A Computationally Efficient Criterion for Antenna Shuffling in DSTTD Systems

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***Abstract*— The application of double space-time transmit diversity (DSTTD) scheme to multicarrier systems, such as orthogonal frequency division multiplexing (OFDM) systems, requires calculating the determinations of the antenna permutation matrices for all subcarriers, resulting in a heavy computation load. In this paper, we show that the signal-to-noise ratio (SNR)-based antenna shuffling criterion for DSTTD systems can be reduced to a simple criterion that evaluates determinants of 2×2 submatrices of the 4×4 equivalent channel matrix. The new criterion can lighten the computational load by about 95%. Furthermore, it is shown that the minimum mean square error (MMSE)-based criterion for antenna permutation can also be reduced to the same criterion.**

***Index Terms*— Double space-time transmit diversity (DSTTD), antenna shuffling, space-time block code, multiple-input multiple-output (MIMO).**

I. Introduction

DSTTD is a multiple-input multiple-output (MIMO) system employing two space-time block code (STBC) encoders at the transmitter [1], [2]. Due to its efficiency in compromising diversity gain with multiplexing, DSTTD with four transmit and two receive antennas has received considerable attention and has become a part of the Broadband Wireless Access standard [3]. This system suggests the use of an antenna shuffling (or grouping) scheme for selecting an appropriate transmit antenna for each STBC encoded data stream. The criteria for antenna shuffling include the minimization of the spatial correlation between transmit antennas [1] and the maximization of the minimum post-processing SNR [2]. It is shown that the latter criterion can yield a smaller bit-error-rate (BER) than the former.

The DSTTD is usually employed for downlink communication (from a base station to a mobile). A mobile station selects a permutation matrix and informs the base station which matrix is chosen. Then the base station antennas are shuffled by multiplying the permutation matrix with the STBC encoded data. The computational load for selecting a permutation matrix may not be heavy. However, in multicarrier systems, such as orthogonal frequency division multiplexing (OFDM), a permutation matrix must be assigned to each subcarrier, and selection of all permutation matrices can require heavy computation. For example, the mobile WiMAX standard [3] suggests an OFDM system with $1,024$ subcarriers with a 5 *msec* frame length. If an identical permutation matrix, which is chosen based on the post-processing SNR, is assigned to each group of 4 successive subcarriers and if 20 subcarrier groups are allocated to one user, then 24.6 million floating point operations (flops) per second are needed to determine all permutation matrices at a mobile station[1]. For such systems, reducing the computational load for selecting the permutation matrix is of practical importance.

In this paper, we show that the SNR-based criterion, which evaluates the eigenvalues of a 4×4 matrix, can be reduced to a simple criterion evaluating the determinants of two 2×2 matrices. The computational saving achieved by the simplified criterion is about 95%. Furthermore, it is shown that the MMSE-based antenna permutation criterion can also be reduced to the same criterion.

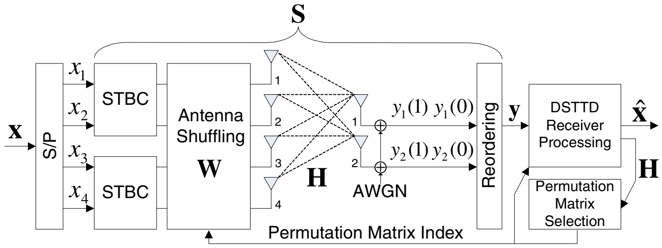

Fig. 1. System model for 2×4 DSTTD employing antenna shuffling.

II. Proposed Antenna Shuffling Criterion

Fig. 1 illustrates the DSTTD system with four transmit and two receive antennas. The MIMO channel is represented by a 2×4 channel matrix $\mathbf{H}$, whose (i, j)-th element $h_{i,j}$ is a channel gain between the j-th transmit and i-th receive antennas. The permutation matrix $\mathbf{W}$, which is employed for antenna shuffling is chosen from the following set:

$$\mathbf{W} \in \mathcal{S}_{\mathbf{W}} = \left\{ \begin{matrix} [\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3, \mathbf{i}_4], [\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_4, \mathbf{i}_3], [\mathbf{i}_1, \mathbf{i}_3, \mathbf{i}_2, \mathbf{i}_4], \\ [\mathbf{i}_1, \mathbf{i}_3, \mathbf{i}_4, \mathbf{i}_2], [\mathbf{i}_1, \mathbf{i}_4, \mathbf{i}_2, \mathbf{i}_3], [\mathbf{i}_1, \mathbf{i}_4, \mathbf{i}_3, \mathbf{i}_2] \end{matrix} \right\}$$

where $\mathbf{i}_k$, $1 \leq k \leq 4$, is a 4×1 vector whose k-th element is 1 and the rest are 0s. Given $\mathbf{H}$, the receiver selects an appropriate permutation matrix from $\mathcal{S}_{\mathbf{W}}$ and informs the transmitter of which matrix is chosen—this requires 3-bit feedback information because $\mathcal{S}_{\mathbf{W}}$ contains six matrices. The input to the STBC encoders and the corresponding received signals are represented in vector form as $\mathbf{x} = [x_1, x_2, x_3, x_4]^T$ and $\mathbf{y} = [y_1(0), y_1^*(1), y_2(0), y_2^*(1)]^T$, respectively. To represent $\mathbf{y}$

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[1]Selecting one permutation matrix using the post-processing SNR requires $1,024$ flops (for details see Section II).

in terms of $\mathbf{H}$ and $\mathbf{W}$, these matrices are block-partitioned as follows:

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1^T & \mathbf{h}_2^T \\ \mathbf{h}_3^T & \mathbf{h}_4^T \end{bmatrix} \text{ and } \mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_2 \\ \mathbf{W}_3 & \mathbf{W}_4, \end{bmatrix},$$

where $\{\mathbf{h}_k\}$ are 2×1 vectors and $\{\mathbf{W}_k\}$ are 2×2 matrices. The received vector can then be written as

$$\mathbf{y} = \mathbf{S}\mathbf{x} + \mathbf{v}, \tag{1}$$

where the equivalent channel matrix

$$\mathbf{S} = \begin{bmatrix} \mathbf{h}_1^T\mathbf{W}_1 + \mathbf{h}_2^T\mathbf{W}_3 & \mathbf{h}_1^T\mathbf{W}_2 + \mathbf{h}_2^T\mathbf{W}_4 \\ (\mathbf{h}_1^H\mathbf{W}_1 + \mathbf{h}_2^H\mathbf{W}_3)\mathbf{J} & (\mathbf{h}_1^H\mathbf{W}_2 + \mathbf{h}_2^H\mathbf{W}_4)\mathbf{J} \\ \mathbf{h}_3^T\mathbf{W}_1 + \mathbf{h}_4^T\mathbf{W}_3 & \mathbf{h}_3^T\mathbf{W}_2 + \mathbf{h}_4^T\mathbf{W}_4 \\ (\mathbf{h}_3^H\mathbf{W}_1 + \mathbf{h}_4^H\mathbf{W}_3)\mathbf{J} & (\mathbf{h}_3^H\mathbf{W}_2 + \mathbf{h}_4^H\mathbf{W}_4)\mathbf{J} \end{bmatrix},$$

$$\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

and the elements of $\mathbf{v}$ are additive white Gaussian noise (AWGN) with variance σ^2. The SNR-based criterion in [2] is given by

$$\arg\max_{\mathbf{W}} \left(\min_i (\lambda_i) \right), \tag{2}$$

where $\{\lambda_i\}$ represent the eigenvalues of the matrix $\mathbf{S}^H\mathbf{S}$ and the superscript H denotes the complex conjugate transpose. When the eigenvalue decomposition method [4] is used for directly evaluating eigenvalues, the SNR-based antenna selection requires $1,024$ flops[2] for one permutation matrix, and for the case of mobile WiMAX described in the previous section, the computational load can reach 24.6 *M* $(1,024 \times 200 \times 6 \times 20)$ flops per second, which may be burdensome in mobile stations[3]. This load can be relieved if a closed-form expression for the eigenvalue λ_i is available. Next, we derive such an expression.

Lemma 1: The eigenvalue λ_i is represented as

$$\lambda_i = \frac{c_1 \pm \sqrt{c_1^2 - 4(\alpha\beta - \eta)}}{2}, \tag{3}$$

where $\alpha = |s_{1,1}|^2 + |s_{1,2}|^2 + |s_{3,1}|^2 + |s_{3,2}|^2$, $\beta = |s_{1,3}|^2 + |s_{1,4}|^2 + |s_{3,3}|^2 + |s_{3,4}|^2$, $c_1 = \alpha + \beta$, and $\eta = (|s_{1,1}|^2 + |s_{1,2}|^2)(|s_{1,3}|^2 + |s_{1,4}|^2) + (|s_{3,1}|^2 + |s_{3,2}|^2)(|s_{3,3}|^2 + |s_{3,4}|^2) + 2\,\mathrm{Re}\{(s_{1,1}^* s_{1,3} + s_{1,2} s_{1,4}^*)(s_{3,1} s_{3,3}^* x + s_{3,2}^* s_{3,4})\} + 2\,\mathrm{Re}\{(s_{1,1}^* s_{1,4} - s_{1,2} s_{1,3}^*)(s_{3,1} s_{3,4}^* - s_{3,2}^* s_{3,3})\}$ ($s_{i,j}$ is the (i,j)-th entry of $\mathbf{S}$.)

Proof: The equivalent channel matrix $\mathbf{S}$ in (1) can be block-partitioned as

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{1,1} & \mathbf{S}_{1,2} \\ \mathbf{S}_{2,1} & \mathbf{S}_{2,2} \end{bmatrix},$$

where $\{\mathbf{S}_{i,j}\}$ are 2×2 matrices given by

$$\mathbf{S}_{i,j} = \begin{bmatrix} \mathbf{h}_{2i-1}^T\mathbf{W}_j + \mathbf{h}_{2i}^T\mathbf{W}_{j+2} \\ \left(\mathbf{h}_{2i-1}^T\mathbf{W}_j + \mathbf{h}_{2i}^T\mathbf{W}_{j+2}\right)\mathbf{J} \end{bmatrix}.$$

It can be seen that each $\mathbf{S}_{i,j}$ is represented in the form of $\left[\begin{smallmatrix} a_1 & a_2 \\ a_2^* & -a_1^* \end{smallmatrix}\right]$ for some complex numbers a_1 and a_2—such a matrix is called an Alamouti matrix [5]. Then

$$\begin{aligned} \mathbf{S}^H\mathbf{S} &= \begin{bmatrix} \mathbf{S}_{1,1}^H\mathbf{S}_{1,1} + \mathbf{S}_{2,1}^H\mathbf{S}_{2,1} & \mathbf{S}_{1,1}^H\mathbf{S}_{1,2} + \mathbf{S}_{2,1}^H\mathbf{S}_{2,2} \\ \mathbf{S}_{1,2}^H\mathbf{S}_{1,1} + \mathbf{S}_{2,2}^H\mathbf{S}_{2,1} & \mathbf{S}_{1,2}^H\mathbf{S}_{1,2} + \mathbf{S}_{2,2}^H\mathbf{S}_{2,2} \end{bmatrix} \\ &= \begin{bmatrix} \alpha\mathbf{I}_2 & \mathbf{U} \\ \mathbf{U}^H & \beta\mathbf{I}_2 \end{bmatrix}, \end{aligned} \tag{4}$$

where $\mathbf{I}_k$ represents a k-dimensional identity matrix, and $\mathbf{U}$ is another Alamouti matrix satisfying $\mathbf{U}\mathbf{U}^H = \mathbf{U}^H\mathbf{U} = \eta\mathbf{I}_2$. In (4), the second inequality comes from the facts that an Alamouti matrix has an orthogonal property[4] and its structure remains invariant under the sum or product of two Alamouti matrices [5]. Then, $\{\lambda_i\}$ satisfy

$$\begin{bmatrix} \alpha\mathbf{I}_2 & \mathbf{U} \\ \mathbf{U}^H & \beta\mathbf{I}_2 \end{bmatrix} \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix} = \lambda_i \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}, \tag{5}$$

where $[\mathbf{z}_1^T \ \mathbf{z}_2^T]^T$ is the eigenvector of $\mathbf{S}^H\mathbf{S}$. Solving this equation results in $\left[\alpha\mathbf{I}_2 - \lambda_i\mathbf{I}_2 - \frac{\eta}{\beta - \lambda_i}\mathbf{I}_2\right]\mathbf{z}_1 = \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]$, and λ_i is derived as in (3). ■

Using Lemma 1, we can simplify the SNR-based criterion.

Property 1: The SNR-based criterion in (2) is simplified to that minimizes κ, where

$$\kappa = \left| \det\left(\begin{bmatrix} s_{1,1} & s_{1,2} \\ s_{3,1} & s_{3,2} \end{bmatrix} \right) + \det\left(\begin{bmatrix} s_{1,3} & s_{1,4} \\ s_{3,3} & s_{3,4} \end{bmatrix} \right) \right|, \tag{6}$$

$\det(\mathbf{A})$ denotes the determinant of matrix $\mathbf{A}$ and $|\cdot|$ represents the magnitude, i.e.,

$$\arg\max_{\mathbf{W}} \left(\min_i (\lambda_i) \right) = \arg\min_{\mathbf{W}} \kappa. \tag{7}$$

Proof: $\arg\max_{\mathbf{W}} (\min_i(\lambda_i)) = \arg\max_{\mathbf{W}} \left(\frac{c_1 - \sqrt{c_1^2 - 4(\alpha\beta - \eta)}}{2} \right) = \arg\max_{\mathbf{W}} (\alpha\beta - \eta) = \arg\max_{\mathbf{W}} (c_2 - c_3 - \kappa) = \arg\min_{\mathbf{W}} \kappa$, where $c_2 = \sum_{i \neq j} |s_{1,i}|^2 |s_{3,j}|^2$ and $c_3 = 2\,\mathrm{Re}\left\{ \sum_{i=1}^4 \left(s_{1,i}^* s_{3,i} (\sum_{j=i+1}^4 s_{1,j} s_{3,j}^*) \right) \right\}$. The second and fourth equalities are true because c_1 and $c_2 - c_3$ are independent of $\mathbf{W}$. In fact, it can be shown that $c_1 = \sum_{j=1}^4 \mathbf{h}_j^T \mathbf{h}_j^*$ and $c_2 - c_3 = \left(\mathbf{h}_1^T\mathbf{h}_1^* + \mathbf{h}_2^T\mathbf{h}_2^*\right)\left(\mathbf{h}_3^T\mathbf{h}_3^* + \mathbf{h}_4^T\mathbf{h}_4^*\right) - |\mathbf{h}_1^T\mathbf{h}_3^* + \mathbf{h}_2^T\mathbf{h}_4^*|^2$ using the following properties of $\mathbf{W}$: $\mathbf{W}_i\mathbf{W}_i^H + \mathbf{W}_{i+1}\mathbf{W}_{i+1}^H = \mathbf{I}_2$ $(i = 1, 2)$, $\mathbf{W}_1\mathbf{W}_3^H + \mathbf{W}_2\mathbf{W}_4^H = \left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right]$, $\forall \mathbf{W} \in \mathcal{S}_{\mathbf{W}}$, and $|s_{2i-1,2j-1}|^2 + |s_{2i-1,2j}|^2 = (\mathbf{h}_{2i-1}^T\mathbf{W}_j + \mathbf{h}_{2i}^T\mathbf{W}_{j+2})(\mathbf{h}_{2i-1}^T\mathbf{W}_j + \mathbf{h}_{2i}^T\mathbf{W}_{j+2})^H$. This completes the proof. ■

As shown in (7), the SNR-based criterion is reduced to a simple criterion evaluating the determinants of two 2×2 submatrices of $\mathbf{S}$. Evaluating κ for each $\mathbf{W}$ in (6) requires only 33 flops, which is less than 4% of the computations (1,024 flops) needed to obtain $\{\lambda_i\}$ by the eigenvalue decomposition.

We now consider the MMSE-based criterion. Assuming an MMSE receiver is employed, the MSE can be represented as

$$\mathrm{E}\left[\|\tilde{\mathbf{x}} - \mathbf{x}\|^2\right] = \sigma^2\,\mathrm{tr}\left(\left[\mathbf{S}^H\mathbf{S} + \sigma^2\mathbf{I}_4\right]^{-1} \right), \tag{8}$$

[2]For a real-valued $m \times n$ matrix, the eigenvalue decomposition needs $4mn^2 - 4n^3/3$ flops [4]. For complex-valued matrices, we approximate the flop count as $24mn^2 - 8n^3$ by treating every complex operation as a complex multiplication.

[3]It is assumed that $\mathbf{S}$ and $\mathbf{S}^H\mathbf{S}$ are pre-determined for all $\mathbf{W} \in \mathcal{S}_{\mathbf{W}}$ once $\mathbf{H}$ is given.

[4]If $\mathbf{A}$ is a $k \times k$ Alamouti matrix, then $\mathbf{A}^H\mathbf{A} = c\mathbf{I}_k$ for some constant c.

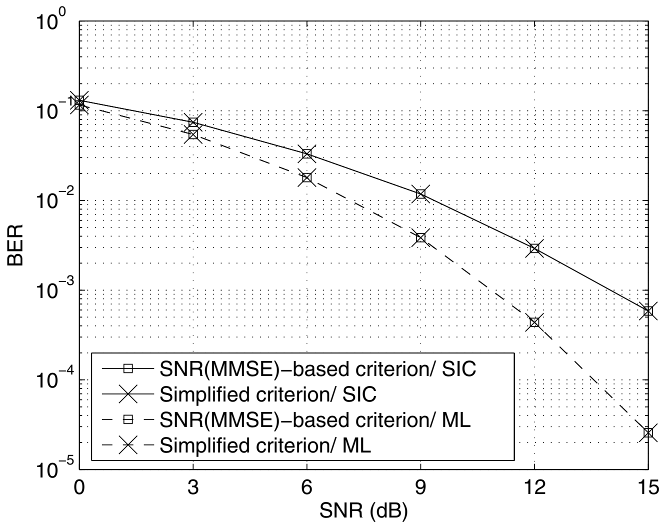

Fig. 2. BER performance comparison.

where $\mathrm{E}[\cdot]$ and $\mathrm{tr}(\cdot)$ denote the expectation and the trace, respectively, and $\tilde{\mathbf{x}}$ is the MMSE output given by $\tilde{\mathbf{x}} = \left(\mathbf{S}^H\mathbf{S} + \sigma^2\mathbf{I}_4\right)^{-1}\mathbf{S}^H\mathbf{y}$. The equivalence between the MMSE criterion and the SNR-based criterion is shown below.

Property 2: The permutation matrix minimizing the MSE in (8) is identical to that which minimizes κ, i.e.,

$$\arg\min_{\mathbf{W}} \mathrm{E}\left[\|\tilde{\mathbf{x}} - \mathbf{x}\|^2\right] = \arg\min_{\mathbf{W}} \kappa.$$

Proof: Using the inversion lemma for partitioned matrices, the MSE in (8) can be rewritten as

$$\begin{aligned}
&\sigma^2 \,\mathrm{tr}\left(\left[\mathbf{S}^H\mathbf{S} + \sigma^2\mathbf{I}_4\right]^{-1}\right)\\
&= \sigma^2\,\mathrm{tr}\left(\left[\begin{bmatrix}\alpha\mathbf{I}_2 & \mathbf{U}\\ \mathbf{U}^H & \beta\mathbf{I}_2\end{bmatrix} + \sigma^2\mathbf{I}_4\right]^{-1}\right)\\
&= \sigma^2\,\mathrm{tr}\left(\left[(\alpha+\sigma^2)\mathbf{I}_2 - \frac{1}{(\beta+\sigma^2)}\mathbf{U}\mathbf{U}^H\right]^{-1}\right)\\
&\quad + \sigma^2\,\mathrm{tr}\left(\left[(\beta+\sigma^2)\mathbf{I}_2 - \frac{1}{(\alpha+\sigma^2)}\mathbf{U}^H\mathbf{U}\right]^{-1}\right)\\
&= \frac{2\sigma^2(\alpha+\beta+2\sigma^2)}{(\alpha+\sigma^2)(\beta+\sigma^2)-\eta}\\
&= \frac{2c_1\sigma^2 + 4\sigma^4}{\sigma^4 + c_1\sigma^2 + \alpha\beta - \eta}.
\end{aligned}$$

Since c_1 and σ are independent of $\mathbf{W}$, the MMSE cost function can be written as

$$\arg\min_{\mathbf{W}} \mathrm{E}\left[\|\tilde{\mathbf{x}} - \mathbf{x}\|^2\right] = \arg\max_{\mathbf{W}} \left(\alpha\beta - \eta\right).$$

This completes the proof (see the proof of Property 1). ■

III. Simulation Results

Computer simulations were conducted to confirm the properties derived in the previous section by showing the BER performances of DSTTD systems that employ the SNR- and MMSE-based shuffling criteria are identical to that employing the simplified criterion. In the simulation, quadrature phase-shift keying (QPSK) is used without channel coding, and information symbols are grouped into frames consisting of $10,000$ symbols. For each frame, a spatially correlated, flat fading channel matrix $\mathbf{H}$ is generated using $\mathbf{H} = \mathbf{R}_R^{1/2}\mathbf{H}_w\mathbf{R}_T^{1/2}$ [6], in which we set $\mathbf{R}_R = \mathbf{I}_2$ and $\mathbf{R}_T = \mathtt{toeplitz}[1,\ 0.9,\ 0.81,\ 0.729]^T$. $\mathbf{H}_w$ is a 4×4 matrix consisting of independent, identically distributed complex Gaussian random variables with a mean of 0 and a variance of 1. Channel $\mathbf{H}$ is fixed during a frame, but it varies independently over frames. Two types of receivers are considered: an MMSE-based successive interference cancelling (SIC) receiver [7] and a maximum-likelihood (ML) receiver [8]. It is assumed that $\mathbf{H}$ is perfectly known at the receiver. The BER values are obtained from $50,000$ independent frames. Fig. 2 shows the BER performances of the DSTTD systems. As expected, the ML receivers outperform the SIC receivers. For both types of receivers, the SNR- and MMSE-based selection schemes and those with the simplified criterion yield identical BER performances.

IV. Conclusion

It is shown that both the SNR- and MMSE-based criteria for antenna shuffling in DSTTD systems can be reduced to a simple criterion evaluating the determinants of two 2×2 matrices. The simplified criterion can dramatically reduce the computational complexity for antenna shuffling without sacrificing BER performance.

References

[1] E. O. Onggosanusi, A. G. Dabak, and T. M. Schmidl, "High rate space-time block coded scheme: performance and improvement in correlated fading channels," in *Proc. IEEE Wireless Communication and Networking Conf.*, Mar. 2002, vol. 1, pp. 194–199.

[2] S. Shim, K. Kim, and C. Lee, "Efficient antenna shuffling scheme for a DSTTD system," *IEEE Commun. Lett.*, vol. 9, no. 2, pp. 124–126, Feb. 2005.

[3] *IEEE Air Interface for Fixed and Mobile Broadband Wireless Access Systems*, IEEE P802.16e/D12, 2005.

[4] G. H. Golub and C. F. Van Loan, *Matrix Computation*, 3rd ed. Baltimore, MD: The Johons Hopkins Univ. Press, 1996.

[5] A. H. Sayed, W. M. Younis, and A. Tarighat, "An invariant matrix structure in multiantenna communications," *IEEE Signal Processing Lett.*, vol. 12, no. 11, pp. 749–752, Nov. 2005.

[6] J. P. Kermoal, L. Schumacher, K. I. Pedersen, P. E. Mogensen, and F. Frederiksen, "A stochastic MIMO radio channel model with experimental validation," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 6, pp. 1211–1226, Aug. 2002.

[7] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, "Simplified processing for high spectral efficiency wireless communication employing multi-element arrays," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 11, pp. 1841–1852, Nov. 1999.

[8] M. O. Damen, H. E. Gamal, and G. Caire, "On maximum-likelihood detection and the search for the closest lattice point," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2389–2402, Oct. 2003.