Optimal Beam Subset and User Selection for Orthogonal Random Beamforming

Tae-Sung Kang and Hyung-Myung Kim, *Senior Member, IEEE*

***Abstract*—The throughput performance of orthogonal random beamforming (ORBF) with a finite number of users is limited due to the increasing amount of residual interference. In this letter, we find the optimal beam subset, the optimal user set, and the optimal number of random beams to maximize the sum throughput of the ORBF. The proposed scheme provides the best trade-off between the multiplexing gain and the multiuser interference by the determination of the optimal number of random beams as well as the beam selection diversity gain due to the selection of the optimal beam subset. In addition, two efficient suboptimal schemes are presented to reduce the computational complexity and the feedback overhead of the optimal method.**

***Index Terms*—Multiuser MIMO, random beamforming, partial CSI, selection diversity.**

I. Introduction

In a downlink multiuser MIMO system, when the base station (BS) transmitter with M antennas communicates K mobile receivers each of which has a single antenna, the sum capacity is achieved using dirty paper coding (DPC) and transmit beamforming schemes [1]-[3] and the sum capacity is linearly increased with $\min(M, K)$. However, since these methods require the condition that the BS transmitter has the perfect channel state information (CSI), it is difficult to satisfy this condition in practice, particularly in frequency division duplexing systems. There have been several studies on a model with partial CSI for the purpose of reducing feedback overhead of the CSI [4],[5]. The orthogonal random beamforming (ORBF) method constructs M orthogonal random beams and selects the user with the highest signal-to-interference-plus-noise-ratio (SINR) for each beam [4]. As the number of users goes to infinity, the sum rate scaling is equal to the sum capacity and, in fact, the multiuser interference is completely removed by the spatial multiuser diversity effect. However, in the case of a finite number of users, the multiuser diversity effect is not large enough to completely remove the multiuser interference. In fact, even though M increases, the sum throughput reduction caused by the increase of multiuser interference can be much larger than the sum throughput increase achieved by an increase in the number of streams. Hence, the sum throughput for a small M can be higher than that for a large M (as shown Figure 1). We can deduce therefore the optimal number of the random beams may not be equal to the number of transmit antennas.

Motivated with this observation, in this letter, we consider the following optimization problem: "How many random beams as well as which beams and which users should be selected for transmitting their data simultaneously to maximize the sum throughput?". The determination of the optimal number of random beams enables the best trade-off between the multiplexing gain and the multiuser interference. Moreover, the selection of the optimal beam subset provides the beam selection diversity gain and the selection of the optimal user set gives the multiuser selection diversity gain. In [5], the optimal number of beams for ORBF was addressed. However, the optimal number of beams in [5] is determined to satisfy target outage probability while that of the proposed scheme is determined to maximize the sum throughput considering beam subset selection. In the optimal method of the proposed scheme, we assume that the total average transmit power is one and that each user feeds back the channel gain magnitudes for all random beams. Since the optimal method requires high computational complexity and large feedback overhead, two efficient suboptimal methods are presented to resolve these problems.

Manuscript received April 24, 2008. The associate editor coordinating the review of this letter and approving it for publication was P. Cotae. This work was supported in part by Samsung Electronics.

The authors are with the school of EECS, KAIST, Daejeon, Korea (email: {tskang, hmkim}@csplab.kaist.ac.kr).

Digital Object Identifier 10.1109/LCOMM.2008.080648.

II. System Model

We consider a downlink multiuser MIMO system with K mobile receivers equipped with a single antenna and a base station (BS) transmitter with M antennas. It is assumed that $K > M$ and that the channel of each user does not vary during the scheduling interval of T. The received signal of the user k is represented as

$$y_k = \sqrt{\rho_k}\mathbf{h}_k\mathbf{x} + w_k, \quad k = 1, ..., K \tag{1}$$

where $\mathbf{h}_k$ is a $1 \times M$ channel gain vector of the user k and the entries of $\mathbf{h}_k$ are independent and identically distributed with zero mean and unit variance, $\mathbf{x}$ is an $M \times 1$ transmitted symbol vector and w_k is complex Gaussian noise with zero mean and unit variance of the user k. Moreover, the scalar ρ_k models the power attenuation due to the path loss and shadowing effect. The total average transmit power is assumed to be 1; that is, $E[||\mathbf{x}||^2] = 1$. Then the average receive signal-to-noise-ratio (SNR) becomes ρ_k.

Let $U^{(Q)} \subset \{1, ..., K\}$ and $S^{(Q)} \subseteq \{1, ..., M\}$ denote a user set and a beam subset with Q elements without repetition, respectively, where $Q \in \{1, ..., M\}$. At each time slot, Q users are scheduled among K users, and then the scheduled Q users' information data are transmitted via a set of Q random beams; that is $\mathbf{x} = \sum_{m \in S^{(Q)}} \mathbf{v}_m s_m$, where $\mathbf{v}_m$ is the mth orthonormal random vector generated according to isotropic distribution [6] and s_m is the mth transmit symbol for $m \in S^{(Q)}$. We assume that the kth mobile receiver knows $\sqrt{\rho_k}\mathbf{h}_k\mathbf{v}_m$ for $m = 1, ..., M$ (which can be estimated by pilot symbols transmitted from the BS) and feeds back its magnitude $c_{k,m} = \sqrt{\rho_k}|\mathbf{h}_k\mathbf{v}_m|$ for $m = 1, ..., M$ to the BS. The SINR value, $\gamma_{k,m}^{(Q)}$, for the mth signal in $S^{(Q)}$ of the kth user in $U^{(Q)}$ is represented as

$$\gamma_{k,m}^{(Q)} = \frac{|\mathbf{h}_k\mathbf{v}_m|^2}{\sum_{i\neq m, i\in S^{(Q)}} |\mathbf{h}_k\mathbf{v}_i|^2 + Q/\rho_k}, \quad m \in S^{(Q)}, k \in U^{(Q)} \tag{2}$$

where the average transmit power per beam is assumed to be $1/Q$, thereby ensuring that the total average transmit power is 1. In (2), there are $\binom{M}{Q}Q!$ permutations of $\gamma_{k,m}^{(Q)}$ for $S^{(Q)}$ because $\gamma_{k,m}^{(Q)}$ depends on the permuted set of $S^{(Q)}$.

III. Proposed Beam Subset and User Set Selection Method

A. Optimal Method

Let $R(U^{(Q)}, S^{(Q)})$ denote the sum throughput when users in user set $U^{(Q)}$ transmit information data via beam subset $S^{(Q)}$. The sum throughput $R(U^{(Q)}, S^{(Q)})$ is expressed as

$$R(U^{(Q)}, S^{(Q)}) = \sum_{k\in U^{(Q)}, m\in S^{(Q)}} \log_2\left(1 + \gamma_{k,m}^{(Q)}\right), \; Q = 1, ..., M \tag{3}$$

The achievable maximum throughput can be represented as

$$R_{\max} = \max_{Q=1,...,M} \max_{U^{(Q)}, S^{(Q)}} R(U^{(Q)}, S^{(Q)}) \tag{4}$$

In (4), we can find the optimal user set $S^{*(Q)}$, the optimal beam subset $U^{*(Q)}$ and the optimal number of random beams Q using exhaustive search method. However, in order to obtain $R_{\max}$, we should compute $R(U^{(Q)}, S^{(Q)})$ for $\binom{K}{Q}$ combinations of $U^{(Q)}$ and $\binom{M}{Q}Q!$ permutations of $S^{(Q)}$ for $Q = 1, ..., M$. Then the total computational complexity of finding the optimal solution is proportional to $\sum_{Q=1}^{M}\binom{K}{Q}\binom{M}{Q}Q!$, that is, $O((KM)^M)$, which is practically difficult to compute for a large K or a large M.

B. Suboptimal Reduced Complexity Method

In this subsection, we propose a suboptimal method to reduce the computational complexity. Because the denominator of $\gamma_{k,m}^{(Q)}$ in (2), namely $\sum_{i\in S^{(Q)}-\{m\}} |\mathbf{h}_k\mathbf{v}_i|^2 + Q/\rho_k$, does not depend on the order of the elements of $S^{(Q)} - \{m\}$ given $m \in S^{(Q)}$, there can be some duplications in $\binom{M}{Q}Q!$ permutations of $\gamma_{k,m}^{(Q)}$. When the duplications of $\gamma_{k,m}^{(Q)}$ are eliminated, there remain Q $\gamma_{k,m}^{(Q)}$ values for $\binom{M}{Q}$ combinations of $S^{(Q)}$. Let the jth combination of $S^{(Q)}$ denote $S_j^{(Q)} = \{m_{j,q} | 1 \leq m_{j,q} \leq M$ and $m_{j,q} \neq m_{j,l}$ for $q \neq l, 1 \leq q, l \leq Q\}$ for $j = 1, ..., \binom{M}{Q}$. The SINR value for the $m_{j,q}$th signal of the kth user can then be expressed as

$$\gamma_{k,j,q}^{(Q)} = \frac{|\mathbf{h}_k\mathbf{v}_{m_{j,q}}|^2}{\sum_{i=1, i\neq q}^{Q} |\mathbf{h}_k\mathbf{v}_{m_{j,i}}|^2 + Q/\rho_k}, \quad \forall q, j, k \tag{5}$$

Since the SINR value in (5) depends only on the beam subset but not on user set, the user set maximizing $R(U^{(Q)}, S^{(Q)})$ can easily be found by selecting Q users with the largest Q SINR values for each beam subset instead of computing $R(U^{(Q)}, S^{(Q)})$ for all $\binom{K}{Q}$ combinations of $U^{(Q)}$. Hence, the procedure of the reduced complexity method can be stated as follows.

Step 1: For $Q = 1, ..., M$, repeat *Step 1-1* and *Step 1-2*.

Step 1-1: Find the best Q user indices for $j = 1, ..., \binom{M}{Q}$

Initialize $Z = \{1, ..., K\}, U_j^{(Q)} = \emptyset, q = 1$.

Repeat the following equations (6), (7) and (8) until $q = Q$

$$k_{j,q} = \arg\max_{k\in Z} \gamma_{k,j,q}^{(Q)} \tag{6}$$

$$U_j^{(Q)} \leftarrow U_j^{(Q)} \cup \{k_{j,q}\} \tag{7}$$

$$Z \leftarrow Z - \{k_{j,q}\}, q \leftarrow q + 1 \tag{8}$$

Step 1-2: Find the optimal beam subset index, j^*

$$j^* = \arg\max_{1\leq j\leq \binom{M}{Q}} \sum_{q=1}^{Q} \log_2\left(1 + \gamma_{k_{j,q},j,q}^{(Q)}\right) \tag{9}$$

Then $U^{*(Q)} = U_{j^*}^{(Q)}, S^{*(Q)} = S_{j^*}^{(Q)}$.

Step 2: Find the optimal number of Q^*:

$$Q^* = \arg\max_{Q=1,...,M} R(U^{*(Q)}, S^{*(Q)}) \tag{10}$$

Then the complexity of the reduced complexity method is reduced to $\sum_{Q=1}^{M} K\binom{M}{Q}Q$. In *Step 1-1*, the user selection procedure is similar to that of the zero-forcing beamformer with user selection in [3].

C. Suboptimal Reduced Feedback Method

Because the optimal method and the reduced complexity method require M channel quality indicators (CQIs) for each user, the feedback overhead is large. However, the following procedure reduces the amount of overhead required for channel feedback. For some training period, using the reduced complexity method as stated in the previous subsection, the sample average of Q^* is obtained. Let Q_{avg}^* denote the sample average of Q^*. Note that because Q_{avg}^* depends on the number of users (as described in numerical results), this training period needs only when the number of users changes. Hence this training period using the reduced complexity method is short compared to the period applying the reduced feedback method. Given Q_{avg}^* and M, the kth mobile sends $\max_{1\leq j\leq \binom{M}{Q_{avg}^*}, 1\leq q\leq Q_{avg}^*} \gamma_{k,j,q}^{(Q_{avg}^*)}$ and the corresponding indices j_k, q_k to the BS. Hence, the feedback overhead is reduced to one CQI and two indices for each user. In the reduced feedback method, *Step 1* of the reduced complexity method is modified as follows.

Step 1: Given Q_{avg}^* and M, classify users according to the users' indices. Let $D_{j,q} = \{k | j_k = j, q_k = q\}$, $q = 1, .., Q_{avg}^*, j = 1, ..., \binom{M}{Q_{avg}^*}$.

Step 1-1: Find the best Q_{avg}^* user indices for $j = 1, ..., \binom{M}{Q_{avg}^*}$

Initialize $U_j^{(Q_{avg}^*)} = \emptyset, q = 1$.

Repeat the following equations until $q = Q_{avg}^*$

$$k_{j,q} = \arg\max_{k\in D_{j,q}} \gamma_{k,j,q}^{(Q_{avg}^*)} \quad \text{if } D_{j,q} \neq \emptyset \tag{11}$$

$$U_j^{(Q_{avg}^*)} \leftarrow U_j^{(Q_{avg}^*)} \cup \{k_{j,q}\} \tag{12}$$

$$D_{j,q+1} \leftarrow D_{j,q+1} - \{k_{j,q}\}, \quad q \leftarrow q + 1 \tag{13}$$

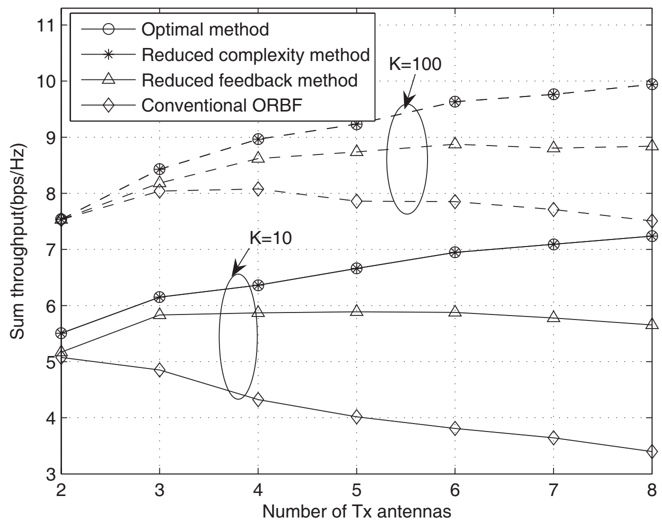

Fig. 1. Performance comparison of the conventional ORBF and the proposed schemes : Sum throughput vs. the number of transmit antennas(M) for $K = 10, 100$ at an SNR of 10dB; solid line: K=10, dashed line: K=100.

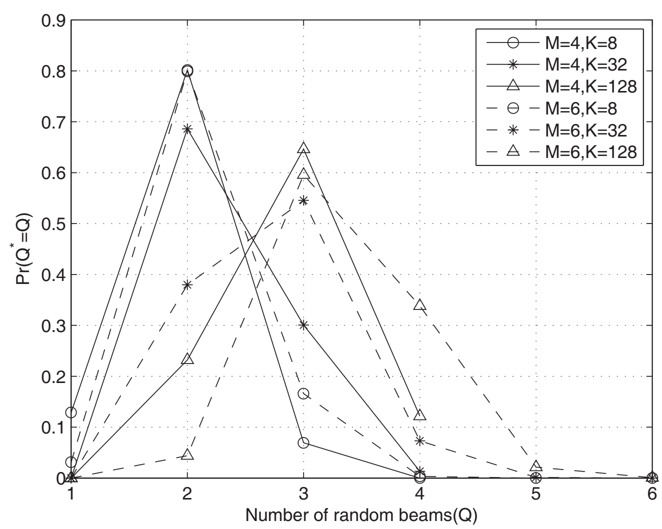

Fig. 2. Probability $\Pr(Q^* = Q)$ of the reduced complexity method for M=4 and 6, K=8,32, and 128 at an SNR of 10dB.

Step 1-2: Find the optimal beam subset index j^*

$$j^* = \arg \max_{1 \le j \le \binom{M}{Q^*_{avg}}} \sum_{q=1}^{Q^*_{avg}} \log_2 \left(1 + \gamma^{(Q^*_{avg})}_{k_{j,q},j,q}\right) \tag{14}$$

Then $U^{*(Q^*_{avg})} = U^{(Q^*_{avg})}_{j^*}, S^{*(Q^*_{avg})} = S^{(Q^*_{avg})}_{j^*}$ Note that we assumed that no user is assigned if $D_{j,q} = \emptyset$ in equation (11). In the conventional ORBF method, the kth user feeds back $\max_{1 \le q \le M} \gamma^{(M)}_{k,q}$ and the corresponding index q_k [4]. Hence, the conventional ORBF can be considered as a case of $Q = M$ in the reduced feedback method.

IV. Numerical Results

We assumed that the channel is invariant during a scheduling interval (2ms)[7] and that the channel of each antenna of each user is an identical and independent Rayleigh distributed channel and that all users have the same SNR of 10 dB. The simulation was performed more than 100,000 runs. In Figure 1, we compare the proposed schemes and the conventional ORBF [4]. Figure 1 shows the sum throughput vs. the number of transmit antennas(M) for $K = 10, 100$. We can see that the throughput of the reduced complexity method is very close to the throughput of the optimal method. The reduced feedback method has a lower throughput than the reduced complexity method because the optimal number of random beams of the reduced feedback method is statistically determined as Q^*_{avg} whereas the optimal number of random beams of the reduced complexity method is instantaneously determined as Q^*. The sum throughput of the conventional ORBF method can decrease with M since the number of random beams is equal to M and it increases the multiuser interference. In contrast, the proposed schemes can increase the sum throughput with M due to the beam selection diversity gain and the determination of Q^* or Q^*_{avg} to provide the best trade-off between the multiplexing gain and the multiuser interference. Note that the proposed schemes require slight increase in the feedback overhead whereas the amount is much smaller than that of the throughput improvement achieved in Figure 1. Figure 2 shows the probability, $\Pr(Q^* = Q)$, of the reduced complexity method for $K = 8, 32, 128$ and $M = 4, 6$. When K is small ($K = 8$), $\Pr(Q^* = 2)$ is dominant; in contrast, when K is large ($K = 128$), $\Pr(Q^* = 3)$ is dominant. As K increases, the $\Pr(Q^* = Q)$ value of a large Q increases. Similarly, the Q^*_{avg} value of the reduced feedback method increases with the number of users ($Q^*_{avg} = 1$ for $K < 8$, $Q^*_{avg} = 2$ for $8 \le K \le 78$, and $Q^*_{avg} = 3$ for $K \ge 78$ when $M = 4$).

V. Conclusion

In this letter, we report on the process of selecting an optimal number of random beams, the optimal beam subset, and an optimal user set to improve the performance of the ORBF with a finite number of users. In future research, we intend to consider the notion of the fairness in a heterogeneous network in which users have different SNRs.

References

[1] M. Costa, "Writing on dirty paper," *IEEE Trans. Inform. Theory*, vol. 29, pp. 439–441, May 1983.
[2] G. Caire and S. Shamai, "On the achievable throughput of a multi-antenna Gaussian broadcast channel," *IEEE Trans. Inform. Theory*, vol. 4, pp. 1691–1706, July 2003.
[3] G. Dimic and N. D. Sidiropoulos, "On downlink beamforming with greedy user selection: performance analysis and a simple new algorithm," *IEEE Trans. Signal Processing*, vol. 53, no. 10, pp. 3857–3868, Oct. 2005.
[4] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channels with partial information," *IEEE Trans. Inform. Theory*, vol. 51, no. 2, pp. 506–522, Feb. 2005.
[5] N. Zorba and A. I. Perez-Neira, "Optimum number of beams in multiuser opportunistic scheme under QoS constraints," in *Proc. IEEE Workshop on Smart Antennas*, 2007.
[6] T. L. Marzetta and B. M.Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. Inform. Theory*, vol. 45, pp. 138-157, Jan. 1999.
[7] 3GPPTR 25.814, "Physical layer aspect for evolved Universal Terrestrial Radio Access (UTRA)," V7.1.0, Oct. 2006.