STRUCTURAL ANALYSIS AND STRAIN MONITORING OF THE FILAMENT WOUND MOTOR CASE

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ABSTRACT

Filament wound structures such as pressure tanks, pipes and motor cases of rockets are widely used in aerospace applications. The determination of a proper winding angle and thickness is very important to decrease manufacturing difficulties and to increase structural efficiency. In this study, possible winding angles considering the slippage between a fiber and a mandrel surface are calculated using the semi-geodesic path equation. In addition, finite element analyses using ABAQUS are performed to predict the behavior of filament wound structures considering continuous change of the winding angle and thickness at the dome part due to fiber built-up near the metallic boss. Water-pressuring tests of a 3rd stage motor case are performed to verify the analysis procedure. The strain gages are attached on the surface in the fiber direction. Progressive failure analysis predicted the burst pressure a little bit higher than the experiment and the weakest region of the motor case very well. The effect of reinforcement is also studied to increase its performance.

Keywords : Filament winding, Semi-geodesic path equation, Finite element analysis, Motor case, Water-pressuring test

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< Nomenclatures >

$S(x,\theta)$: arbitrary surface defined with x, θ coordinate

λ : slippage tendency

$\vec{f}_b$: transverse force

$\vec{f}_n$: normal force

$\vec{c}$: curvature vector towards the center of curvature of the curve

α : filament winding angle

SRC : stiffness reduction coefficient

t_c : thickness of a helical layer in the cylinder part

α_c : winding angle of a helical layer in the cylinder part

1. INTRODUCTION

Fiber reinforced composites are widely used in aircrafts, rockets and automotive structures for their low weight with high strength and stiffness. Good corrosion and chemical resistance of composites allow them to be used in storage and transport of fluid and gases for rockets and automotives. In addition, a designer can utilize the anisotropy produced by building up a laminate from plies with a properly selected fiber/resin combination and orientation/stacking sequence to meet the performance requirements.

Filament winding is one of the production techniques for high performance composites. Continuous filaments are the cheapest and strongest form of fiber reinforcement and can be oriented to match the direction of stresses loaded in a structure. Filament winding also allows the placement of fibers with a highly reproducible degree of precision. However, the problem with filament winding is that the trajectory of the fiber path and the corresponding fiber angles cannot be chosen freely because of the fiber path stability requirement. Fiber path instability induced by fiber slippage on the mandrel surface cannot be predicted because it is affected by many parameters, such as mandrel shape, temperature, surface treatment, fiber/resin combination and so on. Therefore, most design and manufacturing of wound structures are based on manufacturing experience as well as trial and error.

With the increasing use of filament wound structures, analysis of wound structure and winding patterns has drawn more and more attention in recent years. Several authors have investigated various methods to determine stable winding patterns [1-4]. The mandrel shape can be completely predetermined, or the design of the shape can be an integral part of the winding design in the previous studies. In other words, previous methods have calculated dome shape and fiber path simultaneously for a given parameter such as the radius ratio of openings to a cylinder part. Carvalho, *et al.*[5] determined the best suited feasible angle for a given surface geometry with prevention of fiber slippage and bridging over the locally concave surface. Lossie, *et al.*[6] proposed a stable non-geodesic winding often called semi-geodesic winding which can keep the fiber in its proper position without slipping. Both geometry and lay-up were optimized using semi-geodesic winding. Scholliers, *et al.*[7] developed a computer-integrated winding environment including design, production and quality control. A T-shaped piece is wound with tapes and the fiber path is calculated from a semi-geodesic path equation.

In most of the finite element analyses of filament wound pressure tanks, layered shell and axisymmetric solid elements are used [4, 8-12]. For an axisymmetric solid element, the 3-dimensional effective modulus should be calculated for each layer and the transformation of stress and strain to fiber direction should be done. As a result, reduction of material properties in the layer level is not easy in progressive failure analysis. When shell elements are used in the analysis, detailed modeling of metallic boss is impossible without a solid element connected by rigid beams, and stress concentration at the knuckle part is sometimes overestimated due to rapid change of curvature and thickness. Doh, *et al.*[4] performed a progressive failure analysis of filament wound motor case using 8-node degenerated shell elements. It was shown that geometrical and material nonlinearity should be included for filament wound structures due to large deformation induced bending on the dome part and material failure. Several failure modes, such as matrix cracking and fiber breakage are included in the material nonlinear analysis and the strains measured in the pressuring experiments are compared to the results of the analysis. Jeusette, *et al*.[8] developed an isoparametric axisymmetrical element to analyze composite revolution structures. Variation of the fiber angle along the width of a layer in the analysis of the filament wound motor case is taken into account. It is pointed out that the relative difference between fiber directional stresses can reach 40% in the polar region due to the winding angle difference along the width of a winding tow. James, *et al*.[9] studied the local reinforcement using hoop-wound wafers around the polar boss, because the dome/polar boss interface is a very highly-stressed area. The hoop-wound wafers were shown to be very effective in increasing the efficiency of graphite pressure vessels, but several design conditions should be followed to maximize the benefits. Sun, *et al*.[10] performed a nonlinear finite element analysis to calculate the stress and final bursting pressure of rocket motor cases. Maximum stress failure criteria and a stiffness-degradation model were introduced to the failure analysis. The location and burst pressure predicted showed good agreement with experimental results.

In the filament winding literature, typical filament winding-related failure criteria or strength predictions are not often treated, but many researchers have found that quadratic failure criteria, such as Tsai-Wu criterion, give reasonable agreement on filament wound components[13]. In general, Tsai-Wu failure criteria cannot distinguish failure modes, but only predict the occurrence of failure under various stress conditions. A recent syudy to determine failure mode using Tsai-Wu criterion is Reddy's work. Reddy, *et al*.[14] divided the equation of the Tsai-Wu criterion into three groups and evaluated the contribution of each group to failure. According to the group that contributes most to the failure index, failure mode is determined. Then, the failed element is replaced with an equivalent element of degraded properties. In order to simulate gradual degradation, SRC (stiffness reduction coefficient) having a value between 0 and 1 is introduced.

In this study, 3-d solid elements for each layer are used to model a motor case and polar openings using a commercial code, ABAQUS. The user subroutine is coded to consider a winding angle variation over the dome region in the analysis. In addition, a subroutine, UMAT, is also programmed to impose material nonlinearity due to the failure. From the result of the finite element analysis, strains over the motor case are compared to the strains measured by strain gages and fiber optic sensors. Final burst pressure and locations are also predicted in the analysis.

2. FINITE ELEMENT ANALYSIS

2.1. Fiber Path Calculation

The design of a filament wound structures consists of two main parts: designs of the mandrel shape and the fiber paths. In general, the mandrel shape can be determined by imposed design requirements, manufacturing convenience or considering stable winding patterns. When the liner or mandrel surface is given without considering winding patterns, it is required to find a possible winding angle with respect to various slip-conditions. In this study, differential equations are derived for a semi-geodesic path and are solved numerically. More detailed derivation is shown in the reference [15].

An arbitrary surface is defined as $S(x,\theta)$ with x, the axial coordinate and θ, the circumferential coordinate in Figure 1.

$$S(x,\theta) = [x \;\; r(x)\cos\theta \;\; r(x)\sin\theta]^T \qquad (1)$$

Then the slippage tendency, λ, is defined as the ratio between the transverse force, $\vec{f}_b$ and the normal force, $\vec{f}_n$.

$$\lambda = \frac{\|\vec{f}_b\|}{\|\vec{f}_n\|} = \frac{\vec{f}_r \cdot \vec{b}}{\vec{f}_r \cdot \vec{n}} = \frac{\vec{c} \cdot \vec{b}}{-\vec{c} \cdot \vec{n}} \tag{2}$$

The curvature vector is directed towards the center of curvature of the curve and equals :

$$\vec{c} = \frac{d\vec{t}}{ds} = \vec{S}_x \frac{d^2x}{ds^2} + \vec{S}_\theta \frac{d^2\theta}{ds^2} + \vec{S}_{xx}\left(\frac{dx}{ds}\right)^2 + 2\vec{S}_{x\theta}\frac{dx}{ds}\frac{d\theta}{ds} + \vec{S}_{\theta\theta}\left(\frac{d\theta}{ds}\right)^2 \tag{3}$$

where $S_x = \left[\frac{1}{A}\ \frac{r'}{A}\cos\theta\ \frac{r'}{A}\sin\theta\right]^T$, $S_\theta = [0\ -\sin\theta\ \cos\theta]^T$

The normal, $\vec{n} = -\vec{S}_x \times \vec{S}_\theta$ on the surface equals

$$\vec{n} = \left[\frac{-r'}{A}\ \frac{1}{A}\cos\theta\ \frac{1}{A}\sin\theta\right]^T \tag{4}$$

The binormal, $\vec{b}$ can be calculated as follows:

$$\vec{b} = \vec{t} \times \vec{n} = \frac{1}{A}\begin{bmatrix} -\sin\alpha \\ -A\sin\theta\cos\alpha + r'\cos\theta\sin\alpha \\ -r'\sin\theta\sin\alpha - A\cos\theta\cos\alpha \end{bmatrix} \tag{5}$$

where $\vec{t} = \frac{dS}{ds} = \left[\frac{dx}{ds}\ \ r'\frac{dx}{ds}\cos\theta - r\frac{d\theta}{ds}\sin\theta\ \ r'\frac{dx}{ds}\sin\theta + r\frac{d\theta}{ds}\cos\theta\right]^T$

Equation (2) can be simplified as Equation (6) with Equations (3), (4), (5)

$$\lambda = \frac{A^2 r' \sin\alpha + A^3 r \dfrac{d\alpha}{ds}}{A^2 \sin^2\alpha - r r'' \cos^2\alpha} \tag{6}$$

or, in function of $\dfrac{d\alpha}{ds}$

$$\frac{d\alpha}{ds} = \frac{\lambda\left(A^2 \sin^2\alpha - r r'' \cos^2\alpha\right) - A^2 r' \sin\alpha}{A^3 r} \tag{7}$$

Using the chain rule to Equation (7) yields Equation (8).

$$\frac{d\alpha}{dx} = \frac{\lambda(A^2 \sin^2\alpha - r r'' \cos^2\alpha) - r' A^2 \sin\alpha}{r A^2 \cos\alpha} \tag{8}$$

where $\dfrac{dx}{ds} = A\cos\alpha = \dfrac{\cos\alpha}{\sqrt{1+r'^2}}$

By integrating Equation (8) from the known winding angle, $\alpha = 90°$ at the boss, the winding angle, α can be calculated for the entire dome surface. Two assumptions are underlying in the calculation of the thickness in the dome region: fiber volume fraction is maintained consistently and the number of fibers in a cross section is always constant. With these assumptions, the thickness along the longitudinal direction can be derived as follows.

$$(2\pi r)t\cos\alpha = (2\pi r_c)t_c\cos\alpha_c \qquad (9)$$

$$t = \frac{r_c\cos\alpha_c}{r\cos\alpha}\times t_c \qquad (10)$$

where r_c, α_c, t_c are the radius, fiber angle and thickness of the cylindrical region.

As the winding fiber approaches the boss, α becomes 90° and the right-hand side of Equation (10) becomes infinity, as does the thickness theoretically. Thickness divergence is caused by fiber concentration on relatively a small area. That is to say, the fiber rotates repeatedly around the boss in numerical calculation before it winds to the opposite direction. Thus the winding thickness is corrected to be constant from 98 % to 100 % of the meridian length measured from the cylinder-dome junction in this study.

The winding angle and thickness for the first ply are calculated from a given geometry of the motor case using Equations (8) and (10). The winding angle and thickness for the next ply might differ from the first one because of surface changes due to the fiber built-up on the mandrel. In this study, winding angle and thickness are calculated to the last layer ply by ply. The winding angle of the last ply is slightly different from the angle of the first one near the boss region, because fibers are extremely built up at the polar openings.

2.2. Finite Element Modeling

Finite element analyses were performed for the 3rd stage rocket motor case by the commercial code, ABAQUS. Configuration of the rocket motor is shown in Figure 2 and Table 1. The rocket motor is wound with tow prepreg by dry process. The T-800 carbon fibers and Novolac type resin are used. The forward dome has a small opening and is connected to a satellite. The dome shape consists of two hemispherical curves which have different radii and centers. The aft dome has a bigger opening compared to the forward one and is connected to the rocket nozzle. Its surface also is made by two hemispherical curves, but the aft dome shape is very similar to an isotensoid dome. The skirts are wound for reinforcing the knuckle parts and connecting to other structures in the rocket. Epoxy filler is applied between the skirts and the dome to transfer the stress to the skirt. EPDM, an insulating material, is inserted between the metallic boss and composite to prevent de-bonding from each other and to insulate the gas or heat generated during combustion.

Finite element modeling is performed for 1.5° strip of a full motor case using cyclic symmetry boundary condition. The finite element mesh is shown in Figure 3. Z-displacement is restricted at the center nodes of the cylinder part, and radial displacement is fixed at the boss tip. Internal pressure, 6.895 MPa(1000 psi), is applied on the inner surfaces of the elements. A 3-dimentional solid element(C3D27) with 27 nodes is used for each layer. In dome parts, 12 layers(helical winding layers) are inserted with ±22° winding angle and in the cylinder part, several hoop layers(90°) are inserted between helical layers. The helical winding layer is 0.198 mm thick per 2 layers(+22°/−22°) and hoop winding layer is 0.168 mm thick per one layer. The first skirt and second skirts have $[\pm30°_4 / 90°_2 / \pm30°_4 / 90°_2]$ and $[90°_2 / \pm15°_2 / 90°_2 / \pm15°_2 / 90°_6]$ lay-up. Material properties are as follows :

$$E_1 = 142 \text{ GPa},\ E_2 = E_3 = 3.136 \text{ GPa},\ G_{12} = G_{13} = 4.69 \text{ GPa},\ G_{23} = 1.00 \text{ GPa},\ \nu_{12} = \nu_{13} = 0.33,\ \nu_{23} = 0.45$$

$X_T = 2687$ MPa, $X_C = 1441$ MPa, $Y_T = Z_T = 36.36$ MPa, $Y_C = Z_C = 70$ MPa, $R = S =$

T = 59.6 MPa.

In order to impose winding angles on the elements along the dome region, a user subroutine is introduced in the analysis. The subroutine, ORIENT, is coded to consider the local material directions for anisotropic or orthotropic material. First, material directions are calculated from the location of the vertex nodes in each element. Second, fiber angles calculated beforehand along the dome region are read at each integration point and then material directions are transformed to meet the winding angles. More precise fiber angles are imposed at each element compared to methods using single winding angle for one element.

For the purpose of failure analysis, a subroutine, UMAT is coded to define the change of mechanical properties due to the failure. In this study, Tsai-Wu failure criterion is used to assess occurrence of failure. Tsai and Wu developed their failure criterion by considering the interaction of stresses for a composite material. In a three-dimensional stress space, they are as follows.

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j \geq 1 \qquad (11)$$

$$F_1 = \left(\frac{1}{X_T} - \frac{1}{X_C}\right),\ F_2 = \left(\frac{1}{Y_T} - \frac{1}{Y_C}\right),\ F_3 = \left(\frac{1}{Z_T} - \frac{1}{Z_C}\right) \qquad (12)$$

$$F_{11} = \left(\frac{1}{X_T X_C}\right),\ F_{22} = \left(\frac{1}{Y_T Y_C}\right),\ F_{33} = \left(\frac{1}{Z_T Z_C}\right) \qquad (13)$$

$$F_{44} = \left(\frac{1}{R^2}\right),\ F_{55} = \left(\frac{1}{S^2}\right),\ F_{66} = \left(\frac{1}{T^2}\right) \qquad (14)$$

$$F_{12} = -\frac{1}{2}\left(\frac{1}{X_T X_C Y_T Y_C}\right)^{\frac{1}{2}},\ F_{13} = -\frac{1}{2}\left(\frac{1}{X_T X_C Z_T Z_C}\right)^{\frac{1}{2}},\ F_{23} = -\frac{1}{2}\left(\frac{1}{Y_T Y_C Z_T Z_C}\right)^{\frac{1}{2}} \qquad (15)$$

Where X, Y, Z represent the strength of each direction, and the subscript T,

C mean tensile and compressive values respectively. R , S , T denotes YZ, XZ, ZY-plane shear strength.

F_i is a linear coefficient that consists of axial loads and F_{ii} is the quadratic stress term. F_{ij} is the term which include the interaction effect of various normal stresses. To determine the failure mode, terms of Tsai-Wu criterion are divided into three parts shown below.

$$I_1 = F_1\sigma_1 + F_{11}\sigma_1^{\,2} + 2\left(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3\right) \qquad (16)$$

$$I_2 = F_2\sigma_2 + F_{22}\sigma_2^{\,2} + F_{66}\sigma_{12}^{\,2}\sigma_2 + 2\left(F_{12}\sigma_1\sigma_2 + F_{23}\sigma_2\sigma_3\right) \qquad (17)$$

$$I_3 = F_3\sigma_3 + F_{33}\sigma_3^{\,2} + 2\left(F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3\right) + F_{44}\sigma_{23}^{\,2} + F_{55}\sigma_{13}^{\,2} \qquad (18)$$

$$\text{IF } F_i\sigma_i + F_{ij}\sigma_i\sigma_j \geq 1, \text{ then failure mode} = Max(I_1, I_2, I_3) \qquad (19)$$

I_1 mainly depends on the fiber directional stresses, and I_2 is a transverse-direction dominated term. I_3 consists of the stresses related to the thickness direction. When failure occurs at a certain element, the values of three terms are compared with each other to find the largest value. Then the failure mode is determined such that I_1 represents fiber failure, I_2 matrix failure and I_3 delamination failure, respectively.

An element failure is first identified and later the failed element is replaced with a degraded element. However, total stress accumulated till the failure occurs is not removed. In the static analysis, removal of stresses at a certain increment causes a large displacement of nodes because there is no mechanism to consider the energy absorbed by failure or dissipated by sound, etc. Stresses of 14 integration points are averaged before the evaluation of failure. The degradation method should be carefully chosen because results of progressive failure analysis often depend on the mesh size, increment size, degree of reduction, etc. The equivalent properties of the damaged element might exist between solid ones and values of zero. In the present study, the stiffness reduction factor varies from 0.05 to 0.9. In the case of SRC=0.05, the failed element almost lost its load-carrying capacity and the predicted bursting pressure might be taken as a minimum value. When SRC is 0.9, mechanical properties of failed element do not decrease much compared to the solid properties. It could be taken as a maximum bursting pressure.

2.3. Pressuring tests

To verify the procedure of finite element analysis, pressuring tests for the motor case are performed. 2 channels of strain gages are attached to the surface of the motor case. Each channel has 16 strain gages with 5 mm gage length. All strain gages are aligned to the fiber direction except 2 gages attached in the longitudinal direction. Fig 4 shows the location of the strain gages. 5 strain gages per one channel are attached on the aft dome because the meridian length is relatively short compared to the forward dome. The strain gages are stuck along the same prepreg tow. At the cylinder part, 8 gages are attached in the fiber or circumferential direction and 2 gages are bonded in the longitudinal direction. 6 gages are applied on the forward dome per one channel, and totally 32 strain gages are used to monitor the strain.

The arrangements of the specimen and other devices are shown in Fig. 4. The signals of each strain gauge and pressure transducer are measured simultaneously by an A/D converter(LabVIEW Instruments, PCI 6110E) and stored in the computer. Pressure is applied to 6.895 MPa (1000psi) by several steps. The pressure level is held to check the pressure drop that occurs due to leaking water at each step.

3. RESULTS AND DISCUSSION

3.1. Comparison of strains between analysis and experiment

Fig. 6 shows the deformation shape and strain distribution simulated by finite element analysis without any reinforcement on the dome part. Because the forward dome consists of thin helical layers and has a long meridian surface, the swelling of the central dome is large enough for the metallic boss to withdraw from the original position. As a result, bending deformation of the dome induces the stress concentration and large fiber directional stress gradient at the center of the dome. The center region of the dome and the end of the metallic boss show the maximum fiber directional stress. It is well known that all of the internal pressure acting across the polar opening must be transferred to the motor case within the boss flange and the dome/polar boss interface is a very highly stressed area. Therefore, it's required to reinforce the dome and boss flange region. The motor case used in the experiment is reinforced by a unidirectional prepreg over the entire dome region. Fig. 7 shows the stress distribution after the reinforcements are inserted. The stiffness in the circumferential direction is increased so that the swelling of the dome part is restricted and the metallic boss moves forward. As a result, the stress level on the dome part is greatly decreased and stress concentration due to excessive bending of the dome disappears. In addition, the strains measured by strain gages are compared with each other in Fig. 8. They show good agreement at all the regions except the forward opening region. This might be due to the fact that the measuring of the attached location of strain gages has an error and the amount of fiber-built-up near the opening could not be predicted exactly in the analysis. However, the result of the finite element analysis shows reasonable agreement and can be applied to predict the final bursting pressure with a progressive failure routine.

3.2. Progressive failure analysis

Fig. 9 shows the development of transverse stress in a failed element according to the load steps. The first matrix cracking occurred at 8.8 MPa(1276 psi) and the properties related to transverse direction are degraded to 5% of the solid case. As a result, stress development makes slow progress after degradation. All other failed elements show the same tendency. However, fiber directional stress of transversely cracked elements is not affected much by reduction of mechanical properties in the matrix dominant direction. This means that the fibers in the filament wound structure carry most of the applied pressure load and the matrix makes a small contribution to supporting the internal pressure. Fig. 10 locates the first occurrence of each failure mode. Most of the matrix-cracking failures are located near the opening but they do not affect the stability of the structure until the failed region expands to the entire dome part. In case of fiber breakage, failure occurs at the helical layers under the skirt in the knuckle part and instability induced from the reduction of fiber directional modulus causes the solution to diverge. Because local stress concentration caused by the fiber failure could not be distributed to the layers or fibers well, the damage of local fiber bundles causes an entire structure to fail at the same time in the filament wound structures. In the analysis, the final bursting occurs at both the knuckle parts in the pressure value of 32.64 MPa (4732 psi). From the experimental results, the designed motor cases usually failed between 20.69 MPa (3000 psi) to 31.03 MPa (4500 psi) at the knuckle part. The bursting pressure predicted is relatively high compared with the experimental result, but the location of a weak region is well predicted. The discrepancies between experiment and analysis might be produced from the inaccurate strength data, manufacturing defects, and errors induced from improper material degradation model in the progressive failure analysis.

4. CONCLUSIONS

Numerical and experimental analyses of filament wound structure are performed for the third stage rocket motor case. The stable winding angles without fiber slippage are calculated for an arbitrary surface using a semi-geodesic path equation. Fiber built-up near the polar opening produced a change of winding angle through the thickness direction, hence the angle variation in thickness direction should not be neglected for a wound structure which has thick helical layers. Finite element analyses considering geometric and material non-linearity predict the behavior of the wound structure well under internal pressure. The property degradation of the transverse direction rarely increases the fiber directional stress, but affects the solution stability when matrix and delamination failures are accumulated. The predicted bursting pressure is somewhat high compared with experimental results, but the location where final burst occurs is well predicted in the progressive failure analysis. Using the proposed analysis procedure mentioned in this study, the performance of motor cases or filament wound pressure vessels are compared with each other for various design parameters such as winding angles, helical winding thickness, geometries of openings and so on. This could reduce the time required to design filament wound structures.

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