Minimum-weight design of compressively loaded composite plates and stiffened panels for postbuckling strength by Genetic Algorithm

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ABSTRACT: The minimum-weight design of compressively loaded composite plates and composite stiffened panels under constrained Postbuckling strength is addressed herein. A nonlinear finite element code, COSAP (COmposite Structural Analysis Program) was applied to analyze the buckling and postbuckling behaviour. As an optimization technique, a modified Genetic Algorithm was used to find the optimum points. The parallel computing scheme was implemented with MPI (Message Passing Interface) library and realized in a parallel-computing supercomputer and a pc cluster. The optimal design was performed with two chosen examples: a composite plate and a composite stiffened panel. The design variables were the number of plies and the ply angle in each ply. In case of the stiffened panels, the size and the location of the stiffeners were also considered as the design variables. The objective function was defined as the product of the nondimensional weight and the strength. The optimization results showed better performances than conventional designs.

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KEYWORDS: weight-minimization, optimal design, genetic algorithm, composite, postbuckling

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1. INTRODUCTION

Fiber reinforced composite laminated structures have superior characteristics in design, i.e. variable stacking sequence and ply angles, when compared to conventional materials. Therefore optimal design can be achieved by determining the proper stacking sequences and the number of plies. However, there are some difficulties in optimal design such as discreteness of the design values and complexity of the design space. Moreover, it was inevitable to use discrete ply angles such as 0°, 90°, or ±45° for designing realistic composite structures. However, in the early 90's, continuous ply angles were considered [1]. From the mid 90's, there was more interest in the use of discrete ply angles, and some researchers used discrete optimization techniques [2,3]. One of the discrete optimization techniques, Genetic Algorithm was proposed for the optimization of composite structures, and many researchers reported that Genetic Algorithm was a good solution for complex optimization problems such as composite structures [4-7]. Above all, weight minimization is the final goal for many researchers because the major incentive for the use of composites is weight reductions. There are two methods to minimize weight: the first is minimization of sizes and the second is minimization of the number of plies in laminates. The former method was considered by many researchers, but the latter has not received the same attention [7]. A further important issue is the postbuckling behavior. Buckling of aerospace structures does not mean complete structural collapse and it is reasonable to design structures allowing buckling of skins [8,9].

In this paper, the minimum-weight design of compressively loaded laminated plates and stiffened panels was performed using a nonlinear finite element (FE) code and

Genetic Algorithm with parallel computing scheme. FE analysis was chosen for the precise postbuckling analysis. Some modifications and a parallel computing scheme in the Genetic Algorithm were implemented to accelerate the optimization procedure. Two previous papers [8,9] have also performed studies in which optimization was aimed to minimize the weight of stiffened composite structures under load constraints in the postbuckling region. However, ply angles and stacking sequences were included as design variables for the present study, but were not considered in the previous ones. Also, although this study used a nonlinear FE analysis and a progressive failure analysis directly to evaluate performances for chosen structures, previous studies instead used a nonlinear FSM [8] and a trained neural network [9]. These analysis techniques are cost-effective but are not as precise as a nonlinear FE analysis. Therefore, compared to previous studies, this study shows an optimization procedure that is more reliable, and although it is less cost-effective, the problem of calculation cost was overcome by improving the Genetic Algorithm.

2. NONLINEAR FINITE ELEMENT ANALYSIS FOR COMPOSITE STRUCTURES

A nonlinear finite element analysis code for composite structures, COSAP (COmposite Structural Analysis Program) which was developed previously in the papers [10-12], was used to analyze the buckling and postbuckling behavior of composite plates and stiffened panels in this study. A brief formulation procedure of the analysis is stated below.

At an arbitrary *(n+1)*th equilibrium state, the principle of virtual work without body force terms can be rewritten in terms of the second Piola-Kirchhoff stress, S_{ij}, and the Green strain, ε_{ij}, with taking the configuration at the *n*th equilibrium state as the reference one:

$$\iiint_{V^n} \left(\sigma_{ij}^n + \Delta S_{ij}\right)\delta\left(\Delta\varepsilon_{ij}\right)dV - \iint_{S_T^n} \left(T_i^n + \Delta T_i\right)\delta\left(\Delta u_i\right)dS = 0 \qquad (1)$$

where σ_{ij}, e_{ij}, T_i, u_i, and δ are Cauchy stress, infinitesimal strain, surface traction, displacement, and variation operator respectively.

The Green strain, $\Delta\varepsilon_{ij}$ can be divided into the linear term, Δe_{ij}, and the nonlinear term, $\Delta\eta_{ij}$.

$$\Delta\varepsilon_{ij} = \Delta e_{ij} + \Delta\eta_{ij} \qquad (2)$$

By substituting $\Delta\varepsilon_{ij}$ given in Eqn. (2) to Eqn. (1), eliminating second-order terms, and implementing stress-strain relation, the equation can be obtained as

$$\begin{aligned} &\iiint_{V^n} \delta\left(\Delta e_{ij}\right)D_{ijkl}^n \Delta e_{kl}dV + \iiint_{V^n} \sigma_{ij}^n \delta\left(\Delta u_{k,i}\right)\Delta u_{k,j}dV \\ &= \iint_{S_T^n} \left(T_i^n + \Delta T_i\right)\delta\left(\Delta u_i\right)dS - \iiint_{V^n} \sigma_{ij}^n \delta\left(\Delta e_{ij}\right)dV \end{aligned} \qquad (3)$$

where D_{ijkl} is the stress-strain relation matrix in the global coordinate system.

The degenerated shell element with 8 nodes is used for the formulation. Each node has 5 DOFs and the shear deformation was implemented with the first-order shear deformation theory. The strain and the displacement can be expressed with the shape functions of the element and the nodal DOF vector.

$$\{\Delta e\} = \left[B_L^n\right]\{\Delta U_n\},\ \{\Delta u,_k\} = \left[B_{NL}^n\right]\{\Delta U_n\} \tag{4}$$

The finite element equation can be obtained by substituting Eqn. (4) to Eqn. (3) as

$$\left([K_L] + [K_{NL}]\right)\{\Delta U_n\} = -\{\Delta P\} \tag{5}$$

where

$$[K_L] = \iiint_{V^n} \left[B_L^n\right]^T \left[D^n\right]\left[B_L^n\right] dV \tag{6}$$

$$[K_{NL}] = \iiint_{V^n} \left[B_{NL}^n\right]^T \left[\overline{\sigma}^n\right]\left[B_{NL}^n\right] dV \tag{7}$$

$$\{\Delta P\} = \iiint_{V^n} \left[B_L^n\right]^T \{\sigma^n\} dV - \{F_n\} \tag{8}$$

In the previous equations, {Fn} is the nodal force vector and $\{\sigma\}$, $[\sigma]$, and $[\overline{\sigma}]$ are defined as follows.

$$\{\sigma\} = \left[\sigma_x \quad \sigma_y \quad \sigma_z \quad \tau_{yz} \quad \tau_{xz} \quad \tau_{xy}\right]^T \tag{9}$$

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \tag{10}$$

$$[\bar{\sigma}] = \begin{bmatrix} [\sigma] & 0 & 0 \\ 0 & [\sigma] & 0 \\ 0 & 0 & [\sigma] \end{bmatrix} \tag{11}$$

In the iteration process of the finite element equation, the arc-length method was used for the load-increment. To estimate the failure load of the structures, the maximum stress criterion is applied to the average stresses in the principal material directions of each layer in each element. The stress component corresponding to the failure mode is unloaded instantaneously.

3. GENETIC ALGORITHM

Genetic algorithm [13] was used as the optimization method in this study. It simulates the natural evolution so that multiple design points gradually evolve into a global optimum. Its calculation process uses a nondeterministic scheme and is not associated with differentiability or convexity. The most useful advantage is that it is very easy to use the discrete ply angles of composites as design variables because, by nature, the Genetic Algorithm uses discrete design variables.

3.1 Parallel Computing Technique

Genetic algorithm is very suitable for the parallel computing scheme because multiple design points should be evaluated in a calculation step. In other words, the algorithm can be programmed so that multiple design points in a generation may be divided into some sub-populations and the corresponding calculation of each sub-population is allocated to one processor in a parallel computer. The programming was coded with MPI (Message Passing Interface) library in this study. Its schematic diagram is shown in Figure 1. The computing system used were CRAY-T3E in the KISTI Supercomputing Center in Korea and a pc cluster with 16 Pentium-4 processors.

3.2 Modification of Genetic Algorithm for Acceleration

In Genetic Algorithm process, the population aggregates to an optimum as convergence is accomplished and some design values that are the same to the ones in the previous generation are re-evaluated, which means the waste of computing time and resources. It becomes a serious problem when the Genetic Algorithm considers postbuckling analyses which take a large amount of computing time for a single case. Thus, it is necessary to avoid waste by modifying the algorithm. There can be many kinds of modifications possible. However, herein is implemented a novel idea to solve the problem. The procedure can be explained briefly as follows:

1. Write all the fitness evaluation results onto a file.
2. Make new generation considering the fitness of the population.
3. In the new generation, find out which design point is the same to the previous one that is written in the file.
4. Read the written results from the file for the overlapped design points that are found in Step 2.
5. Do the real evaluation for newly generated design points only.
6. Append the fitness evaluation results in Step 3 to the file.
7. Go to Step 2 and repeat the procedure.

This method dramatically reduces the number of real evaluations of fitness values. However, the searching process (step 3) might be a time-consuming work, so the method should be applied carefully. In this study, the nonlinear finite element analysis for the fitness evaluation is used. The time consumption of the fitness evaluation is enormous and cannot be compared with the method stated above. Therefore, a modification of Genetic Algorithm was required. One more implementation in the modification was the variable population size. The population size automatically increases in order to guarantee that the minimum number of the real evaluations of fitness is greater than or equal to a particular number which the designer designates. The utility of the variable population size makes the algorithm robust even in the undesirable cases where the initial population size is relatively small and too early convergence is induced.

4. OPTIMAL DESIGN OF COMPOSITE PLATES AND STIFFENED PANELS

4.1 Problem Definition

The optimal shapes, stacking sequences, and ply angles were searched for some composite structures with the modified Genetic Algorithm stated previously. The objective of the optimization was to find the minimum weights of the structures for a given design strength. The objective function was defined as:

$$f = \begin{cases} \dfrac{W_{\max}}{W} \dfrac{11P_{fail}}{10N_{cr} + P_{fail,design}} & , \quad P_{fail} \geq P_{fail,design} \\ \dfrac{P_{fail}}{P_{fail,design}} & , \quad P_{fail} < P_{fail,design} \end{cases} \tag{12}$$

where W_{max} is the possible heaviest weight and W is the weight of the design point. $P_{fail,design}$ and P_{fail} are the design strength and the strength of the design point, respectively. The strength was defined as the load at the moment of the first fiber failure.

4.2 Optimal Design of Composite Plates

The shape of the plate, the boundary conditions, and the loading conditions are shown in Figure 2. The possible maximum number of plies was set as 16, and only 8 plies were used as the design variables because all laminates were assumed to be symmetric. The usable ply angles were limited to 0°, ±45°, 90° for the practical application. The material properties of the composite material are shown in Table 1. The design failure load was fixed at 30kN for this case.

The optimization result showed that the optimal stacking sequence was $[0/90/0_2/90]_S$. The optimized plate has the strength of 34.9kN which is 16% greater than the design value and is 37.5% lighter than the possible heaviest plate. Figure 3 shows the load-deflection curves of three different plates of the same weight: the optimized plate, a quasi-isotropic plate with a stacking sequence of $[0/\pm36/\pm72]_S$, and a unidirectional plate with a stacking sequence of $[0_{10}]_T$. It shows that only the optimized plate endures the design strength.

4.3 Optimal Design of Composite Stiffened Panels

The shape of the stiffened panel, the boundary conditions, and the loading conditions are shown in Figure 4. The design variables were selected as shown in Table 2. The material properties of the composite material are the same as shown in Table 1. In order to determine the reference design strength, a composite stiffened panel, which has the shape and the stacking sequence shown in Table 3, was selected and the postbuckling analysis was performed for this reference stiffened panel. As the result of the analysis, the strength of the panel was 65.5kN, and the design strength was set to be a slightly smaller value, 60kN.

The optimization result is shown in Table 4. The strength is 64.5kN which is almost the same as the reference design strength; however, the weight is 15.4% smaller than the weight of the reference stiffened panel. Figures 5 and 6 show the comparison of the load-deflection curves and the deformed shapes of the reference stiffened panel in Table 3 and the optimized result. Readers may wonder why the optimized shape seems unconventional, and why the stiffener spacing is too small and the cap size is too large. It is mainly because of the uncommonness of the problem. We would like to mention several reasons how this unconventional shape was induced. First thing that we would like to point out is that the skin size and the number of stiffeners were fixed. In real structure, the decrease of the stiffener spacing induces a weight increase because the number of stiffeners increases. However, in this optimization problem, the stiffener spacing does not have direct relationship to the weight. In addition, we think that the optimal shape in this problem is a box beam and the skin sections on both sides have less contribution to the load resistance than the other sections because of the free boundary conditions at the edges on both sides. It can thus be seen that two stiffeners and the middle skin section followed a tendency to form a box beam shape. Although the structure was evolved to a box beam, the cap size was restricted to a certain value because the larger cap size would induce a further increase in weight. For simulating a more realistic situation, modeling of the skin with variable size that assumes a local section of a whole structure is necessary.

Figure 7 shows the changes of the population size as the generation proceeded. In this figure, although the population size was increasing, the number of fitness evaluation by FEM was kept to be slightly greater than or equal to 16. It was intentionally sustained because the total number of processors used in the parallel computing system was 16. If the population size is less than the number of processors, it is inferred that some processors are not used for the fitness evaluation. Thus, the minimum population size was sustained to be sure of using all processors in each generation. In addition, the most important point in this figure is that a reasonably large population can be handled with the small number of real calculations. For example, although the population size was 21, the real calculation was 16 which is smaller than the population size. The last two figures, Figure 8 and Figure 9, show the changes of the variables and some parameters during the optimization procedure.

5. CONCLUSION

In this paper, the weight optimization of composite structures was conducted by considering the postbuckling behavior. In order to estimate the postbuckling behavior, a nonlinear finite element analysis code was applied and the modified Genetic Algorithm was used as the optimization method. The optimization was performed for composite plates and stiffened panels. The optimization results showed that the optimal designs presented in this paper have better performance than conventional designs and the modification of the algorithm was highly effective.

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Figure Captions

Fig. 1. Schematic diagram of Genetic Algorithm with parallel computing.

Fig. 2. Shape, boundary conditions, and loading conditions of composite plates.

Fig. 3. Load-deflection curves of three different plates of same weight: the optimized plate, a quasi-isotropic plate, and a unidirectional plate.

Fig. 4. Shape, boundary conditions, and loading conditions of composite stiffened panels.

Fig. 5. Comparison of the load-deflection curves of the reference and the optimized stiffened panel.

Fig. 6. Comparison of the deformed shapes of the reference and the optimized stiffened panel.

Fig. 7. Changes of the population size as generations proceed.

Fig. 8. Changes of some parameters of the best-fitted designs during the optimization procedure.

Fig. 9. Changes of geometries of the best-fitted designs during the optimization procedure.

Table 1. The material properties of the composite material

property	value	property	value
E_1	130.0 GPa	ν_{12}	0.31
E_2	10.0 GPa	ν_{13}	0.31
E_3	10.0 GPa	ν_{23}	0.52
G_{12}	4.85 GPa	ν_{21}	0.024
G_{13}	4.85 GPa	ν_{31}	0.52
G_{23}	3.62 GPa	ν_{32}	0.024
X_T	1933 Mpa	Y_T	51 Mpa
X_C	1051 MPa	Y_C	141 MPa
S	61 MPa	ply thickness	0.125 mm

Table 2. Definition of design variables of composite stiffened panel design

Skin size	L (mm)	250
	W (mm)	160
Stiffener type		I
Design failure load, $P_{fail.design}$ (N)		60000
Stiffener Location, s	s_{min} (mm)	25
	s_{max} (mm)	56
	bits*	5
Flange size, f (mm)		24 (fixed)
Web size, w	w_{min} (mm)	15
	w_{max} (mm)	78
	bits*	6
Cap size, c	c_{min} (mm)	15
	c_{max} (mm)	78
	bits*	6
Max. number of the skin plies		8×2
Max. number of the stiffner plies		8×2
Bits for a degree		3
Total bits		65
Number of possible designs		3.7×10^{19}

* : number of bits for s, w, c.

Table 3. Shape and stacking sequence of the reference composite stiffened panel

Stiffener location, s (mm)	Web height, w (mm)	Cap width, c (mm)	Skin stacking sequence	Stiffener stacking sequence	Ultimate failure load, P_{fail} (N)	Weight, W (kg)
30	25	20	$[0/90/\pm45]_S$	$[0/90/\pm45]_S$	65524	0.1192

Table 4. Optimal design results of composite stiffened panels

Stiffener location, s (mm)	Web height, w (mm)	Cap width, c (mm)	Skin stacking sequence	Stiffener stacking sequence	Ultimate failure load, P_{fail} (N)	Weight, W (kg)
52	21	43	$[0/90/90]_S$	$[90/0/0]_S$	64514	0.1008