Cascade/Parallel Form FIR Filters with Powers-of-Two Coefficients

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ABSTRACT

A prefilter-equalizer structure for cascade form 2PFIR filtering is proposed. This structure consists of a prefilter made up of cascaded cyclotomic polynomial(CP) filters, and an FIR equalizer having signed powers-of-two coefficients. It is shown that this prefilter-equalizer is often easier to design, and can be perform better than direct and cascade form 2PFIR filters.

I. Introduction

Due to their simplicity in implementation, FIR filters with powers-of-two coefficients which are often referred to as 2PFIR filters have received considerable attention in digital signal processing. [1]-[5]

Most 2PFIR filters have direct form structures [1]-[3]. In an effort to improve the performance of 2PFIR filters, cascade form filters which cascade two direct form 2PFIR filters have been designed in [4] and [5]. It has been observed that the cascade form filters can achieve much smaller peak ripple than can direct form filters. The design of the former, however, is considerably more difficult than that of the latter.

In this paper, we introduce an alternative structure for cascade form 2PFIR filtering. The proposed structure consists of a "prefilter" made up of one or more cascaded *cyclotomic polynomial*(CP) filters [6], followed by an FIR "equalizer" having signed powers of two coefficients (Fig. 1). The prefilter consists of cascaded multiplierless subsections, where each subsection transfer function equals a CP in z^{-1}[7], [8]. Following the procedure proposed in [6], we choose CP's from the set of the first 104 CP's which contain only the coefficients { −1, 0, 1}. After obtaining this prefilter, an equalizer with signed powers of two coefficients, which will be called 2PFIR equalizer, is designed such that the cascaded prefilter-equalizer meets a set of desired filter specifications. It will be shown that this prefilter-equalizer is often easier to design and can produce smaller peak ripple as compared with the cascaded 2PFIR filters in [4] and [5].

II. Designing the Prefilter-Equalizer

Given a set of desired filter specifications, we search for eligible CP's as follows [6] : examine each of the first 104 CP's having coefficients { −1, 0, 1 } and keep only those CP's containing zeros within the stopband or within some user-specified intrusion into the transition band (Fig. 2). This procedure eliminates all CP's with passband zeros, and typically retains only a few CP's. In our design examples, which will be presented in the following section, we consider all possible combinations of the eligible CP's as candidates for the prefilter.

The design of a 2PFIR equalizer is a discrete optimization problem of finding signed powers of two coefficients, so that the overall frequency response, say $H(e^{j\omega})$, of the cascaded prefilter-equalizer is a best approximation to the desired frequency response, $H_d(e^{j\omega})$. In contrast to the optimization for designing the cascade form filter in [3], this problem involves no nonlinear constraints on the coefficients, since the prefilter is pre-determined and only the equalizer is to be designed. It can be solved by directly applying some standard techniques such as *mixed integer linear programming* (MILP) [1], [3]. We shall design 2PFIR equalizers by using the MILP algorithm proposed in [3]. The cost function to be minimized is the maximum weighted ripple given by

$$\max_{\omega\in F}\left|W(\omega)[H_d(e^{j\omega})-H(e^{j\omega})]\right|$$

where F is the closed set { ω | $\omega \in$ passbands, $\omega \in$ stopbands } and $W(\omega)$ is the weighting function.

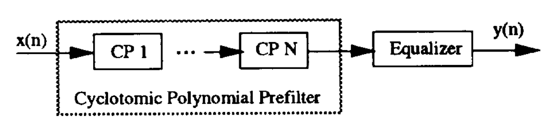

Fig. 1. Prefilter-equalizer cascade structure

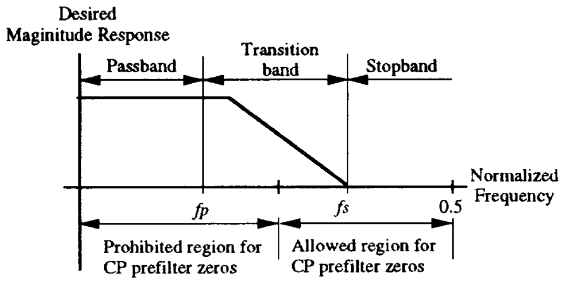

Fig. 2. Example of reation between the desired filter magnitude response and the prefilter zeros. (fp : passband edge, fs : stopband edge)

III. Filter Design Examples

In order to demonstrate the performance of the cascaded prefilter-2PFIR equalizer, we present two design examples. The lowpass design in Example 1 was computed for comparison with the results of previous methods in [4] and [5]. The bandpass design in Example 2 is presented to show the advantage of the prefilter-equalizer over the direct-form 2PFIR structure. In these examples, each coefficient of 2PFIR equalizers and of 2PFIR filters is a signed powers-of-two value selected from $\{ 0, \pm 2^{-1}, \pm 2^{-2}, ..., \pm 2^{-8}, \pm 2^{-9} \}$. Note that each multiplication of the 2PFIR equalizers and filters can be implemented by using a hard-wired shifter. Thus the number of additions is a good measure of complexity for implementing these filters. We set the weighting function $W(\omega)=1$ in the design. The filter performance is examined by comparing the normalized peak weighted ripple δ/b where δ is the peak weighted ripple and b is the mean value of the passband gain.

Example 1: This example designs the lowpass filter that has been considered in [4] and [5] to illustrate the behavior of the cascaded 2PFIR filters. Thus it is a good example for purpose of comparison. The lowpass filter has passband edge at f_p=0.15 and stopband edge at f_s=0.22 where f_p and f_s are normalized passband and stopband frequencies, respectively.

Using a 20% transition band intrusion criterion, we obtained the following three eligible CP's for this filter: $(1+z^{-1})$, $(1+z^{-1}+z^{-2})$ and $(1+z^{-2})$. Candidates for the prefilter are:

$P_1(z)=(1+z^{-1})$, $\quad P_2(z)=(1+z^{-1}+z^{-2})$

$P_3=(1+z^{-2})$, $\quad P_4(z)=(1+z^{-1})(1+z^{-1}+z^{-2})$

$$P_5(z)=(1+z^{-1})(1+z^{-2})=\frac{(1-z^{-4})}{(1-z^{-1})}$$

$$P_6(z)=(1+z^{-1}+z^{-2})(1+z^{-2})=\frac{(1+z^{-2})(1-z^{-3})}{(1-z^{-1})}$$

$$P_7(z)=(1+z^{-1})(1+z^{-1}+z^{-2})(1+z^{-2})=\frac{(1-z^{-3})(1-z^{-4})}{(1-z^{-1})^2}$$

Note that the prefilter $P_5(z)$, $P_6(z)$ and $P_7(z)$ can be implemented recursively. This recursive implementation reduces the number of additions associated with the prefilters at expense of additional delays. Since the coefficients of the CP's are 0 or ±1, exact pole-zero cancellation is guaranteed in this recursive implementation [6].

For each candidate prefilter $P_i(z)$, i=1, 2, ..., 7, we designed six equalizers of different length while setting the number of additions required for realizing the prefilter-equalizer at 20, 22, 24, 26, 28 and 30. Tables I (a)-(g) summarize the normalized peak ripple values (δ/b) of the resulting prefilter-equalizers. For comparison, data from [4] and [5] which are δ/b values of the direct and cascade form 2PFIR filters are reported in Table I (h). It is seen that the cascade form filters outperform the direct form filters. The prefilter-equalizer structures with $P_4(z)$, $P_6(z)$ and $P_7(z)$ are better than the cascaded 2PFIR filters, and the one with $P_7(z)$ performs the best. The magnitude responses of $P_7(z)$, the equalizer(L_e=27) cascaded with it and the cascaded $P_7(z)$-equalizer are illustrated in Fig. 3.

Example 2: Design a bandpass filter with f_p=(0.25, 0.35) and f_s=(0.15, 0.45). Using a 20% transition band intrusion criterion, we obtained the following eligible CP's: $(1-z^{-1})$, $(1+z^{-1})$ and $(1-z^{-1}+z^{-2})$. Candidates for the prefilter are:

$P_1(z)=(1-z^{-1})$ $\quad P_2(z)=(1+z^{-1})$

$P_3(z)=(1-z^{-1}+z^{-2})$ $\quad P_4(z)=(1-z^{-1})(1+z^{-1})=(1-z^{-2})$

$P_5(z)=(1-z^{-1})(1-z^{-1}+z^{-2})$

$P_6(z)=(1+z^{-1})(1-z^{-1}+z^{-2})=(1+z^{-3})$

$P_7(z)=(1-z^{-1})(1+z^{-1})(1-z^{-1}+z^{-2})=(1-z^{-1})(1+z^{-3})$

We again designed six equalizers for each $P_i(z)$, i=1, 2, 3, ..., 7. The results are summarized in Table II (a)-(g). The δ/b values associated with the direct form 2PFIR filters are also evaluated and listed in Table II (h). We can see that prefilter-equalizer structures outperform the direct form with the exception of those associated with $P_2(z)$ and $P_3(z)$. The one $P_7(z)$ performs the best. The magnitude responses of $P_6(z)$, the equalizer(L_e=27) and the cascaded $P_6(z)$-equalizer are shown in Fig. 4.

IV. Conclusion

We proposed a prefilter-equalizer structure for cascade form 2PFIR filtering. This structure consists of a CP prefilter cascaded with a 2PFIR equalizer. Lowpass and bandpass design examples demonstrated the advantages of this prefilter-equalizer over direct and cascade form 2PFIR filters. The further work in this direction will concentrate on developing some systematic way of constructing an efficient prefilter from a set of eligible CP's.

References

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Table 1. Normalized peak weighted ripple values of the lowpass prefilter-2PFIR equalizers, cascaded 2PFIR filters and direct-form 2PFIR filters. N is the number of additions required for implementing the prefilter-equalizer and L_e is the length of the equalizer. Note that $N-(L_e-1)$ is the number of additions required for prefiltering. In (h) N is the required number of additions for implementing cascade- and direct-form 2PFIR filters

N	L_e	δ/b
20	20	27.5
22	22	27.7
24	24	29.2
26	26	29.6
28	28	30.5
30	30	32.2

(a) $P_1(z)$ cascaded with 2PFIR equalizers

N	L_e	δ/b
20	19	25.8
22	21	26.7
24	23	26.8
26	25	28.1
28	27	28.4
30	29	28.4

(b) $P_2(z)$

N	L_e	δ/b
20	20	22.5
22	22	22.9
24	24	23.9
26	26	25.2
28	28	25.5
30	30	25.8

(c) $P_3(z)$

N	L_e	δ/b
20	18	27.7
22	20	28.9
24	22	32.7
26	24	32.8
28	26	33.1
30	28	35.6

(d) $P_4(z)$

N	L_e	δ/b
20	19	26.0
22	21	26.5
24	23	28.5
26	25	28.5
28	27	28.9
30	29	29.0

(e) $P_5(z)$

N	L_e	δ/b
20	20	22.5
22	22	22.9
24	24	23.9
26	26	25.2
28	28	25.5
30	30	25.8

(f) $P_6(z)$

N	L_e	δ/b
20	17	26.9
22	29	30.1
24	21	33.0
26	23	33.4
28	25	33.4
30	27	35.2

(g) $P_7(z)$

	δ/b		
N	Direct	Cascade from [3]	Cascade from [4]
20	18.0	24.8	23.3
22	19.0	28.3	24.7
24	19.3	29.1	27.2
26	19.7	29.2	27.7
28	20.8	30.4	29.5
30	20.9	32.2	32.8

(h) Cascade- and direct-form 2PFIR filters

Table 2. Normalized peak weighted ripple values of the bandpass prefilter-2PFIR equalizers and direct-form 2PFIR filters.

N	L_e	δ/b
20	20	28.7
22	22	31.4
24	24	31.7
26	26	32.2
28	28	32.2
30	30	32.2

(a) $P_1(z)$ cascaded with 2PFIR equalizers

N	L_e	δ/b
20	20	20.4
22	22	21.7
24	24	23.5
26	26	24.6
28	28	24.9
30	30	26.6

(b) $P_2(z)$

N	L_e	δ/b
20	19	21.4
22	21	21.7
24	23	23.4
26	25	23.7
28	27	24.2
30	29	25.1

(c) $P_3(z)$

N	L_e	δ/b
20	20	27.3
22	22	27.9
24	24	28.2
26	26	29.2
28	28	29.6
30	30	30.5

(d) $P_4(z)$

N	L_p	δ/b
20	18	27.1
22	20	27.4
24	22	27.4
26	24	30.2
28	26	30.3
30	28	33.9

(e) $P_5(z)$

N	L_p	δ/b
20	20	24.4
22	22	26.0
24	24	26.1
26	26	27.6
28	28	27.8
30	30	28.7

(f) $P_6(z)$

N	L_p	δ/b
20	19	29.1
22	21	30.9
24	23	31.9
26	25	32.2
28	27	32.8
30	29	34.9

(g) $P_7(z)$

N	δ/b
20	23.2
22	23.4
24	25.1
26	25.1
28	25.5
30	26.4

(h) Direct-form 2PFIR filters

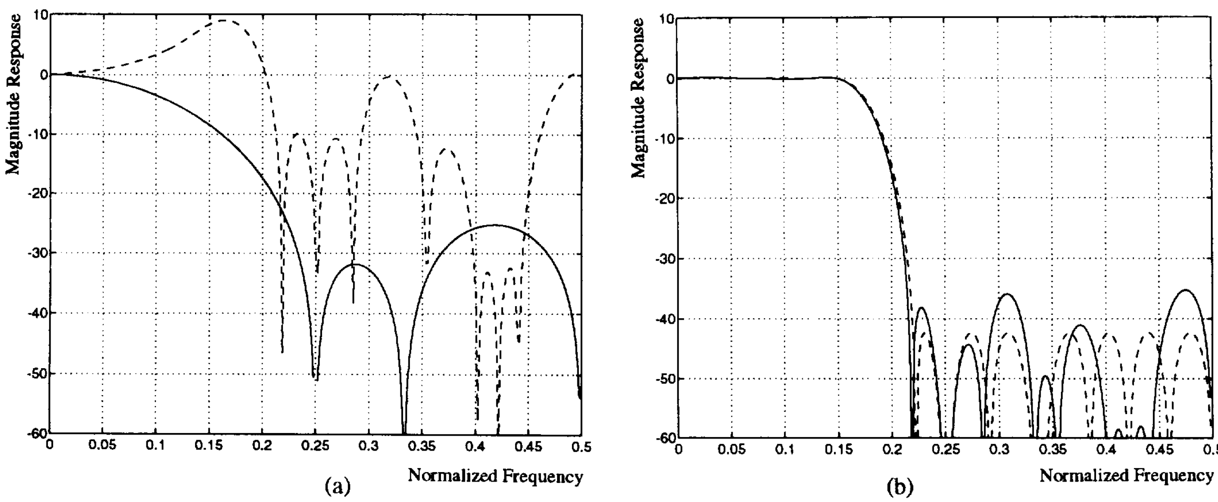

Figure 3. Magnitude response of lowpass (a) prefilter(sold line) and 2PFIR equalizer(dotted line).
(b) The cascaded filter performance comparison between 2PFIR(solid line)
and infinite precision(dotted line) equalizer

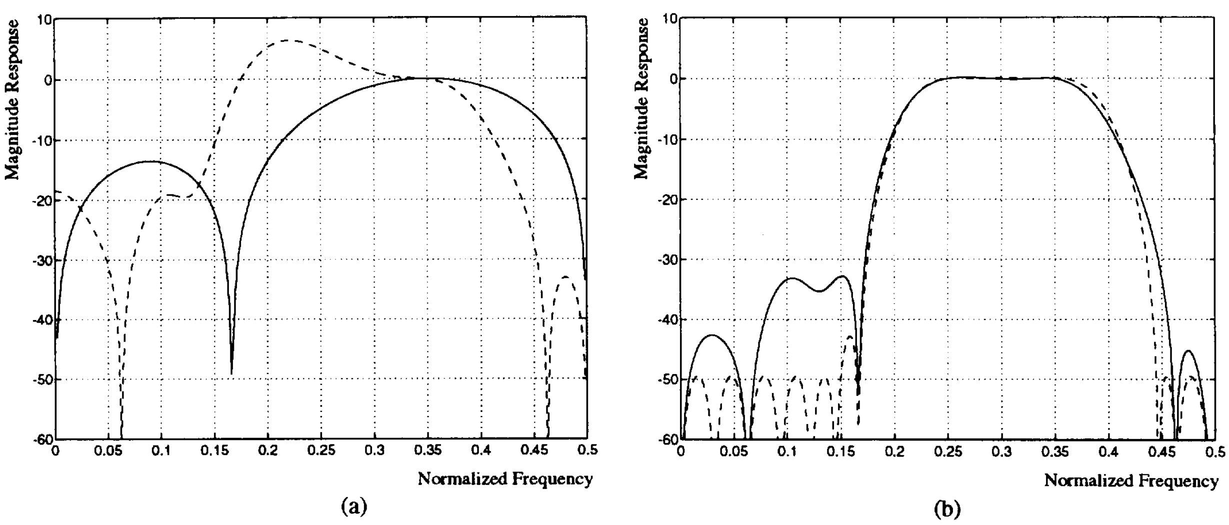

Figure 4. Magnitude response of bandpass (a) prefilter(sold line) and 2PFIR equalizer(dotted line).
(b) The cascaded filter performance performance comparison between 2PFIR(solid line)
and infinite precision(dotted line) equalizer