A Variable Rate LDPC Coded V-BLAST System

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***Abstract*— In this paper, we propose V-BLAST system based on variable rate LDPC codes to improve performance of receiver, when QR decomposition interference suppression combined with interference cancellation is used over independent Rayleigh fading channel. The different rate LDPC codes can be made by puncturing some rows of a given parity check matrix. The performance can be improved by assigning stronger LDPC codes in lower layer than upper layer, because the poor SNR of first detected data streams makes error propagation. Keeping the same overall code rates, the V-BLAST system with different rate LDPC codes has better performance (in terms of Bit Error Rate) than with constant rate LDPC codes. The BER performance has been evaluated in independent Rayleigh fading channel through computer simulation.**

I. INTRODUCTION

In wireless communications environment, the use of multiple antennas at the transmitter and receiver are known to provide higher capacity and therefore higher throughput than single link wireless communication system [1]. Several schemes have been proposed to employ the potential of Multiple Input Multiple Output (MIMO) system which includes Space-Time Block Coding (STBC), Space-Time Trellis Coding (STTC) and Layered Space-Time (LST) architectures. The Bell-laboratory LAyered Space-Time (BLAST) architecture proposed by Foschini in [2] is one of the transmit-receiver architectures using spatial and temporal diversity. The first BLAST architecture proposed in the literature is Diagonal-BLAST (D-BLAST) [2] that maximizes the throughput by diagonally transmitting independent substreams using multiple antennas. D-BLAST can obtain both spatial multiplexing gain and transmit/receive diversity gain, but it has performance loss by redundancy and is too complex to implement due to diagonal processing. On the other hand, Vetical-BLAST (V-BLAST) a simplified version [3] of the BLAST architecture is commonly used in MIMO systems due to the fact that it has relatively low complexity. It provides spatial multiplexing gain and receiver diversity gain.

Low Density Parity Check (LDPC) codes [6], first proposed by Gallager in the 1960s, and later rediscovered by MacKay and Neal [7], [8], have been of great academic interest recently. LDPC codes can achieve near Shannon limit error performance [9] and represent a very promising prospect for error control coding. LDPC codes appear as a class of codes which can yield very good performance on the binary symmetric channel (BSC) as well as on the additive white Gaussian noise (AWGN) channel. In the AWGN channel, a rate one-half LDPC code of block length 10^4 requires an E_b/N_0 of roughly 1.4 dB to achieve a bit error probability of 10^{-5}.

The (n, j, k) binary LDPC codes which were presented by Gallager [6] are specified by a sparse parity check matrix containing mostly zeros and only a relatively small number of ones. An (n, j, k) LDPC code is a block code of length n with a parity check matrix $\mathbf{H}$, where each column consists of j ones and each row consists of k ones (where $k > j$). Since both j and k are small compared to the length of the codeword, the density of $\mathbf{H}$ is very low. For this reason, $\mathbf{H}$ is said to be a low density parity check matrix and its kernel space is called an LDPC code. The LDPC code defined as above is called a regular LDPC code. If not all the rows or all the columns of the parity check matrix $\mathbf{H}$ have the same number of ones, an LDPC code is said to be irregular. In general, the BER performance of an irregular LDPC code surpass a regular LDPC code and Turbo code.

In this paper, we use LDPC codes for multiple antenna systems over a flat fading channel. We consider a simple detection technique which is called the QR decomposition interference suppression combined with interference cancellation [5]. The channel matrix $\mathbf{G}$ can be factored by the QR decomposition into an orthonormal matrix $\mathbf{Q}$ and an upper triangular matrix $\mathbf{R}$ such that $\mathbf{G} = \mathbf{QR}$. When QR decomposition interference suppression combined with interference cancellation is used over independent Rayleigh fading channel, V-BLAST suffers from the problem of error propagation [3] resulting in the poor SNR of first detected data streams. This motivates us to assign LDPC codes with different code rate to each antennas. Error propagation can be reduced by assigning stronger LDPC codes in lower layer. Therefore we can expect that the performance will be improved.

The remainder of the paper is organized as follows. After introducing the system model of proposed system in Section II, we describe the construction of variable rate LDPC codes in Section III. Then a variable rate LDPC coded V-BLAST system is presented in Section IV. In Section V, we discuss simulation results and conclusions are given in Section VI.

II. SYSTEM MODEL

We consider MIMO system with n_T transmit and n_R receive antennas in a flat Rayleigh fading channel as shown in Fig. 1. It is assumed uncorrelated Rayleigh fading and channel transfer function is unknown at the transmitter but known to the receiver. Also total transmitter power P is equally divided

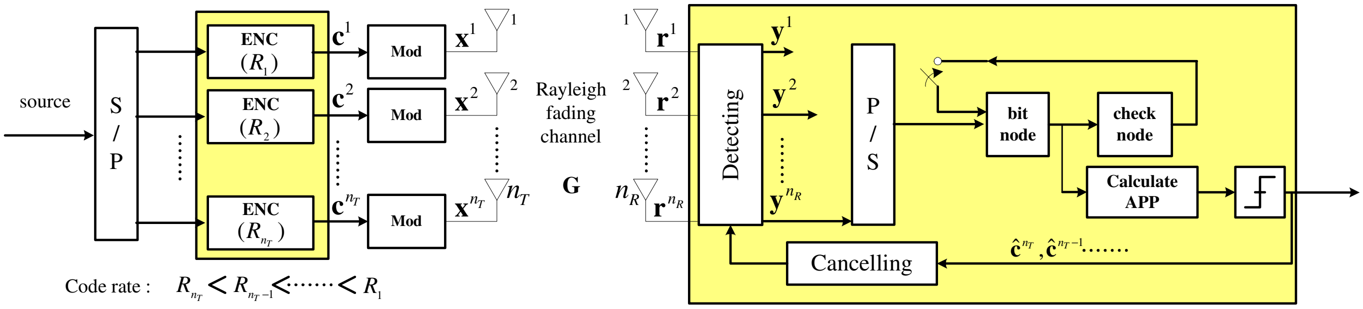

Fig. 1. Block diagram of (n_T, n_R) V-BLAST

at each transmit antenna. Therefore the received signal can be described as follows

$$\mathbf{r} = \sqrt{\frac{P}{n_T}}\mathbf{G}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{r}$ is a vector of the received signal samples at the output of the receiver, $\mathbf{G}$ is a $n_R \times n_T$ channel gain matrix having identically independent complex distribution which has zero-mean, unit variance and $\mathbf{n}$ is a complex n_R dimension Gaussian with zero mean and variance of N_0.

At the trasmitter, the primitive data streams are demultiplexed into n_T data streams of different length. And each data stream is encoded by LDPC codes with various code rate. Each encoded data stream is modulated and then is dispersed along vertical in space-time.

III. The Construction of Variable Rate LDPC Codes

Consider a parity check code of length N as a code whose codewords all satisfy a set of M constraints. Such a code is uniquely defined by its $M \times N$ parity check matrix $\mathbf{H}$. A low-density parity check code is defined by a parity check matrix that is sparse [6]. A (j,k) regular LDPC code has an $M \times N$ parity check matrix having exactly j ones in each column and exactly k ones in each row, where $j < k$ and both are small compared to N. An $M \times N$ regular (j,k) LDPC code can often (but not always) be conveniently expressed in terms of the following shorter matrix H_0. We can construct a regular (j,k) LDPC code by stacking j column permutations of $\mathbf{H}_0$ one atop another:

$$\mathbf{H}_0 = \begin{bmatrix} \underbrace{111\cdots1}_{k} & & & \\ & \underbrace{111\cdots1}_{k} & & \\ & & \ddots & \\ & & & \overbrace{111\cdots1}^{k} \end{bmatrix} \tag{2}$$

$$\mathbf{H} = \left[\ \pi_1(\mathbf{H}_0)\ |\ \ \pi_2(\mathbf{H}_0)\ |\ \ \cdots |\ \ \pi_j(\mathbf{H}_0)\ \right]^T, \tag{3}$$

where $\pi_i(\mathbf{H}_0)$ denotes a matrix whose columns are a permuted version of the columns of $\mathbf{H}_0$. By greedy algorithm, the parity check matrix $\mathbf{H}$ can be transformed into the approximated lower triangular form as Fig. 2. An irregular LDPC code is a generalization of LDPC codes for which the parity-check matrix is still sparse, but for which not all rows have the same weight and not all columns have the same weight. In decoding of LDPC codes, the Belief Propagation (BP) algorithm is commonly used. This algorithm performs iteratively the update process with the extrinsic information. Note that if the weights of row and column of a parity check matrix are less than 2, then such a matrix is not suitable for BP algorithm. We produce higher rate LDPC codes by using the submatrices of "mother" parity check $\mathbf{H}$. These new matrices have the weights of all columns and rows are greater than and equal to 2. In general, these new LDPC codes are irregular LDPC codes with the same block length N.

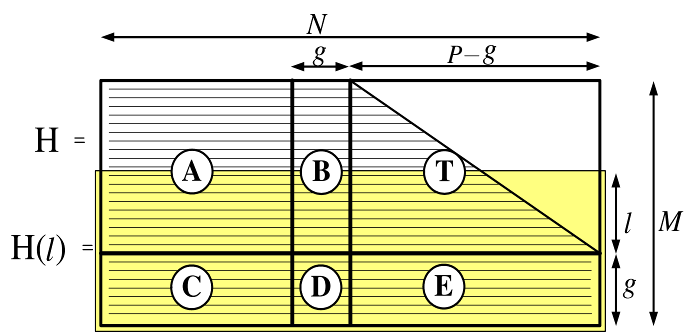

Fig. 2. Construction of variable rate LDPC codes

Let l be the design factor and P be the length of parity part of the codeword. For any $0 < l \le M - g$, we can define the submatrix $\mathbf{H}(l)$ of $\mathbf{H}$ as shown in Fig. 2. The code rate $R(l)$ of the code generated by $\mathbf{H}(l)$ can be calculated as following:

$$R(l) = \frac{N - (g + l)}{N}. \tag{4}$$

IV. A Variable Rate LDPC coded V-BLAST System

The transmitted signal can be represented by

$$\mathbf{x} = \left[\ \mathbf{x}^1\quad \mathbf{x}^2\quad \cdots\quad \mathbf{x}^{n_T}\ \right]^T, \tag{5}$$

where each $\mathbf{x}^k$ with block length L represents the transmitted signal with code rate of R_k at antenna k. Therefore transmitted signal can be rewritten as

$$\left[\begin{array}{cccc} x_0^1 x_1^1 \cdots x_{L-1}^1, & x_0^2 x_1^2 \cdots x_{L-1}^2, & \cdots, & x_0^{n_T} x_1^{n_T} \cdots x_{L-1}^{n_T} \end{array}\right]^T. \quad (6)$$

V-BLAST receiver processing consists of nulling signal and cancelling interference. Assuming that the received signal is $\mathbf{r} = \sqrt{\frac{P}{n_T}}\mathbf{G}\mathbf{x} + \mathbf{n}$, $\sqrt{\frac{P}{n_T}}\mathbf{G}$ can be factored into $\mathbf{QR}$ by QR decomposition. Multiplying Hermitian matrix of $\mathbf{Q}$ by received signal, the nulled signal $\mathbf{y}$ can be represented by

$$\mathbf{y} = \mathbf{Q}^H \mathbf{r} = \mathbf{R}\mathbf{x} + \mathbf{v}, \quad (7)$$

where $(\cdot)^H$ is Hermitian operation, $\mathbf{Q}$ is a unitary matrix and $\mathbf{R}$ is a upper triangular matrix,

$$\mathbf{R} = \begin{bmatrix} R_t^{1,1} & R_t^{1,2} & \cdots & R_t^{1,n_T} \\ 0 & R_t^{2,2} & & R_t^{1,n_T} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & R_t^{n_T,n_T} \end{bmatrix}. \quad (8)$$

Since $\mathbf{Q}$ is a unitary matrix, the statistical properties of the noise term $\mathbf{v} = \mathbf{Q} \cdot \mathbf{n}$ is unchanged and t-th symbol of $\mathbf{y}$ becomes:

$$y_t^k = R_t^{k,k} x_t^k + v_t^k + \sum_{i=k+1}^{n_T} R_t^{k,i} x_t^i, \quad (9)$$

where $R^{i,j}$ is (i,j)th element of matrix $\mathbf{R}$ and v_t^k is white Gaussian noise. This received signal consists of the current desired signal plus noise and interference from other transmit antennas. In uncoded V-BLAST, from Eq. (9) the estimate on the transmitted symbol x_t^k is given by

$$\hat{x}_t^k = Z\left(\frac{y_t^k - \sum_{i=k+1}^{n_T} R_t^{k,i} \hat{x}_t^i}{R_t^{k,k}}\right) \text{ and} \quad (10)$$

$$\hat{\mathbf{x}}^k = \left[\begin{array}{cccc} \hat{x}_0^k & \hat{x}_1^k & \cdots & \hat{x}_{L-1}^k \end{array}\right], \quad (11)$$

where $Z(\cdot)$ denotes the hard decision [5].

In LDPC coded V-BLAST schemes, we have used LDPC decoder applying the Belief Propagation algorithm instead of the hard decision $Z(k)$.

The following steps are composed of signal detection in LDPC coded V-BLAST schemes.

- step 1. Calculate soft decision values through cancelling process as follows

$$\tilde{x}_t^k = \frac{y_t^k - \sum_{i=k+1}^{n_T} R_t^{k,i} \hat{x}_t^i}{R_t^{k,k}}, \quad (12)$$

  where $k = n_T$. Since $\mathbf{R}$ is an upper triangular matrix, it is obvious that the nulled signal $y_t^{n_T}$ at n_Tth receive antenna has no interference and has only the effect of fading.
- step 2. Accumulate L symbols (block length = $L \log_2 K$) to process LDPC decoding

$$\tilde{\mathbf{x}}^k = \left[\begin{array}{cccc} \tilde{x}_0^k & \tilde{x}_1^k & \cdots & \tilde{x}_{L-1}^k \end{array}\right], \quad (13)$$

- step 3. Perform LDPC decoding applying the BP algorithm

$$\hat{\mathbf{c}}^k = Q\left(\tilde{\mathbf{x}}^k\right), \quad (14)$$

  where $Q(\cdot)$ denotes LDPC decoder.
- step 4. Modulate the LDPC decoded value to perform cancelling process

$$\hat{\mathbf{x}}^k = M\left(\hat{\mathbf{c}}^k\right), \quad (15)$$

  where $M(\cdot)$ denotes modulation.
- step 5. $k = k - 1$ and step 1 is repeated until $k = 1$.

Therefore the detecting of $\mathbf{x}^{n_T}$ is first performed and then $\mathbf{x}^{n_T-1}$ and so on. Due to the cancelling operation, the poor SNR of the first detected data stream can make possible resulting error propagation. Also the degree of freedom in diagonal of $\mathbf{R}$ decreases as k increases. The error propagation can be reduced if an error correcting capability of the lower layer is stronger than that of the upper layers. Therefore we use V-BLAST system by allocating different code rate to each layer under same throughput.

V. Simulation Results

In this section, we present simulation results of a variable rate LDPC coded V-BLAST system. We consider channel environment in independent Rayleigh fading changing every symbol. We also have used QPSK modulation. Detection scheme is QR decomposition interference suppression combined interference cancellation. We have used randomly generated LDPC codes [8] and set the maximum iteration number of iterations to 50 in our simulation. A constant rate 0.625 regular LDPC codes for V-BLAST is used. In case of $n_T = n_R = 2$, code rates are 0.665, 0.58 for transmit antennas 1 and 2 respectively. For the system with $n_T = n_R = 4$, code rates are 0.75, 0.665, 0.58, 0.5 for transmit antenna 1, 2, 3 and 4 respectively. Fig.3 shows bit error rate (BER) performance of the system considered with block length 200 at $n_T = n_R = 2$ and $n_T = n_R = 4$. In case of $n_T = n_R = 2$, V-BLAST system based variable rate LDPC codes is 1.2 dB better in E_b/N_0 than with constant rate for BER of 10^{-4}. Also for $n_T = n_R = 4$, system with variable rate is 1.6 dB better in E_b/N_0 than with constant rate. And BER performance of the system with block length 1008 is shown in Fig.4. For $n_T = n_R = 4$ and $n_T = n_R = 2$ the performance improvement of E_b/N_0 is about 1.4 dB, 1.1 dB in E_b/N_0 for BER of 10^{-4}. This shows that error propagation is reduced by employing variable rate LDPC codes under same throughput.

VI. Conclusion

We have used a variable rate LDPC codes based V-BLAST system when using method of QR decomposition interference suppression combined with interference cancellation in a flat fading channel. Due to the cancelling process, the poor SNR of the first detected data stream makes possible resulting error propagation. In order to prevent error propagation of detected data streams and obtain more reliable data streams in lower layer, LDPC codes with variable code rate is employed. The

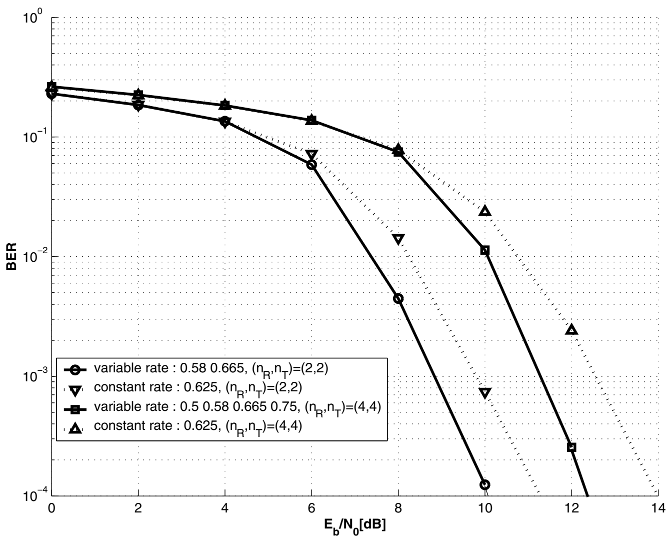

Fig. 3. The BER performance comparison between variable and constant rate LDPC code based V-BLAST with $n_T = n_R = 2$ and $n_T = n_R = 4$ (block length=200)

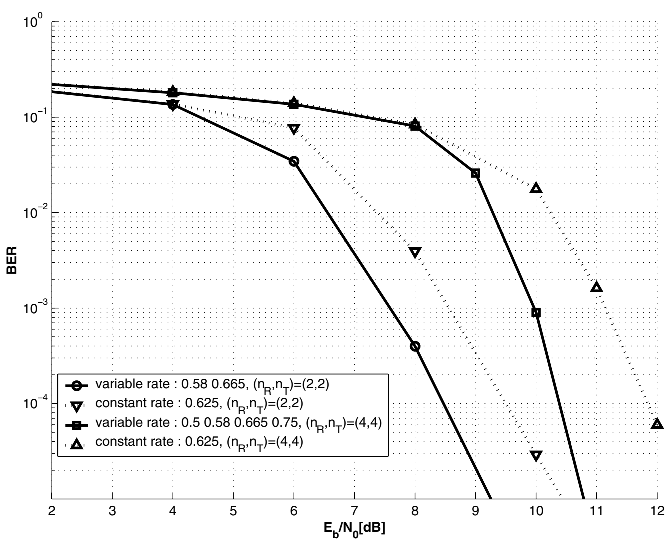

Fig. 4. The BER performance comparison between variable and constant rate LDPC code based V-BLAST with $n_T = n_R = 2$ and $n_T = n_R = 4$ (block length=1008)

error propagation is reduced by allocating stronger codes in the lower layer than that of the upper layers under same throughput. Simulation results show that the performance of V-BLAST system based variable rate LDPC codes is 1.6 dB (block length = 200), 1.4 dB (block length = 1008) better in E_b/N_0 than with constant rate. We summarize our simulation results in Table. I.

TABLE I
PERFORMANCE IMPROVEMENT

(n_T, n_R)	block length = 200	block length = 1008
$(2,2)$	1.2 dB	1.1 dB
$(4,4)$	1.6 dB	1.4 dB

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