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Bias Estimation of Magnetometer using Genetic Algorithm

Eunghyun Kim[1] and Hyo-Choong Bang[2]

[1] Korea Aerospace Research Institute, Daejeon, Korea
(Tel : +82-42-860-2433; E-mail: ekim@kari.re.kr)
[2] Department of Aerospace Engineering, KAIST, Daejeon, Korea
(Tel : +82-42-869-3722; E-mail: hcbang@fdcl.kaist.ac.kr)

Abstract: A number of algorithms have been proposed for estimating the magnetometer bias without the spacecraft attitude knowledge. The simplest is the scalar checking which minimizes the differences in the squares of the magnitudes of the measured and modeled magnetic fields. This approach is that the cost function is presented as forth order equation with respect to the magnetometer bias and therefore is required to find global minimum. Magnetometer bias estimation using the genetic algorithm was proposed. New estimation method was the same structure like TWOSTEP. The genetic algorithm provides the initial estimate to find the final solution. The proposed bias estimation using genetic algorithm was compared with other optimization algorithm like the gradient method and TWOSTEP.

Keywords: Magnetometer, Bias, Estimation, Genetic Algorithm

1. INTRODUCTION

Three-axis magnetometer is widely used for onboard spacecraft operations. Magnetometer is useful because it provides both the direction and magnitude of the magnetic field. However the attitude obtained using magnetometer measurements is not accurate. The on-board accuracy using a three-axis magnetometer can be improved through the calibration of magnetometer biases.

The simple method for estimating the magnetometer bias without the spacecraft attitude knowledge is the scalar checking which minimizes the differences in the squares of the magnitudes of the measured and modeled magnetic fields. The scalar checking is based on the principle that scalars, such as the magnitude of a measurement vector, do not depend on the coordinate system [1]. As shown Fig. 1, a vector measured from the magnetometer at any time has the same magnitude in both different coordinates, while the components of a vector have not the same value. Therefore this method does not require an attitude estimate for the bias estimation. However this approach has the disadvantage that the cost function is presented as quartic equation with respect to the magnetometer bias and therefore is required to find global minimum. To overcome this disadvantage, many algorithms have been proposed and Alonso *et al.* [2] examined the several algorithms for in-flight magnetometer bias determination without knowledge of the attitude. Also the new method, which is called TWOSTEP, was proposed. TWOSTEP combines the convergence in a single step of a heuristic algorithm with the correct treatment of the statistics of the measurement [3].

In the present paper magnetometer bias estimation using genetic algorithm was proposed. The method, which we call TWOSTEP using genetic algorithm, seeks to the same computational step with TWOSTEP. The difference from TWOSTEP is to use the genetic algorithm to find the initial estimate.

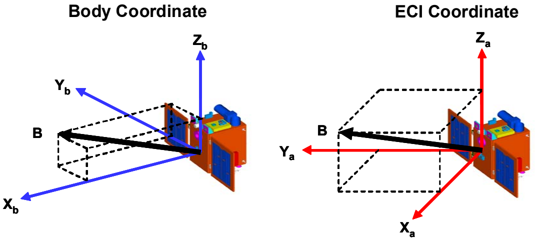

Fig. 1 Vector magnitude in different coordinates.

2. FORMULATION

2.1 Measurement Model

The magnetometer measurements can be modeled as the below equation.

$$\vec{B}_k = A_k \vec{H}_k + \vec{b} + \varepsilon_k, \qquad k = 1,\cdots,N, \tag{1}$$

where $\vec{B}_k$ is the measurement of the magnetic field by the magnetometer at time t_k, $\vec{H}_k$ is the corresponding value of the geomagnetic field with respect to an Earth fixed coordination, A_k is the unknown attitude matrix of the magnetometer with respect to the Earth-fixed coordinates, $\vec{b}$ is the magnetometer bias vector, and ε_k is the measurement noise vector assumed to be a zero-mean Gaussian process.

An attitude independent observation and the effective measurement noise can be formulated as the below equations.

$$z_k = \|\vec{B}_k\|^2 - \|\vec{H}_k\|^2, \tag{2}$$

$$v_k = 2(\vec{B}_k - \vec{b}) \cdot \varepsilon_k - \|\varepsilon_k\|^2, \tag{3}$$

Therefore,

$$z_k = 2\vec{B}_k \cdot \vec{b} - \|\vec{b}\|^2 + v_k, \qquad k = 1,\ldots,N, \tag{4}$$

This is the starting point for the derivation of all of the algorithms. Assuming that the measurement noise ε_k on the magnetometer measurement is white and Gaussian with $\varepsilon_k \sim \mathrm{N}(0,\Sigma_k)$, it follows to very good approximation that the effective scalar noise satisfies

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$$v_k \sim \mathrm{N}(\mu_k, \sigma^2{}_k), \tag{5}$$

where

$$\mu_k = -tr(\Sigma_k), \tag{6}$$

$$\sigma^2{}_k = 4(\vec{B}_k - \vec{b})^T \Sigma_k (\vec{B}_k - \vec{b}) + 2[tr(\Sigma^2{}_k)], \tag{7}$$

2.2 Cost Function Formulation

Given the statistical model above and used the maximum likelihood estimation, the negative log likelihood function for the magnetometer bias is given by the following equation.

$$J(\vec{b}) = \frac{1}{2}\sum_{k=1}^{N}\left[\frac{1}{\sigma^2{}_k}(z_k - 2\vec{B}_k \cdot \vec{b} + \|\vec{b}\|^2 - \mu_k)^2 + \log\sigma^2{}_k + \log 2\pi\right], \tag{8}$$

The maximum likelihood estimate maximizes the likelihood of the estimate, which is the probability density of the measurements given as a function of the magnetometer bias [3]. The likelihood estimate minimizes the negative-log-likelihood function which provides the cost function.

3. BIAS ESTIMATION METHODS

3.1 Gradient Method

We already know the negative-log-likelihood function as the cost function and can obtain the solution of this cost function by scoring. Classical gradient method like Newton- Raphson or Gauss-Newton can be applied to the scoring. We replace the Hessian matrix of the cost function by its expectation to the Fisher information matrix *F*. As the sample number *N* approaches the infinite, Fisher information matrix is as following.

$$\frac{\partial^2 J}{\partial\vec{b}\partial\vec{b}^T} \equiv F_{bb} = \sum_{k=1}^{N}\frac{1}{\sigma^2{}_k}4(\vec{B}_k - \vec{b})(\vec{B}_k - \vec{b})^T, \tag{9}$$

The scoring procedure becomes now as the below equations.

$$\vec{b}_0^{GN} = 0, \tag{10}$$

$$\vec{b}_{i+1}^{GN} = \vec{b}_i^{GN} - F_{bb}^{-1}\frac{\partial J}{\partial\vec{b}}(\vec{b}_i^{GN}), \tag{11}$$

If we are sufficiently close to the maximum likelihood estimate for some *i*, the *i* th bias estimate will become the maximum likelihood for the unknown bias vector.

3.2 TWOSTEP

TWOSTEP is extended from RESIDG algorithm of Gambir [2]. At first in order to avoid the minimization of a quartic cost function in Eq. (8), the centering operation is executed. Thus the centering measurement equation is led as Eq. (12)

$$\tilde{z}_k = 2\tilde{\vec{B}}_k \cdot \vec{b} + \tilde{v}_k, \qquad k = 1, \ldots, N, \tag{12}$$

Therefore using the centered measurements we can solve for the centered maximum-likelihood estimate in a single iteration of the Newton-Raphson method or Gauss-Newton method.

The second step consists of using the centered estimate as an initial value and computing the corrected estimate by applying the Gauss-Newton method to the full negative-log-likelihood function.

3.3 TWOSTEP using Genetic Algorithm

Genetic algorithm (GA) is search algorithm based on the natural selection and natural genetics [4]. A simple genetic algorithm consists of the following stages: reproduction, crossover, and mutation. Reproduction is the process by which individual chromosomes are being reproduced according to their fitness function or likelihood function. Thus, more likely chromosomes will have higher probability of contributing offspring in the next generation. Crossover is the process of exchanging gene between two reproduced chromosomes. A simple crossover is carried out by selecting two chromosomes randomly and swapping their characters from a randomly selected position. Whether or not to perform a crossover is dictated by the probability of crossover. Even though reproduction and crossover improve the population, they can become overzealous and lose potentially important genetic information. Mutation protects against such an irrecoverable loss by simply altering each gene with a probability of mutation.

TWOSTEP using genetic algorithm uses the genetic algorithm to find the initial estimate. The centered estimate in TWOSTEP using genetic algorithm computes from the genetic algorithm unlike original TWOSTEP algorithm.

4. SIMULATED DATA RESULT

We assumed that a spacecraft attitude is a three axis stabilization type with Earth nadir pointing. The spacecraft orbit chose to be sun-synchronous circular orbit with an altitude 685km and an inclination 98.13 deg. The geomagnetic field simulated using the International Geomagnetic Reference Field model. For the purpose of simulation, we assumed that the magnetometer measurement error was an effective white Gaussian with isotropic error distribution. A standard deviation per axis was assumed as 0.5 mG in Case 1 and 10 mG in Case 2. The geomagnetic field data were used for the reference magnetic field data and spanned for 2 orbits with 10 second time interval. Bias error modeled as [10 20 30] mG.

Reproduction using tournament method, direction based crossover, and uniform mutation was applied to the genetic algorithm. Crossover probability was 25% and mutation probability was 1% during the simulation.

Fig. 2 and Fig. 3 show the simulation results using the genetic algorithm when each measurement is 200 second and 2000 second span. At both simulations the population size was 40. According to Fig. 3, the bias estimate was converged into the true value when using measurement of more than 200 second spans.

Fig. 4 and Fig. 5 show the simulation result by the variation of the population size. In Fig. 4 the population size was 10 and the bias estimate did not converge into the modeled value until 100 generation. As shown Fig. 5, the bias estimate converged into when the population size was more than 20.

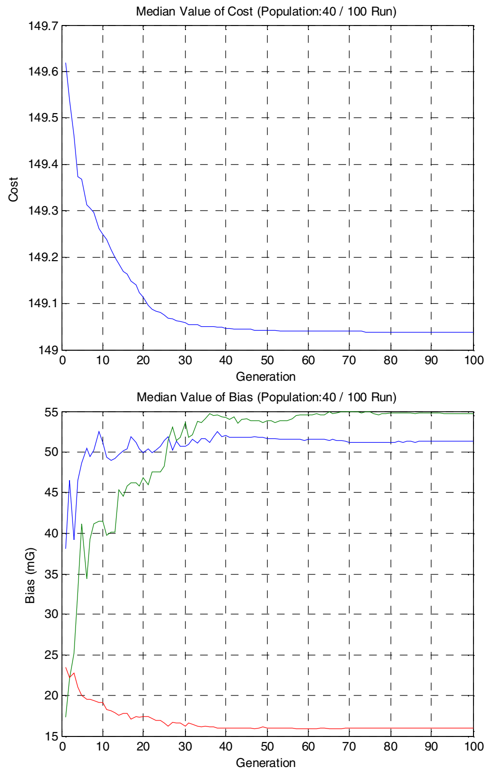

Fig. 2 Median value using 200 sec measurement.

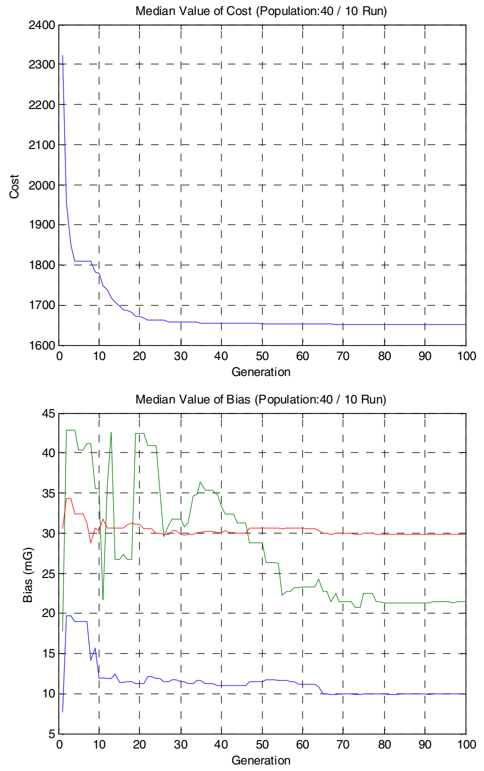

Fig. 3 Median value using 2000 sec measurement.

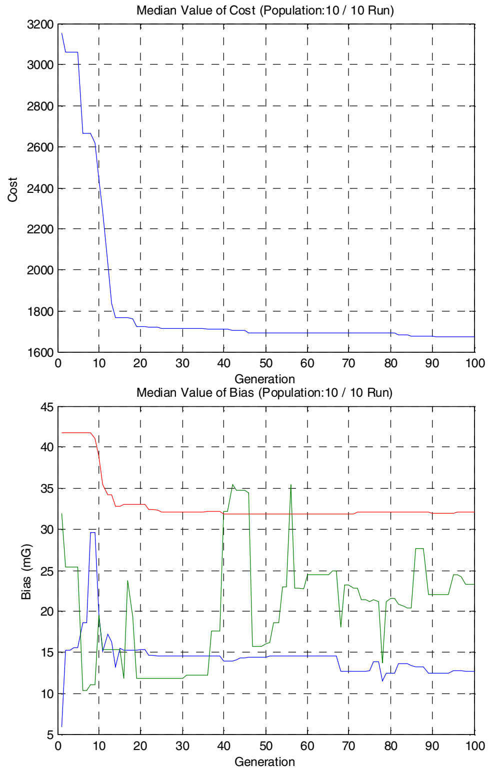

Fig. 4 Simulation result as population size 10.

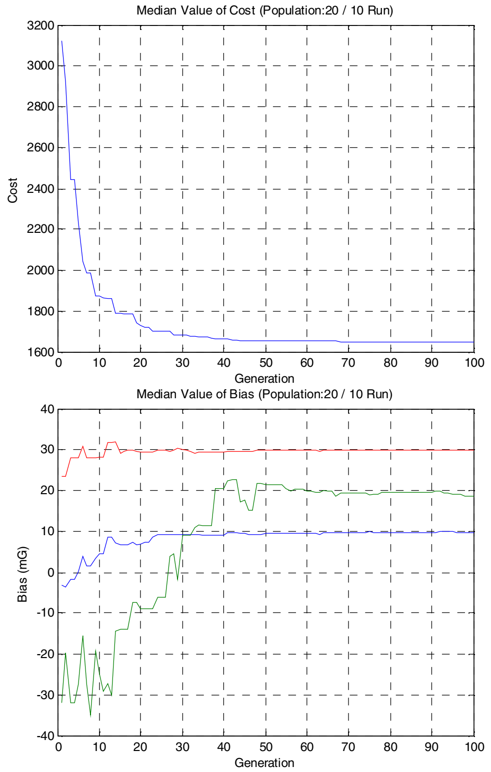

Fig. 5 Simulation result as population size 20.

Table 1 and Table 2 shows that TWOSTEP using genetic algorithm has a good performance. Table 1 shows the result using the magnetometer measurement with 0.5 mG standard deviation error. In Table 2, the measurement error had 10 mG standard deviation per axis. Population size and the measurement for genetic algorithm were determined from the previous genetic algorithm experiments. As shown Table 1 and Table 2, TWOSTEP using genetic algorithm acquired an approximate estimate with only two computational steps.

Table 1 Result of TWOSTEP using GA (Case 1).

Bias axis	b_1	b_2	b_3
Model bias (mG)	10	20	30
1st Step Estimate	9.72	18.77	29.85
2nd Step Estimate	9.95	19.63	30.00

Table 2 Result of TWOSTEP using GA (Case 2).

Bias axis	b_1	b_2	b_3
Model bias (mG)	10	20	30
1st Step Estimate	10.70	5.68	30.97
2nd Step Estimate	10.13	18.82	29.92

Table 3 shows the comparison TWOSTEP using genetic algorithm with original TWOSTEP and classical gradient method. TWOSTEP and TWOSTEP using genetic algorithm performed well while the result of the gradient method had small deviation. The simulated data with nadir pointing attitude caused more inaccurate estimate in y-axis of body coordinate.

Table 3 Comparison of algorithm performance.

Bias axis	b_1	b_2	B_3
Model bias (mG)	10	20	30
Gradient Method	9.81	14.80	29.83
TWOSTEP	10.11	18.87	29.90
TWOSTEP using GA	10.13	18.82	29.92

5. CONCLUSION

Three-axis magnetometer bias estimation using genetic algorithm was performed. This method is the same approach with Alonso's TWOSTEP except for substituting genetic algorithm for the centering operation. This method gives more accurate solution than gradient method even when the measurement includes large noise.

REFERENCES

[1] Lerner, G. M., "Scalar Checking," *Spacecraft Attitude Determination and Control*, edited by J. R. Wertz, Kluwer Academic, Dordrecht, The Netherlands, 2002, Chap. 9.3, pp. 328~331.

[2] R. Alonso and M. D. Shuster, "Attitude Independent Magnetometer Bias Determination: A Survey," *The Journal of the Astronautical Sciences*, American Astronautical Society, Vol. 50, No. 4, pp. 453-475, 2002.

[3] R. Alonso and M. D. Shuster, "TWOSTEP: A Fast Robust Algorithm for Attitude Independent Magnetometer Bias Determination," *The Journal of the Astronautical Sciences*, American Astronautical Society, Vol. 50, No. 4, pp. 433-451, 2002.

[4] Yaakov Oshman and Avishy Carmi, "Attitude Estimation from Vector Observations Using Genetic-Algorithm-Embedded Quaternion Particle Filter," Journal of Guidance, Control, and Dynamics, AIAA, Vol. 29, No. 4, pp. 879-891, 2006.

[5] G.-G. Jin, *Genetic Algorithms and Their Applications*, Kyo Woo Sa, Seoul, 2002