PERFORMANCE ANALYSIS OF GENERALIZED MODIFIED ORDER STATISTICS CFAR DETECTORS

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ABSTRACT

Generalized Order Statistic Cell Averaging (GOSCA), Generalized Order Statistic Greatest Of (GOSGO), and Generalized Order Statistic Smallest Of (GOSSO) CFAR detectors are proposed. Each of them has its own advantages according to radar environment situations so that most effective radar detector can be chosen at any unpredictive situation. Their performance formulas in terms of false alarm probability and detection probability are derived. From the performance analysis, the GOSCA CFAR detector is the best in homogeneous situation, the GOSGO CFAR detector in clutter region near the clutter edges, and the GOSSO in clear region close to the clutter boundary and interfering targets situation. A new window structure to eliminate the fatal problem in GOSSO CFAR detector in clutter regions is also proposed. The false alarm probability of the proposed window structure performance is compared with conventional window structure.

1. INTRODUCTION

Constant false alarm rate (CFAR) detector is used to regulate the false alarm probability to be a desired level in varying background environments [2]. The proposed Generalized Modified Order Statistic CFAR detectors are shown in Figure 1. The modified CFAR structure splits the reference window into two: lagging and leading windows. By selecting a proper logical function among three, this generalized CFAR detector serves as any one of GOSCA, GOSGO, and GOSSO CFAR detectors. This structure is computationally efficient. In the following section, the exact formulas for three CFAR detectors in homogeneous and nonhomogeneous environments are derived and their performances are shown.

To get rid of the fatal drawback of the GOSSO CFAR detector, the structure of data transfer from two reference windows to two ordering windows is changed. The dashed lines denote the new method, that is, half of samples in lagging reference window go into leading ordering window, and those of the leading reference window move into lagging ordering window. The performance of CFAR detector with the new window structure is shown in section 5.

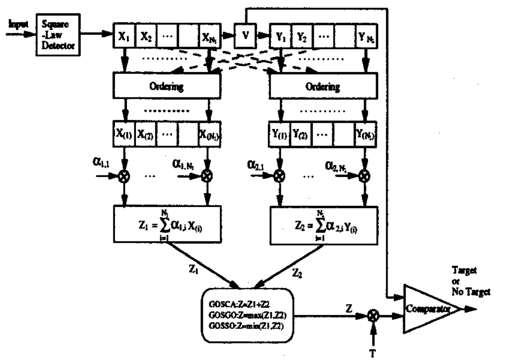

Figure 1: Generalized modified Order Statistics CFAR Detectors

2. GOSCA CFAR DETECTOR

The output Z_1 of the leading generalized order statistic (GOS) [3] window of length N_1 and Z_2 of the lagging GOS window of length N_2 are added to estimate the background noise level as shown in Figure 1.

The output sequence of the squared-law detector shift serially into the reference window which consists of lagging and leading windows. The samples in two separate sub-windows are first ordered, and then the ordered outputs are multiplied by coefficient $\{\alpha_{i,j}\}$ to produce Z_X and Z_Y by adding all weighted ordered samples in the lagging and leading windows, respectively. Finally, we obtain a statistic Z as

$$Z = Z_1 + Z_2, \tag{1}$$

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where

$$Z_1 = \sum_{i=1}^{N_1} \alpha_{1,i} X_{(i)}, \;\; Z_2 = \sum_{i=1}^{N_2} \alpha_{2,i} Y_{(i)}, \qquad (2)$$

where $X_{(i)}$ and $Y_{(i)}$ are the i-th smallest samples among N_1 and N_2 samples in a lagging or leading reference, respectively, and $\vec{\alpha}_{N_k} = \{\alpha_{k,i}\}_{i=1}^{N_k}$, $k = 1, 2$, are the sets of the weighting coefficients for the lagging or leading OS processors. By changing the weighting coefficients of the GOSCA CFAR detector, various CFAR detectors are obtained.

The processing time of the proposed GOSCA CFAR detector is less than half that of the GOS CFAR detector because the cells in the lagging and leading windows are independently ordered according to their own levels.

We assume that the target returns are Swerling I target model and the background cells have Gaussian distributions. The MGF of Z_k, $k = 1$ or 2, is given by [4, 6]

$$M_{Z_k}(s; \vec{\alpha}_{N_k}, \vec{\beta}_{N_k}) = \sum_{all\ N_k!\ inverses} H(\vec{\alpha}_{N_k}, \vec{\beta}_{N_k}) \times \prod_{i=1}^{L_k} \left(s + C_{k,i}(\vec{\alpha}_{N_k}, \vec{\beta}_{N_k})\right)^{-1} \qquad (3)$$

where

$$H(\vec{\alpha}_{N_k}, \vec{\beta}_{N_k}) = \left(\prod_{i=1}^{L_k} \frac{1}{\beta_{k,(i)}}\right) \left(\prod_{i=L_k+1}^{N_k} \frac{1}{\beta_{k,(i)} B_{k,i}(\vec{\beta}_{N_k})}\right) \left(\prod_{i=1}^{L_k} \frac{1}{A_{k,i}(\vec{\alpha}_{N_k})}\right), C_{k,i}(\vec{\alpha}_{N_k}, \vec{\beta}_{N_k}) = \frac{B_{k,i}(\vec{\beta}_{N_k})}{A_{k,i}(\vec{\alpha}_{N_k})},$$
$$B_{k,i}(\vec{\beta}_{N_k}) = \sum_{j=i}^{N_k} \frac{1}{\beta_{k,(j)}}, \;\; A_{k,i}(\vec{\alpha}_{N_k}) = \sum_{j=i}^{N_k} \alpha_{k,j},$$
$$L_k = \max_{1 \le j \le N_k} \{j \mid \alpha_{k,j} \neq 0\}, k = 1, 2. \qquad (4)$$

The MGF of Z is given by [4, 5]

$$M_Z^{GOSCA}(s; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}, \vec{\alpha}_{N_2}, \vec{\beta}_{N_2}) = M_{Z_1}(s; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}) M_{Z_2}(s; \vec{\alpha}_{N_2}, \vec{\beta}_{N_2}). \qquad (5)$$

The false alarm probability of the GOSCA CFAR detector is given by [4] and (5)

$$P_{fa} = M_Z^{GOSCA}(T; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}, \vec{\alpha}_{N_2}, \vec{\beta}_{N_2}). \qquad (6)$$

The detection probability of the GOSCA CFAR detector is given by

$$P_d = M_Z^{GOSCA}(T/(1+S); \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}, \vec{\alpha}_{N_2}, \vec{\beta}_{N_2}), \qquad (7)$$

where S is the SNR of a target. The ADT, defined by [1], of the GOSCA CFAR detector is written as

$$ADT = T\{ N_1! H(\vec{\alpha}_{N_1}, \vec{1}_{N_1}) \times \sum_{j=1}^{D_{N_1}} \left[C_j(\vec{\alpha}_{N_1}, \vec{1}_{N_1})^{-2} \prod_{i=1, i\neq j}^{D_{N_1}} C_i(\vec{\alpha}_{N_1}, \vec{1}_{N_1})^{-1} \right] \times M_{Z_2}(0; \vec{\alpha}_{N_2}, \vec{1}_{N_2}) + N_2! H(\vec{\alpha}_{N_2}, \vec{1}_{N_2}) \times \sum_{j=1}^{D_{N_2}} \left[C_j(\vec{\alpha}_{N_2}, \vec{1}_{N_2})^{-2} \prod_{i=1, i\neq j}^{D_{N_2}} C_i(\vec{\alpha}_{N_2}, \vec{1}_{N_2})^{-1} \right] \times M_{Z_1}(0; \vec{\alpha}_{N_1}, \vec{1}_{N_1}) \}. \qquad (8)$$

3. GOSGO CFAR DETECTOR

The background noise power is estimated from the larger of two separate order statistics obtained from the leading and lagging GOS windows in Figure ?? by

$$Z = max(Z_1, Z_2). \qquad (9)$$

The MGF of Z is given by

$$M_Z^{GOSGO}(s; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}, \vec{\alpha}_{N_2}, \vec{\beta}_{N_2}) = d_1(s; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}, \vec{\alpha}_{N_2}, \vec{\beta}_{N_2}) + d_2(s; \vec{\alpha}_{N_2}, \vec{\beta}_{N_2}, \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}), \qquad (10)$$

The value of $d_1(s; \cdot)$ can be obtained as follows;

$$d_1(s; \cdot) = (-1)^{L_1+1} \sum_{all\ N_1!\ inverses} \sum_{i=1}^{P} H(\vec{\alpha}_{N_1}, \vec{\beta}_{N_1}) \frac{1}{(m_i-1)!} \sum_{j=0}^{m_i-1} \binom{m_i-1}{j} R_i(u-s)^{(m_i-1-j)} \times PK_2(u)^{(j)} \Big|_{u=s+D_i(\vec{\alpha}_{N_1}, \vec{\beta}_{N_1})}, \qquad (11)$$

where

$$R_i(s; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1})^{(m)} = \left(\frac{1}{\prod_{j=1}^{L_1-m_i} (s - Q_i[j; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}])} \right)$$
$$= \sum_{k_{M-1}=0}^{q} \binom{q}{k_{M-1}} \frac{(q-k_{M-1}+1)!(-1)^{q-k_{M-1}}}{(s - Q_i[M; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}])^{q-k_{M-1}+1}}$$
$$\times \sum_{k_{M-2}=0}^{k_{M-1}} \binom{k_{M-1}}{k_{M-2}} \frac{(k_{M-1}-k_{M-2}+1)!(-1)^{k_{M-1}-k_M}}{(s - Q_i[M-1; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}])^{k_{M-1}-k_{M-}}}$$
$$\vdots$$
$$\times \sum_{k_1=0}^{k_2} \binom{k_2}{k_1} \frac{(k_2-k_1+1)!(-1)^{k_2-k_1}}{(s - Q_i[2; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}])^{k_2-k_1+1}}$$
$$\times \frac{(k_1+1)!(-1)^{k_1}}{(s - Q_i[1; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}])^{k_1+1}},$$

$$PK_2(s)^{(m)} = \sum_{j=0}^{m} \binom{m}{j} A_2(s)^{(m-j)} G(s)^{(j)}, \qquad (13)$$

$$G(s)^{(l)} = (-1)^l\, l!\, s^{-(l+1)}. \quad (14)$$

$D_i(\vec{\alpha}_{N_1}, \vec{\beta}_{N_1})$ in (11) is the distinguished value among the values of $C(s; (\vec{\alpha}_{N_1}, \vec{\beta}_{N_1}))$, $Q_i[1, \cdots, L_1 - m_i; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}] = array\{C_j(\vec{\alpha}_{N_1}, \vec{\beta}_{N_1}) | C_j(\vec{\alpha}_{N_1}, \vec{\beta}_{N_1}) \neq C_i(\vec{\alpha}_{N_1}, \vec{\beta}_{N_1})\}_{j=1}^{L_1}$. m_i is the number of multiple poles for $D_i(\vec{\alpha}_{N_1}, \vec{\beta}_{N_1})$. Note that M in (12) is $L_1 - m_i$. When $L_1 - m_i = 0$, $R_i(s)^{(m)} = 1$ and $m = 0$. If $M - 1 = 1$, then $k_1 = q$. The values of $A_k(s)$ in (13) can be obtained as

$$A_k(s)^{(k_M)} = \sum_{k_{M-1}=0}^{k_M} \binom{k_M}{k_{M-1}} I_k(s)^{(k_M - k_{M-1})} \sum_{k_{M-2}=0}^{k_{M-1}} \binom{k_{M-1}}{k_{M-2}} I_k(s)^{(k_{M-1} - k_{M-2})} \cdots \sum_{k_1=0}^{k_2} \binom{k_2}{k_1} I_k(s)^{(k_2 - k_1)} I_k(s)^{(k_1)}, \; k = 1, 2, \quad (15)$$

where

$$I_k(s)^{(q)} = \sum_{allN_k!inverses} H(\vec{\alpha}_{N_k}, \vec{\beta}_{N_k}) \sum_{k_{L_k-1}=0}^{q} \frac{q!\,(-1)^{q-k_{L_k-1}}}{k_{L_k-1}!(s + C_{k,L_k}(\vec{\alpha}_{N_k}, \vec{\beta}_{N_k}))^{q-k_{L_k-1}+1}} \sum_{k_{L_k-2}=0}^{k_{L_k-1}} \frac{k_{L_k-1}!\,(-1)^{k_{L_k-1}-k_{L_k-2}}}{k_{L_k-2}!(s + C_{k,L_k-1}(\vec{\alpha}_{N_k}, \vec{\beta}_{N_k}))^{k_{L_k-1}-k_{L_k-2}+1}} \cdots \sum_{k_1=0}^{k_2} \frac{k_2!\,(-1)^{k_2-k_1}}{k_1!(s + C_{k,2}(\vec{\alpha}_{N_k}, \vec{\beta}_{N_k}))^{k_2-k_1+1}} \frac{k_1!\,(-1)^{k_1}}{(s + C_{k,1}(\vec{\alpha}_{N_k}, \vec{\beta}_{N_k}))^{k_1+1}}, \quad k = 1, 2, \quad (16)$$

and (q) denotes the q-th derivative, and $k_1 = q$ when $L_k = 1$. $d_2(s; \cdot)$ can be obtained from $d_1(s; \cdot)$ by exchanging the subscripts 1 and 2. Therefore, the P_{fa} and P_d are derived from (6), (7), and (10). The ADT of the GOSGO CFAR detector is given by

$$ADT = -T \left[\frac{d}{ds} d_1(s; \vec{\alpha}_{N_1}, \vec{1}_{N_1}, \vec{\alpha}_{N_2}, \vec{1}_{N_2}) + \frac{d}{ds} d_2(s; \vec{\alpha}_{N_2}, \vec{1}_{N_2}, \vec{\alpha}_{N_1}, \vec{1}_{N_1}) \right] \Bigg|_{s=0}. \quad (17)$$

4. GOSSO CFAR DETECTOR

In the GOSSO CFAR detector the noise power estimation is the smallest of the order statistics Z_1 and Z_2 as depicted in Figure 1, *i.e.*,

$$Z = min(Z_1, Z_2). \quad (18)$$

The MGF of Z is given by

$$M_Z^{GOSSO}(s; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}, \vec{\alpha}_{N_2}, \vec{\beta}_{N_2}) = M_{Z_1}(s; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}) + M_{Z_2}(s; \vec{\alpha}_{N_2}, \vec{\beta}_{N_2}) - M_Z^{GOSGO}(s; \vec{\alpha}_{N_1}, \vec{\beta}_{N_1}, \vec{\alpha}_{N_2}, \vec{\beta}_{N_2}). \quad (19)$$

From (6),(7), and (19), the P_{fa} and P_d are derived. The ADT of the GOSSO CFAR detector is given by

$$ADT = -T \left\{ \frac{d}{ds} M_{Z_1}^{GOS}\left(s; \vec{\alpha}_{N_1}, \vec{1}_{N_1}\right)\Big|_{s=0} \right\} - T \left\{ \frac{d}{ds} M_{Z_2}^{GOS}\left(s; \vec{\alpha}_{N_2}, \vec{1}_{N_2}\right)\Big|_{s=0} \right\} - ADT_{GOSGO}. \quad (20)$$

5. PERFORMANCE RESULTS AND DISCUSSION

GOS**(a,b) represents a generalized modified OS CFAR detector with $\{\alpha_{k,i}\}_{i=a}^{b} = 1$ and other weighting values as zeros. Table 1 shows T and ADT for various GOSCA, GOSGO, and GOSSO CFAR detectors when the desired P_{fa} is 10^{-6} and $N_1 = N_2 = 12$. The important factor determining detection probability for a CFAR detector in homogeneous background is ADT [1]. The smaller ADT results in a better detection probability in a CFAR detector. It can be observed from the Table 1 that the GOSCA CFAR detectors perform better than the GOSGO amd GOSSO CFAR detector. The optimum weighting coefficients of the CFAR detectors are obtained and compared with the non-optimum CFAR detectors. For example, GOS**(9,10) CFAR detectors have larger ADT than the optimum CFAR detectors , whose coefficients are $\alpha_{k,9} = 3.229$ and $\alpha_{k,10} = 3$.

In homogeneous and interfering target situations, the detection performances are given in Table 2 and 3. In homogeneous case, the GOSCA CFAR performs the best among them as expected from the explanation of the ADT. The GOSSO CFAR detectors show better performance than the GOSCA and GOSGO CFAR's whenever the number of interfering targets becomes larger.

False alarm probabilities for three CFAR detectors are shown in Figure 2. The CNR of clutter data is assumed $10dB$, and the test cell is in the high-power-region. The number of clutter cell is assumed to be greater than 12. The GOSGO CFAR detector performs best among them because the false alarm rate is most close to the desired P_{fa}. This means that the excessive false alarm rate can be decreased by adopting the GOSGO CFAR detector. One can observe that the GOSSO CFAR's have worst performance in that region, and bring out too heavy false alarm rate. On the other hand, when the test cell is in low-power regions, the GOSSO

Table 1: Values of T and ADT for the proposed CFAR detectors when $P_{fa} = 10^{-6}$

$\vec{\alpha}_{N_1} = \vec{\alpha}_{N_2}$	GOSCA		GOSGO		GOSSO	
	T	ADT	T	ADT	T	ADT
1 1 1 1 1 1 1 1 1 1 1 1 1	0.7782	18.6768	1.4003	19.5120	2.3497	23.6517
0 0 0 0 0 0 0 0 0 1 1 0 0	3.5682	20.5040	6.3578	21.6574	11.8922	27.8248
0 0 0 0 0 1 1 1 1 1 1 0 0	1.8756	20.1292	3.3449	21.1742	6.1950	27.2699
0 0 0 1 1 1 1 1 1 1 1 0 0	1.6070	20.1257	2.8656	21.1639	5.3110	27.2886
0 0 1 1 1 1 1 1 1 1 1 0 0	1.5411	20.1457	2.7479	21.1884	5.0975	27.3302
0 1 1 1 1 1 1 1 1 1 1 0 0	1.5028	20.1693	2.6794	21.2165	4.9747	27.3724
1 1 1 1 1 1 1 1 1 1 1 0 0	1.4855	20.1845	2.6486	21.2374	4.9201	27.4002
0 0 0 0 0 0 0 0 0 3.229 3 0 0	1.1504	20.5005	2.049	21.6454	3.7952	27.5390
0 0 0 0 0 2.8766 1 1 1 3 0 0	1.0209	20.0071	1.8214	21.0204	3.36	27.0675
0 0 0 2.2612 1 1 1 1 1 3 0 0	1.0009	19.9281	1.7866	20.9172	3.287	26.9423
0 1.4566 1 1 1 1 1 1 1 3 0 0	0.9957	19.9071	1.776	20.8819	3.268	26.9107
1 1 1 1 1 1 1 1 1 1 3 0 0	0.9952	19.9054	1.7752	20.8799	3.266	26.9054

Table 2: Detection probabilities for various interfering target situations

SNR	GOSCA(6,10)		GOSGO(6,10)		GOSSO(6,10)	
	Number of interfering targets (Lagging window, Leading window)					
	(0,0)	(3,0)	(0,0)	(3,0)	(0,0)	(3,0)
6	0.0256	0.0071	0.0231	0.0053	0.0109	0.0065
8	0.0763	0.0290	0.0700	0.0216	0.0384	0.0252
10	0.1742	0.0862	0.1629	0.0665	0.1042	0.0754
12	0.3139	0.1918	0.2988	0.1554	0.2182	0.1723
14	0.4699	0.3363	0.4539	0.2865	0.3667	0.3111
16	0.6146	0.4924	0.6004	0.4387	0.5214	0.4667
18	0.7324	0.6338	0.7212	0.5852	0.6580	0.6117
20	0.8202	0.7472	0.8120	0.7083	0.7655	0.7301
22	0.8818	0.8308	0.8762	0.8021	0.8438	0.8185
24	0.9235	0.8890	0.9197	0.8691	0.8979	0.8807
26	0.9509	0.9282	0.9484	0.9148	0.9341	0.9227
28	0.9687	0.9540	0.9671	0.9452	0.9578	0.9504
30	0.9801	0.9707	0.9791	0.9650	0.9732	0.9684

CFAR detectors show very excellent performance over others. This means that the target missing can be minimized by using the GOSSO CFAR's in radar systems.

If the GOSSO CFAR detector has the different structure, the fatal problem can be resolved. The data transfer scheme from two reference windows to two ordering windows is changed as shown in Figure 1 with dashed line. In Figure 2, the GOSSO CFAR detector with the new window structure, GOSSO(6,10)-W, shows the drastic reducing of the false alarm rate in high power region near clutter edges compared with the GOSSO(6,10) CFAR detector.

As radar background situation is changed, the most available CFAR detectors can be selected from the derived formulas of proposed CFAR detectors and the new window scheme.

Table 3: Detection probabilities for various interfering target situations

SNR	GOSCA(9,10)		GOSGO(9,10)		GOSSO(9,10)	
	Number of interfering targets (Lagging window, Leading window)					
	(0,0)	(3,0)	(0,0)	(3,0)	(0,0)	(3,0)
6	0.0245	0.0056	0.0221	0.0041	0.0104	0.0062
8	0.0738	0.0233	0.0674	0.0171	0.0369	0.0239
10	0.1698	0.0714	0.1579	0.0537	0.1009	0.0719
12	0.3082	0.1645	0.2921	0.1287	0.2129	0.1659
14	0.4639	0.2991	0.4467	0.2453	0.3604	0.3024
16	0.6093	0.4521	0.5940	0.3889	0.5152	0.4574
18	0.7283	0.5973	0.7161	0.5357	0.6528	0.6034
20	0.8172	0.7179	0.8083	0.6657	0.7616	0.7235
22	0.8798	0.8091	0.8736	0.7690	0.8410	0.8137
24	0.9221	0.8739	0.9180	0.8452	0.8960	0.8773
26	0.9500	0.9181	0.9473	0.8983	0.9328	0.9204
28	0.9681	0.9473	0.9664	0.9342	0.9570	0.9489
30	0.9797	0.9664	0.9786	0.9578	0.9726	0.9674

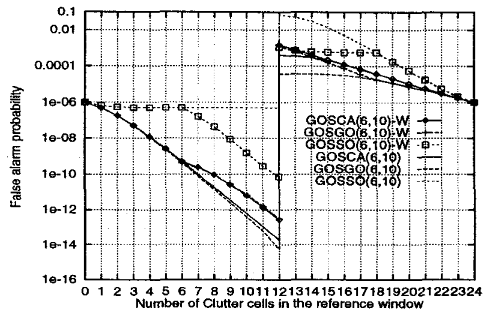

Figure 2: False alarm probabilities of the detectors with a new window structure in clutter edges, CNR = 10dB

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