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A Signal Detector for Weak Composite Signals in Additive and Signal-Dependent Noise

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Abstract

When the noise has both additive and signal-dependent components, locally optimum detector test statistics are obtained for detection of weak composite signals using the generalized Neyman-Pearson lemma. In order to consider the non-additive noise as well as purely-additive noise, a generalized observation model is used in this paper. The locally optimum detector test statistics are derived for several different cases according to the relative strengths of the known signal component, the random signal component, and the signal-dependent noise component. Schematic diagrams of the locally optimum detector structures are also included.

1. Introduction

In various areas of signal processing research including study of signal detection problems, the purely-additive noise (PAN) model has been most widely used because the PAN model is relatively easy to handle mathematically and to obtain explicit structures for detection processors in a variety of applications [1,2]. Moreover, the PAN model produces quite acceptable results in many cases, where the level of the contribution of higher order statistics or of nonlinearity is not very significant.

There are some other cases, however, in which we are forced to use a non-additive noise model to produce more realistic and reasonable approximations [3,4]. For example, the effects of delayed signals from multipath or reverberation phenomena and the actions of automatic gain control circuits or of nonlinearities acting on additive signal and noise components may all be modeled using non-additive (*e.g.*, signal-dependent) as well as purely-additive noise components.

Locally optimum (LO) detectors, which are known to be optimum when the signal strength approaches zero, have bases in the generalized Neyman-Pearson lemma of statistical hypothesis testing [5,6]. Instead of maximizing the detection probability, the LO detectors maximize the slope of the power function for a given false-alarm probability. The LO detector structure is in addition easier to implement than that of other detectors including uniformly most powerful (UMP) and optimum detectors.

In this paper, we employ a generalized observation model in which the effects of both signal-dependent and purely-additive noise components may simultaneously be reflected. In addition, the signal is assumed to be composite signals which have both known (deterministic) and random (stochastic) signal components. The purpose of this paper is thus to obtain the test statistics of the LO detector for detection of composite signals in the signal-dependent noise model.

2. An Observation Model

2.1. The model

The widely-used observation model including only PAN may be described by

$$X_i = \theta Q_i + W_i, \quad i=1,2,\cdots,n, \tag{2.1}$$

where θ is a signal strength parameter, Q_i is either a known signal component or a random signal component, and W_i is the PAN. Although the PAN model has been widely used, there are some cases where the PAN model is not an appropriate approximation to the mechanism producing noisy observations as discussed in Introduction.

Let us now introduce a more general and realistic observation model which may be used in a broader range of situations. Let us consider the model describing the observations X_i for $i=1,2,\cdots,n$, by

$$X_i = \alpha(\tau)e_i + \beta(\tau)S_i + \gamma(\tau)N_i + W_i. \tag{2.2}$$

In (2.2), e_i is the known signal component and S_i is the random signal component with known probability density function (pdf) at the i-th sampling instant. The random signal component S_i is a zero mean random variable which has variance σ_i^2 and pdf f_{S_i}, $i=1,2,\cdots,n$. The functions $\alpha(\tau)$ and $\beta(\tau)$ are the signal strength functions of the known signal components and the random signal components, respectively. The additional term $\gamma(\tau)N_i$ is a signal-dependent noise term with amplitude $\gamma(\tau)$, where the parameter τ also controls the signal strengths through $\alpha(\tau)$ and $\beta(\tau)$. We will assume that $\alpha(\tau)$, $\beta(\tau)$, and $\gamma(\tau)$ are nondecreasing functions of $\tau>0$ and that $\alpha(0)=\beta(0)=\gamma(0)=0$. The signal-dependent noise sequence $\{N_i\}_{i=1}^n$ and the additive noise sequence $\{W_i\}_{i=1}^n$ are both assumed to be independent and identically distributed (i.i.d.) random variables with pdfs f_N and f_W, respectively. It is also assumed that $\{N_i\}_{i=1}^n$ and $\{S_i\}_{i=1}^n$ are independent. Finally we will denote by f_{NW} the common joint pdf of the (N_i, W_i), which are i.i.d. bivariate random variables for $i=1,2,\cdots,n$.

2.2. Hypotheses and definitions

With the observation model (2.2), it is now possible to express our problem of composite signal detection by a statistical hypothesis testing problem of choosing between a null hypothesis H_0 and an alternative hypothesis H_1. More specifically, under H_0 we have $\tau=0$ or

$$H_0 : X_i = W_i, \quad i=1,2,\cdots,n, \tag{2.3}$$

and under H_1 we have $\tau>0$ or

$$H_1 : X_i = \alpha(\tau)e_i + \beta(\tau)S_i + \gamma(\tau)N_i + W_i, \quad i=1,2,\cdots,n. \tag{2.4}$$

Before we proceed further with these hypothesis, let us introduce some definitions for notational convenience. Let us define LO nonlinearities as

$$g_1(x) = -\frac{f_W'(x)}{f_W(x)}, \tag{2.5}$$

$$g_2(x) = -\frac{u'(x)}{f_W(x)}, \tag{2.6}$$

$$h_1(x) = \frac{f_W''(x)}{f_W(x)}, \tag{2.7}$$

and

$$h_3(x) = \frac{v''(x)}{f_W(x)}, \tag{2.8}$$

where

$$u(x) = \int n\, f_{NW}(n,w)\, dn \tag{2.9}$$

$$= f_W(x) E\{N \mid W=x\}$$

and

$$v(x) = \int n^2 f_{NW}(n,w)\, dn \tag{2.10}$$

$$= f_W(x) E\{N^2 \mid W=x\}$$

are weighted conditional mean function and weighted conditional variance function.

2.3. Reparametrization of the model

In deriving the LO detector test statistics, it is convenient to reparametrize the observation model (2.2). Because of the assumptions on $\alpha(\tau)$, $\beta(\tau)$, and $\gamma(\tau)$ that they are nondecreasing functions of $\tau>0$ with values 0 at $\tau=0$, we have

$$\lim_{\tau\to 0^+} \frac{\alpha(\tau)}{\delta\tau^p} = 1, \tag{2.11}$$

$$\lim_{\tau\to 0^+} \frac{\beta(\tau)}{\varepsilon\tau^q} = 1, \tag{2.12}$$

and

$$\lim_{\tau\to 0^+} \frac{\gamma(\tau)}{\eta\tau^r} = 1, \tag{2.13}$$

where $p, q, r, \delta, \varepsilon$, and η are all positive numbers. With the six numbers defined by (2.11)-(2.13) let us define two parameters Δ_1 and Δ_2 as follows:

$$\Delta_1 = \frac{p}{r} \tag{2.14}$$

and

$$\Delta_2 = \frac{p}{q}. \tag{2.15}$$

The values of Δ_1 and Δ_2 indicate the strength of the known signal component with respect to that of the signal-dependent noise component and the strength of the known signal component with respect to that of the random signal component, respectively. It is thus easy to see that the parameters Δ_1 and Δ_2 would play an important role in determining if the signal-dependent noise component is dominant over the two signal components.

The reparametrization of the observation model (2.2) is accomplished by applying one of the following three rules:

A) $a(\theta)=\theta$, $b(\theta)=\beta(\tau)$, $c(\theta)=\gamma(\tau)$ with $\theta=\alpha(\tau)$,
B) $b(\theta)=\theta$, $c(\theta)=\gamma(\tau)$, $a(\theta)=\alpha(\tau)$ with $\theta=\beta(\tau)$,
or
C) $c(\theta)=\theta$, $a(\theta)=\alpha(\tau)$, $b(\theta)=\beta(\tau)$ with $\theta=\gamma(\tau)$.

Application of a specific reparametrization rule among the above three rules is determined according to the values of Δ_1 and Δ_2 as follows:

Case 1 : $\Delta_2<2$

i) When $\Delta_1 \le 1$, we apply reparametrization rule A).
ii) When $\Delta_1 \ge 2$, we apply reparametrization rule C).
iii) When $1<\Delta_1<2$, we apply reparametrization rule A) if $E\{N \mid W\} \equiv 0$ and reparametrization rule C) if $E\{N \mid W\} \ne 0$.

Case 2 : $\Delta_2 \ge 2$

i) When $\Delta_2 \ge 2\Delta_1$, we apply reparametrization rule B).
ii) When $\Delta_2 \le \Delta_1$, we apply reparametrization rule C).
iii) When $\Delta_1<\Delta_2<2\Delta_1$, we apply reparametrization rule B) if $E\{N \mid W\} \equiv 0$ and reparameterization rule C) if $E\{N \mid W\} \ne 0$.

Above reparametrization of the observation model (2.2) does not change the structure of the LO detector for composite signal detection; rather it relieves us from unnecessarily excessive mathematical operations when we derive the test statistics of the LO detectors as explained in [7,8,9].

In the observation model after the reparametrization, the observation X_i is represented by

$$X_i = a(\theta)e_i + b(\theta)S_i + c(\theta)N_i + W_i, \tag{2.16}$$

where at least one of the three amplitude functions $a(\theta)$, $b(\theta)$, and $c(\theta)$ is θ.

3. Detector Test Statistics and Structures

3.1. Test statistics

Since we can find the detailed discussions on the general theory of LO detection in many other studies, we will proceed here directly to the derivation of the LO detector test statistics. Because the noise and random signal components are assumed to be independent, the joint pdfs of the observation set are

$$f_0(x) = \prod_{i=1}^{n} \int f_{NW}(n_i, x_i)\, dn_i \tag{3.1}$$

under H_0 and

$$f_1(x) = \int f_S(s) \prod_{i=1}^{n} \int f_{NW}(n_i, x_i - (a(\theta) e_i + b(\theta) s_i + c(\theta) n_i)) \, dn_i \, ds \tag{3.2}$$

under H_1, where f_S is the joint pdf of $S_1, S_2, \cdots, S_n$. Applying the generalized Neyman-Pearson lemma, we get the test statistic of the LO detector,

$$T_{LO}(x) = \frac{f_1^{(\nu)}(x)|_{\theta=0}}{f_0(x)}, \tag{3.3}$$

where ν is the first non-zero derivative of $f_1(x)$ at $\theta=0$.

Using (3.3) the test statistics of the LO detectors for the observation model (2.16) are obtained to be as follows.

1) When $\Delta_2 < 2$ or when $\Delta_2 \geq 2$ and $\Delta_1 > \Delta_c$, the test statistic is

$$T_{LO}(X) = \sum_{i=1}^{n} \{e_i \lambda_1(X_i) + \lambda_2(X_i)\}. \tag{3.4}$$

2) When $\Delta_2 \geq 2$ and $\Delta_1 \leq \Delta_c$, the test statistic is

$$T_{LO}(X) = \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} K_S(i,j) g_1(X_i) g_1(X_j) + \sum_{i=1}^{n} \{\sigma_i^2 h_1(X_i) + e_i \lambda_1(X_i) + \lambda_2(X_i)\}, \tag{3.5}$$

where

$$\lambda_1(x) = \begin{cases} \frac{2\delta}{\varepsilon^2} g_1(x), & \text{when } \Delta_2 \leq 2 \text{ and } \Delta_1 \leq \Delta_c \\ 0, & \text{when } \Delta_2 > 2 \text{ or when } \Delta_1 > \Delta_c \end{cases} \tag{3.6}$$

and

$$\lambda_2(x) = \begin{cases} \frac{2\eta}{\varepsilon^2} g_2(x), & \text{when } E\{N \mid W\} \neq 0 \text{ and } \Delta_1 \geq 1, \\ \frac{\eta^2}{\varepsilon^2} h_3(x), & \text{when } E\{N \mid W\} \equiv 0 \text{ and } \Delta_1 \geq 2, \\ 0, & \text{when } \Delta_1 < \Delta_c \end{cases} \tag{3.7}$$

with

$$\Delta_c = \begin{cases} 1, & \text{when } E\{N \mid W\} \neq 0, \\ 2, & \text{when } E\{N \mid W\} \equiv 0. \end{cases} \tag{3.8}$$

The results 1) and 2) are tabulated in Table 1, where we showed the test statistics in terms of the functions g_1, h_1, λ_1, and λ_2.

From 1) to 2), we can make the following observations:

(a) When $\Delta_2 < 2$ and $\Delta_1 < \Delta_c$, we observe that the LO test statistic is exactly the same as the known signal LO detector test statistic [1]. When $\Delta_2 > 2$ and $\Delta_1 < \Delta_c$, on the other hand, the LO detector test statistic is exactly the same as that for the random signal LO detector test statistic [2]. It is also observed that when $\Delta_1 = \Delta_c$ or when $\Delta_2 = 2$ and $\Delta_1 \leq \Delta_c$, the test statistic is a combined form of these three test statistics. For example, when $E\{N \mid W\} \neq 0$, $\Delta_1 = 1$, and $\Delta_2 = 2$ (*e.g.*, (p,q,r) = (2,1,2)) or when $E\{N \mid W\} \equiv 0$, $\Delta_1 = 1$, and $\Delta_2 = 2$ (*e.g.*, (p,q,r) = (2,1,1)), the known signal components, the random signal components, and the signal-dependent noise components have effects on the test statistics.

(b) The critical value of Δ_2, from which we can say whether the known signal components are dominant or the random signal components are dominant, is 2. In other words, when $\Delta_2 < 2$ the known signal components are relatively strong, and when $\Delta_2 > 2$ the random signal components are dominant. When $\Delta_2 = 2$ both the known signal components and the random signal components have effects on the LO detector test statistic.

(c) The critical value of Δ_1, from which we can say whether the signal components are dominant or the signal-dependent noise components are dominant, is Δ_c. In other words, when $\Delta_1 < \Delta_c$ the signal components are relatively strong, and when $\Delta_1 > \Delta_c$ the signal-dependent noise components are dominant. When $\Delta_1 = \Delta_c$ both the signal components and the signal-dependent noise components have effects on the LO detector test statistic.

3.2. Structures of the locally optimum detectors

Let us now show the schematic diagrams of the structures of the LO detectors obtained in Section 3.1.

3.2.1. Case 1 : When $\Delta_2 < 2$ or when $\Delta_2 \geq 2$ and $\Delta_1 > \Delta_c$

A block diagram of the structure of the LO detector in this case is shown in Figure 1. The structure of the LO detector in this case is almost the same as that of the LO detector for known signals in the PAN model.

3.2.2. Case 2 : When $\Delta_2 \geq 2$ and $\Delta_1 \leq \Delta_c$

Let us first assume that the random signal component is a white random process; that is, $K_S(i,j) = 0$ for $i \neq j$. Then the LO detector test statistic of Equation (3.5) can be simplified as

$$T_{LO}(X) = \sum_{i=1}^{n} \{\sigma_i^2 h_1(X_i) + e_i \lambda_1(X_i) + \lambda_2(X_i)\}, \tag{3.9}$$

for which a block diagram of the corresponding locally optimum detector is shown in Figure 2, where $\lambda_1(x)$ and $\lambda_2(x)$ are defined in Equation (3.6) and Equation (3.7), respectively.

To find a structure of the LO detector for the correlated signal case, let us assume that the random signal components are wide-sense stationary, and

$$K_S(i,j) = \begin{cases} K(|i-j|), & \text{for } |i-j| \leq m, \\ 0, & \text{for } |i-j| > m, \end{cases} \tag{3.10}$$

where m is some finite integer and $m \ll n$.

Let a discrete-time filter with impulse response $\{c_i, i = 0, \pm 1, \pm 2, \cdots\}$ have a frequency response $H(w)$ satisfying $|H(w)|^2 = \Phi_S(w)$ where $\Phi_S(w)$ is the signal power spectral density, then we have

$$\Phi_S(w) = \left| \sum_{l=-\infty}^{\infty} c_l e^{-jwl} \right|^2. \tag{3.11}$$

Under these assumptions it can be shown that

$$T_{LO}(X) = \sum_{j=-\infty}^{\infty} \left| \sum_{i=1}^{n} g_1(X_i) c_{j-i} \right|^2 + \sum_{i=1}^{n} [\sigma_i^2 \{h_1(X_i) - g_1^2(X_i)\} + e_i \lambda_1(X_i) + \lambda_2(X_i)]. \tag{3.12}$$

A structure of the corresponding LO detector can be obtained as in Figure 3.

4. Conclusion

In this paper, we have derived the locally optimum detector test statistics for composite signals in a generalized noisy signal model. Under the observation model we investigated the effect of the signal-dependent noise as well as that of the additive noise on the test statistics.

It was shown that the ratio of the decay parameter of the signal-dependent noise strength to that of the known signal strength together with the ratio of the decay parameter of the random signal strength to that of the known signal strength were important factors to obtain the locally optimum detector test statistics. Structures of the locally optimum detectors were obtained.

As a future investigation, it would be natural to examine the performance characteristics of the locally optimum detectors for composite signals in the signal-dependent noise model. The asymptotic performance and the finite sample-size performance of the locally optimum detectors are under investigation.

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	$\Delta_2 < 2$	$\Delta_2 \geq 2$
$\Delta_1 \leq \Delta_c$	$\sum_{i=1}^{n} \{e_i \lambda_1(X_i) + \lambda_2(X_i)\}$	$\sum\sum_{i \neq j}^{n} K_s(i,j) g_1(X_i) g_1(X_j) + \sum_{i=1}^{n} \{d_i h_1(X_i) + e_i \lambda_1(X_i) + \lambda_2(X_i)\}$
$\Delta_1 > \Delta_c$		

Table 1. The Locally Optimum Detector Test Statisitics

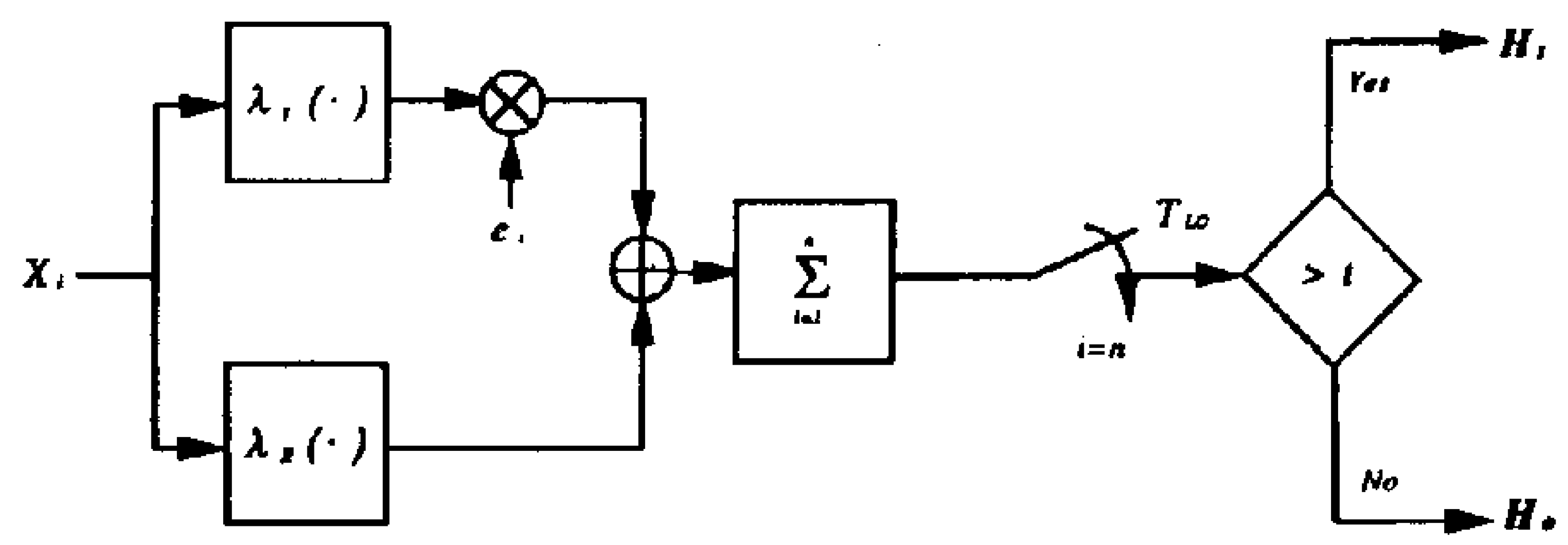

Figure 1. A Block Diagram of the Locally Optimum Detector

When $\Delta_2 < 2$ or When $\Delta_2 \geq 2$ and $\Delta_1 > \Delta_c$

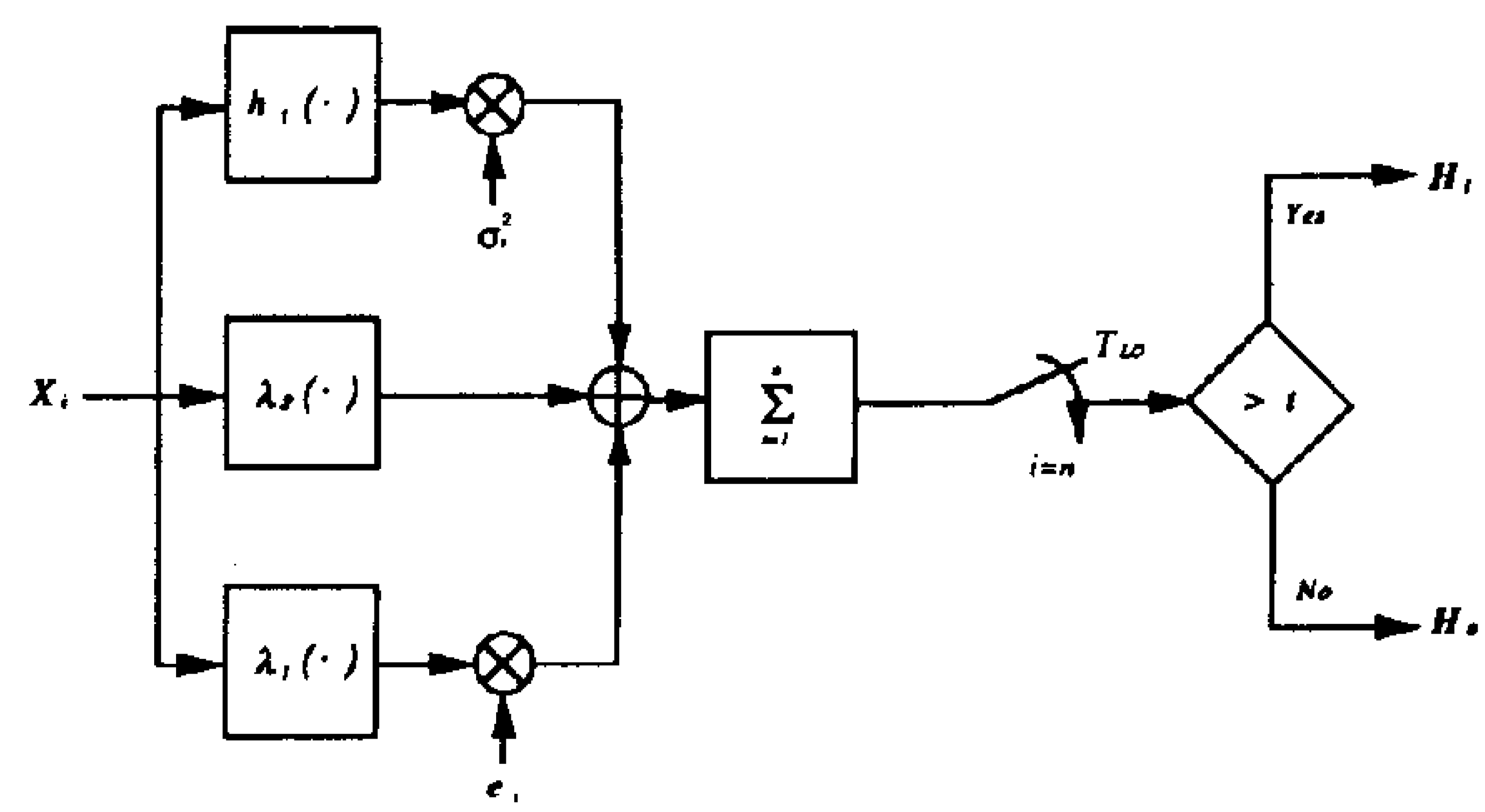

Figure 2. A Block Diagram of the Locally Optimum Detector

When $\Delta_2 \geq 2$ and $\Delta_1 \leq \Delta_c$ for White Random Signal Components

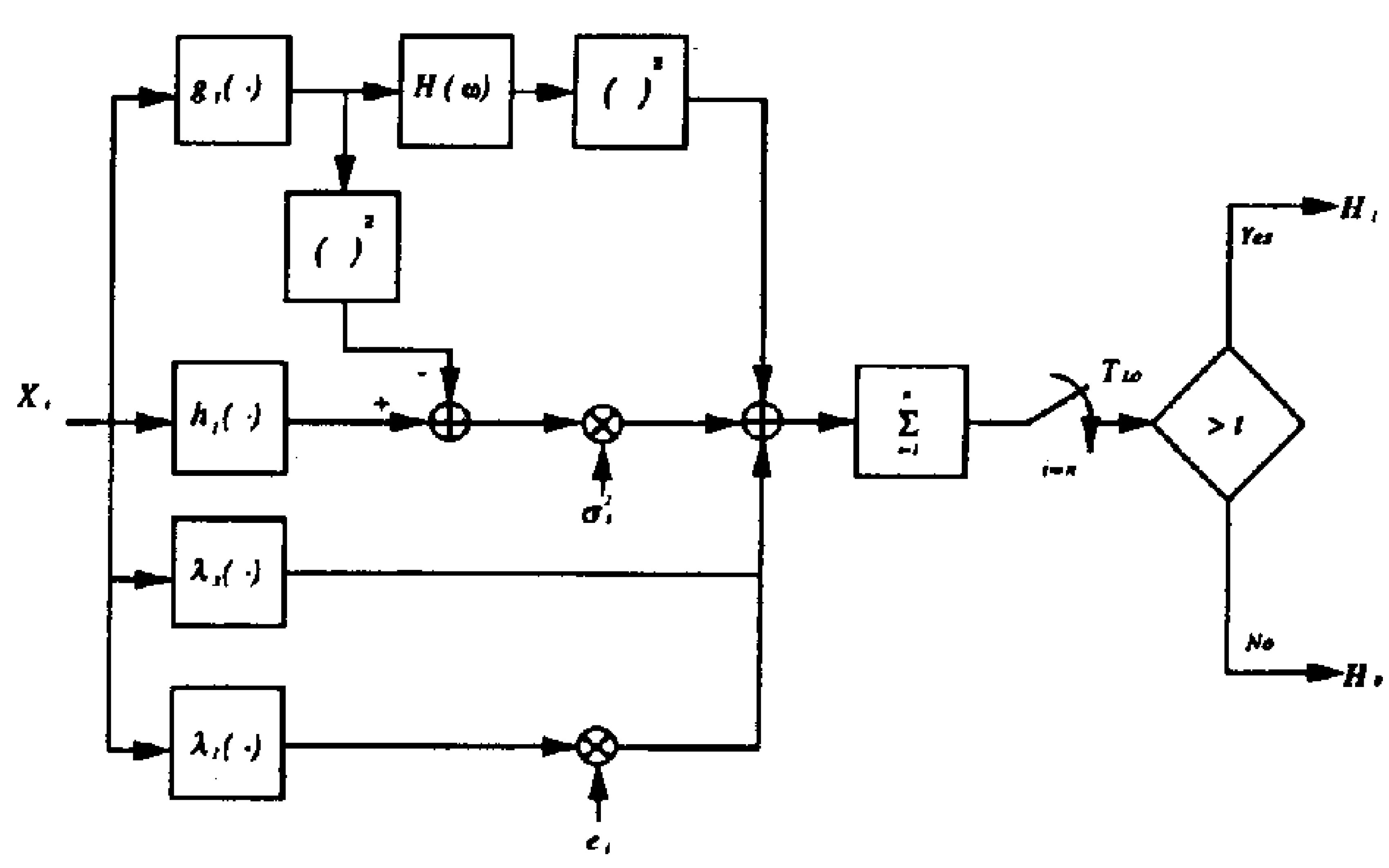

Figure 3. A block Diagram of the Locally Optimum Detector

When $\Delta_2 \geq 2$ and $\Delta_1 \leq \Delta_c$ for Correlated Random Signal Components