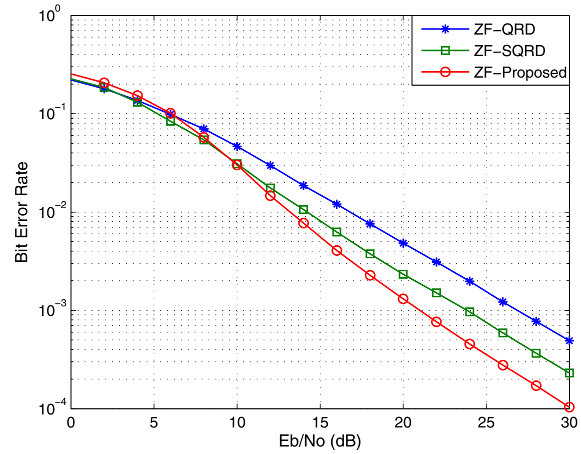

Fig. 3. BER Performance of proposed system for ZF detection with $M = 4$ and $N = 4$ antennas

Fig. 3 shows the BER performance of the proposed transmit strategy compare to the conventional SQRD based V-BLAST system. The ZF detection criterion is employed with $M = 4$ transmit and $N = 4$ receive antennas. It is observed that the proposed scheme gives about 3dB SNR gain at the BER performance of 10^{-3} compare to the SQRD based V-BLAST system. The performance gain is came from the optimal power allocation at the proposed system while conventional SQRD based system provides limited performance for a given channel with equal power allocation.

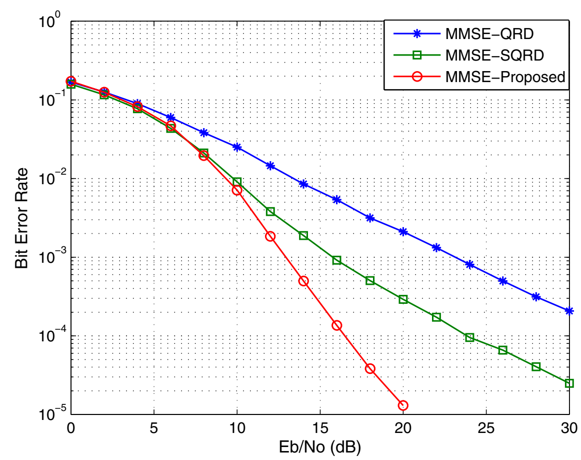

Fig. 4. BER Performance of proposed system for MMSE detection with $M = 4$ and $N = 4$ antennas

The BER performance of the proposed strategy for MMSE detection criterion is presented in Fig. 4. It is shown that the BER performance of the system is also incredibly improved with MMSE criterion. The proposed scheme provides not only SNR gain but also additional diversity order increase. Due to the increase of diversity order, performance of the proposed system is remarkable in high SNR region.

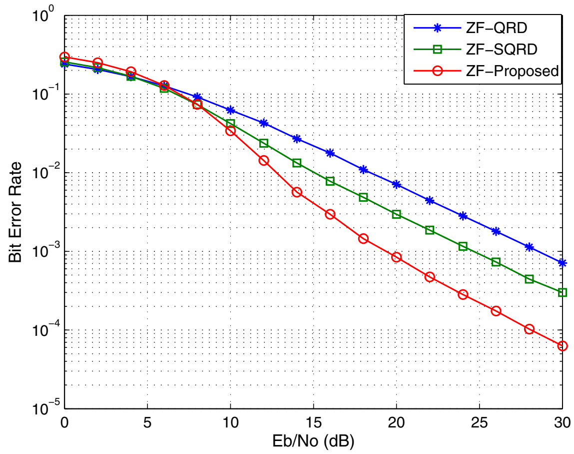

Fig. 5. BER Performance of proposed system for ZF detection with $M = 8$ and $N = 8$ antennas

In Fig. 5, the BER performance of the proposed system with $M = 8$ transmit and $N = 8$ receive antenna is presented. Compare to the results in Fig. 3, we can observe an additional performance gain with increased number of transmit and receive antennas. The SNR gain at the BER performance of 10^{-3} is about 5dB against the conventional SQRD based V-BLAST system. The reason for the additional performance gain is that as the number of transmit antenna increases, the possibility to choose wrong detection ordering is increased for the SQRD based V-BLAST system while the proposed system provides always the optimal performance.

V. Conclusion

In this paper, we have proposed an efficient transmit strategy with partial channel information feedback for the QRD based V-BLAST systems. With the proposed scheme, we significantly improved the BER performance compare to the conventional SQRD based V-BLAST systems. In the QRD based system equipped with 4 transmit and receive antennas, SNR gain of 3dB is obtain at BER of 10^{-3} by using ZF detection criterion. With increased number of antennas, we can get additional SNR gains. With the MMSE detection criterion, we can observe remarkable diversity order increase. Also, a simple transmit power allocation at the transmitter with only a fraction of channel feedback has a remarkable benefit on system implementation compare to the conventional power control approaches. In addition, by employing the optimal transmit power allocation, the receiver complexity of the proposed system can be reduced to less than that of the SQRD based V-BLAST system through eliminating the sorting process at the receiver.

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