Resonant transmission of self-collimated beams through coupled zigzag-box resonators: slow self-collimated beams in a photonic crystal

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Abstract: The resonant transmission of self-collimated beams through zigzag-box resonators is demonstrated experimentally and numerically. Numerical simulations show that the flat-wavefront and the width of the beam are well maintained after passing through zigzag-box resonators because the up and the down zigzag-sides prevent the beam from spreading out and the wavefront is perfectly reconstructed by the output zigzag-side of the resonator. Measured split resonant frequencies of two- and three-coupled zigzag-box resonators are well agreed with those predicted by a tight binding model to consider optical coupling between the nearest resonators. Slowing down the speed of self-collimated beams is also demonstrated by using a twelve-coupled zigzag-box resonator in simulations. Our work could be useful in implementing devices to manipulate self-collimated beams in time domain.

OCIS codes: (230.4555) Coupled resonators; (260.2030) Dispersion; (260.5950) Self-focusing; (230.5298) Photonic crystals.

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1. Introduction

A self-collimation phenomenon, diffractionless propagation of a light beam in photonic crystals (PCs), has been of great interest in recent years because it could provide a new way to manipulate light propagation in PCs [1–5]. It has been experimentally demonstrated that self-collimated beams can be well guided in PCs without the use of any physical boundary and effectively routed by employing the bends and splitters [6–10]. Moreover, self-collimated beams can be crossed without cross talk [11], hence optical devices based on the self-collimated beams have intrinsic potential for high density photonic integrated circuits.

A resonator has been of fundamental and practical interest in optical devices such as cavities, waveguides, filters, couplers, and so on. A mirror is necessary to make a resonator. A 45-degree mirror to give rise to total internal reflection of a self-collimated beam has been proposed [6,8]. However, it is difficult to make a resonator composed of 45-degree mirrors because the beams reflected from 45-degree mirrors can hardly make destructive interference. Thus, to realize a resonator of self-collimated beams, it is necessary to make a high quality mirror to reflect strongly self-collimated beams into a backward direction.

Designing a mirror of self-collimated beams can start from reviewing fundamental properties of a self-collimated beam. One of the properties is that a self-collimated beam has a flat-wavefront perpendicular to a propagation direction. One can reasonably conjecture that an appropriate geometry to break the flat-wavefront may cause strong back-reflection of self-collimated beams. A zigzag-shape line-defect can be one of structures to break the flat-wavefront of a self-collimated beam. We will show that a zigzag-shape line-defect can reflect strongly incident self-collimated beams into a backward direction and act as a practical mirror of self-collimated beams.

A self-collimated beam has an uncertainty of a wavevector along a direction perpendicular to the beam propagation direction due to the finite beam size. The uncertain wavevector components will make the beam spread out along the direction perpendicular to the propagation direction in the resonator. However, the resonant phenomenon requires that the beam size of an incident self-collimated beam should be preserved after passing through a resonator. In addition, for no spreading of the beam, an incident self-collimated beam broken by one side of a resonator should be reconstructed by another one. Thus, a resonator composed of zigzag-shape line-defects should be designed carefully and have a two-dimensional shape, like a zigzag-box.

In this paper, we experimentally and numerically demonstrated the resonant transmission of self-collimated beams through a designed zigzag-box resonator. Numerical simulations show that the flat-wavefront and the width of the beam are well maintained after passing through a zigzag-box resonator because the up and the down zigzag-sides prevent the beam from spreading out and the wavefront is perfectly reconstructed by the output zigzag-side of the resonator. We have also investigated resonant transmission characteristics of two- and three-coupled zigzag-box resonators. Split resonant frequencies of the coupled resonators are well agreed with those predicted by a tight-binding (TB) model to consider optical coupling between the nearest resonators. Slowing down the speed of self-collimated beams was also demonstrated by using a twelve-coupled zigzag-box resonator in simulations.

2. Results and discussion

It has been demonstrated that microwave experiments are very useful in testing properties of newly designed PC devices before applications in infra-red or visible wavelength ranges. In this study, we employ a square lattice PC composed of cylindrical alumina rods with dielectric constant of 9.7 in air. The lattice constant and radius of the rods are $a = 5$ mm and $r = 0.4\ a = 2$ mm, respectively. Two parallel aluminum plates with periodically drilled holes are used to hold the alumina rods vertically. E-polarized microwaves (the electric-field parallel to the rod axes) of frequencies around 12.5 GHz can propagate with almost no diffraction along the ΓM direction inside the PC [9, 10]. An HP 8720C network analyzer and two horn antennas are uesed in experiments. Numerical simulations are performed by using a finite-difference time-domain (FDTD) method with a perfectly matched layer absorbing boundary condition [12]. A freely available FDTD software package MEEP [13] was employed. The spatial resolutions are

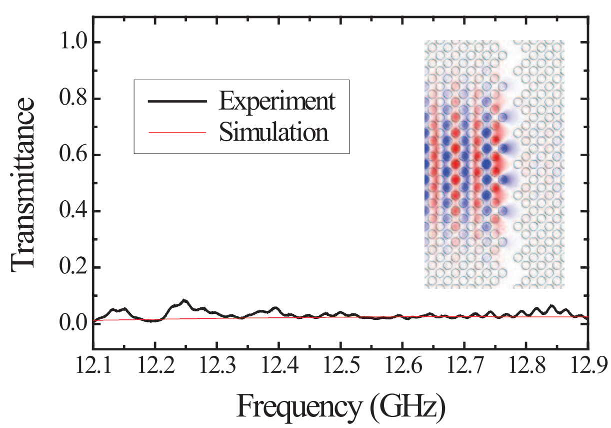

Fig. 1. Experimental and FDTD simulated transmittances of self-collimated beams through the zigzag-shape mirror in a range of frequency from 12.1 to 12.9 GHz. An inset depicts a zigzag-shape mirror to reflect strongly an incident self-collimated beam with a frequency of 12.5 GHz.

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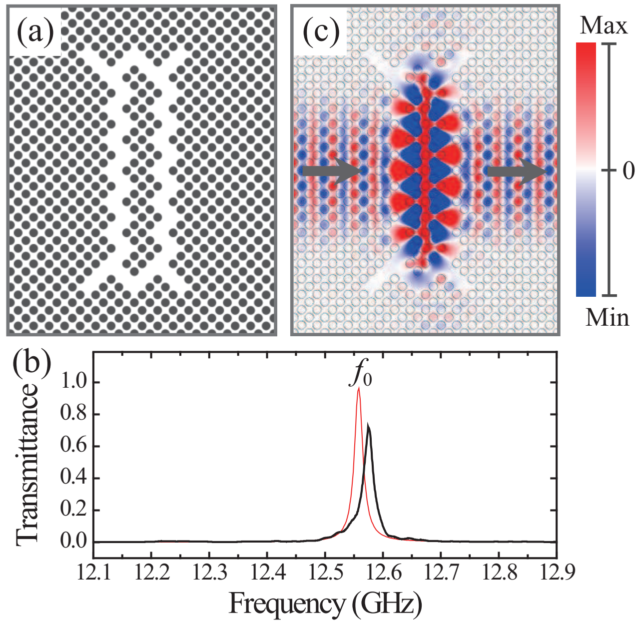

Fig. 2. (a) Top view of a two-dimensional zigzag-box resonator. Gray circles denote alumina rods. (b) Transmission spectra of self-collimated beams through the resonator. Black-thick and red-thin lines indicate experimental and FDTD simulated results, respectively. (c) Simulated spatial distribution of the electric-field of a resonant self-collimated beam at $f_0 = 12.559$ GHz. Arrows denote a propagation direction of the beam.

$\triangle x = \triangle y = a/32$. The discrete time step is set to $\triangle t = S\triangle x/c$, where the Courant factor S is chosen to be 0.5 for stable simulations. To obtain transmission spectra, a Gaussian pulse with a waist of $w = 4\ a$ is launched into the PC and the transmitted power $P_t(\omega)$ is computed at the end of the PC. $P_t(\omega)$ is normalized to an incident power $P_i(\omega)$ calculated at near a source plane without the PC structure. Antireflection structures are employed to eliminate unwanted reflection at the PC-air interfaces [14, 15].

We first investigated transmission properties of self-collimated beams through a zigzag-shape mirror (ZSM) created by missing rods as shown in the inset in Fig. 1. One can clearly see that the flat-wavefront of a self-collimated beam cannot be maintained at the zigzag-shape PC-air interface and the beam is strongly reflected into a backward direction. The flat-wavefront of the reflected beam gets distorted as the height of the zigzag step increases and the reflectance decreases. When the length of the mirror increases, the flat-wavefront of the reflected beam is well maintained and the reflectance increases slightly. Figure 1 represents measured and simulated transmission spectra of self-collimated beams through a ZSM in a frequency range from 12.1 to 12.9 GHz. Measured (simulated) transmittance of self-collimated beams through the ZSM is less than 9 % (3 %), and thus the proposed ZSM can act clearly as a back-reflection mirror for self-collimated beams.

We next designed a two-dimensional zigzag-box resonator as shown in Fig. 2(a). Measured (simulated) transmission spectra plotted in Fig. 2(b) shows the resonant transmission of a self-collimated beam of the frequency $f_0 = 12.575$ GHz (12.559 GHz). The measured frequency is slightly lower than the simulated one. The discrepancy between them may come from the small uncertainty of dielectric constant of the alumina rod since the simulated frequency was matched to the measured one when the dielectric constant of the alumina rod is 9.68, which is slightly

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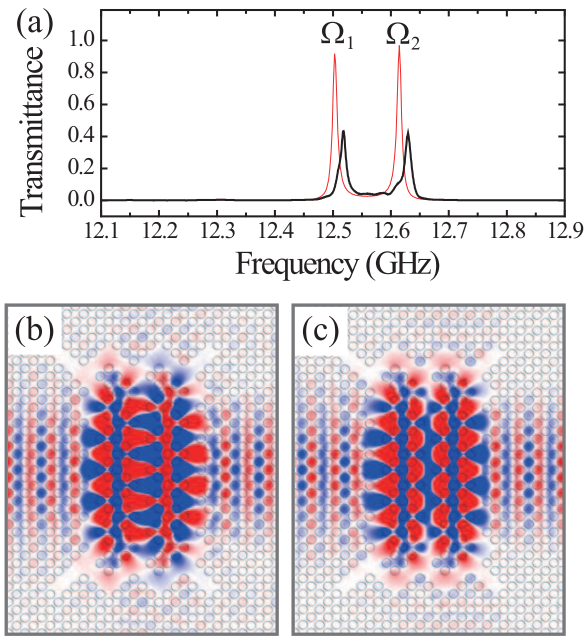

Fig. 3. Transmission spectra of self-collimated beams through a two-coupled zigzag-box resonator with two resonant frequencies of Ω_1 and Ω_2 (a). Black-thick and red-thin lines indicate experimental and FDTD simulated results, respectively. The simulated electric-field distributions of the resonant modes with resonant frequencies of Ω_1 (b) and Ω_2 (c).

smaller than 9.7. The Q-factor estimated from the relation of $f_0/\Delta f$ in the measurement (the FDTD simulation), where Δf is a full width half maximum, is about 607 (718). The simulated electric-field distribution of the beam at $f_0 = 12.559$ GHz represented in Fig. 2(c) shows that the flat-wavefront and the width of the resonant self-collimated beam are well maintained after passing through the zigzag-box resonator because the up and the down zigzag-sides prevent the beam from spreading out and the wavefront is perfectly reconstructed by the output zigzag-side of the resonator.

We also investigated coupled zigzag-box resonators to exhibit multi-resonant transmission peaks due to the evanescent coupling between individual resonator modes. Figure 3(a) shows the measured and the simulated transmission spectra of self-collimated beams through a two-coupled zigzag-box resonator with the inter-distance $a = 15\sqrt{2}$ mm. The measured (simulated) resonant frequencies are $\Omega_1 = 12.518$ GHz (12.502 GHz) and $\Omega_2 = 12.630$ GHz (12.615 GHz). The simulated electric-field distributions of the resonant modes with resonant frequencies of Ω_1 and Ω_2 are represented in Fig. 3 (b) and 3(c), respectively. One can see that the resonant mode with Ω_1 (Ω_2) mimics an odd (even) symmetry mode, even though it has been known that a coupling between two identical resonant modes in coupled high dielectric cavities makes their frequency split into a lower frequency of even mode and a higher frequency of odd mode. Kee and Lim have demonstrated that the parity of split resonant modes in two-coupled resonators in a 2D PC could be switched due to the correlation of the inter-distance between resonators and the period of oscillatory decaying evanescent fields of resonant modes. [16].

It is well-known that a tight binding model is very useful in predicting resonant transmission characteristics of coupled resonant systems. According to the TB model under the consideration of the interaction between the nearest resonators only, two resonant frequencies of a two-

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coupled resonator are given by $\Omega_{1,2}^2 \simeq f_0^2(1\pm\beta)/(1\pm\alpha)$, where α and β are TB parameters and f_0 is a resonant frequency of single resonator. The physical meanings of the TB parameters are described in detail in Ref. [17]. When the number of resonators increases, a transmission band is formed due to the evanescent coupling of individual resonant modes. Physical quantities such as the bandwidth Δf, the dispersion relation $f(k)$ and the group velocity $v_g(k)$ of propagation modes are determined by coupling coefficient κ between the nearest resonators, $\Delta f \simeq 2f_0|\kappa|$, $f(k) \simeq f_0[1+\kappa\cos(ka)]$ and $v_g(k) \simeq -2\pi f_0 a\kappa\sin(ka)$, where the coupling coefficient is defined as $\kappa = \beta - \alpha$ [17, 18].

From the measured (simulated) resonant frequencies f_0, Ω_1 and Ω_2, we obtained the TB parameters and coupling coefficient $\alpha = -0.0085$ (0.0022), $\beta = -0.0174$ (-0.0067), and $\kappa = -0.0089$, respectively. From the TB parameters, one can expect that the group velocities of self-collimated beams should be less than $2\pi f_0 a|\kappa| \simeq 0.05\,c$ in the whole transmission band ($\Delta f \simeq$ 0.22 GHz from 12.45 to 12.67 GHz) and approach to zero at band edges, provided that the TB model is valid. To check the validity of the TB model of the coupled zigzag-box resonator system for self-collimated beams, we compared theoretical resonant frequencies of a three-coupled zigzag-box resonator predicted by the TB model, $\Gamma_2 \simeq f_0$, $\Gamma_{1,3}^2 \simeq f_0^2(1\pm\sqrt{2}\beta)/(1\pm\sqrt{2}\alpha)$, with the measured (simulated) three resonant frequencies obtained from transmission spectra. Table 1 shows that the measured (simulated) three resonant frequencies well coincide with the frequencies predicted by the TB model. Hence, slowing down self-collimated beams could be possible by using coupled systems composed of numbers of zigzag-box resonators. In recent years, slowing the speed of light down to a remarkably low velocity has been of great interest due to its potential applications such as strong light-matter interactions, optical delay lines, optical buffers, and optical storage [19–21].

Table 1. Three resonant frequencies of a three-coupled resonator obtained from measured (simulated) transmission spectra. $TB_{mea.}$ ($TB_{sim.}$) frequencies were calculated by the measured (simulated) TB parameters obtained from the measured (simulated) two resonant frequencies of a two-coupled resonator. The unit of frequency is GHz.

	Measured	$TB_{mea.}$	Simulated	$TB_{sim.}$
Γ_1	12.492	12.494	12.481	12.479
Γ_2	12.575	12.575	12.559	12.559
Γ_3	12.652	12.652	12.637	12.638

To verify the slow propagation of self-collimated beams, we investigated the transmission properties of self-collimated beams through a couple system composed of twelve zigzag-box resonators. The consecutive resonators were made in a PC of length $L_{pc} = 270\sqrt{2}$ mm and the total length of the resonator region is $L_{res} = 195\sqrt{2}$ mm. As shown in Fig. 4(a), the simulated transmission band extending from 12.45 GHz to 12.67 GHz agrees well with the band predicted by the TB model. To obtain the group velocity of light in a media, it is essential to find the dispersion relation because the group velocity of light is given by $v_g(f) = 2\pi[df(k)/dk]$, where k is a wave vector in the media.

In this study, the dispersion relation of the slow light modes has been obtained from the fact that the total phase difference $\Delta\phi$ between a light propagating through a PC with thickness L and air is given by $(k_{pc} - k_{air})L$, where k_{pc} and k_{air} are wave vectors in a PC and air, respectively [22]. After phase determinations for three different cases, (1) $\phi_{air}(f)$ for air, (2) $\phi_{pc}(f)$ for a PC without a resonator and (3) $\phi_{res}(f)$ for a PC with the twelve-coupled zigzag-box resonator,

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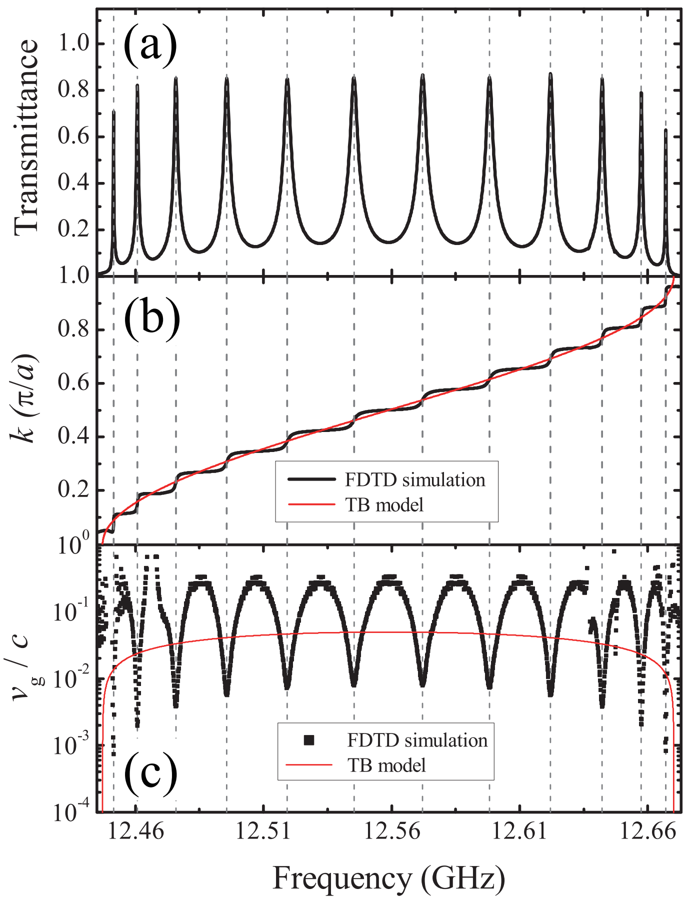

Fig. 4. (a) Transmission spectrum of self-collimated beams through a twelve-coupled zigzag-box resonator. (b) Dispersion relations of the transmission band obtained from the phase calculations (black-thick line) and the TB model (red-thin line). (c) Group velocities obtained from the FDTD simulations (black-solid rectangle) and the TB model (red-thin line). Dashed vertical lines represent the resonant frequencies of a twelve-coupled zigzag-box resonator.

the dispersion relation was determined by the relation,

$$k(f) = \frac{\phi_{res} - \phi_{pc}}{L_{res}} + \frac{\phi_{pc} - \phi_{air}}{L_{pc}} + 2\pi\frac{f}{c}. \tag{1}$$

Figure 4(b) shows the dispersion relations obtained from the phase calculations (black-thick line) and the TB model (red-thin line) in the transmission band. Overall tendency of the measured dispersion relation is in good agreement with the calculated one by the TB model. The step-like behavior of the measured dispersion relation comes from the finite number of cavities. In the simulations, we first recorded the time varying electric-fields at the end of the PC and then performed Fast Fourier Transforms to obtain the phase values.

The group velocities of self-collimated beams were obtained by taking the derivative of the calculated dispersion relation, $v_g(f) = 2\pi/[dk(f)/df]$ and the results were plotted in Fig. 4(c) with the theoretical curve from the TB model. The calculated group velocities (black-solid square) oscillate around the theoretical curve (red-thin line) and has local minimum values at

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resonant frequencies where the transmission exhibits peak values. The oscillatory behavior of the group velocity will disappear and the group velocity curve will be close to the theoretical curve, if the number of cavities becomes infinite [23]. The group velocity noticeably decreases to values less than $c/100$ as a frequency approaches to edges of the transmission band.

It would be challengeable to make zigzag-box resonators in 2D PC slabs operating at optical frequencies, even though we demonstrated their performances in a microwave range. The coupled zigzag-box resonators fabricated in optical wavelength scales could be useful in designing devices to manipulate self-collimated beams in time domain such as delay lines and optical storages in optical communications.

3. Conclusion

In conclusion, the resonant transmission of self-collimated beams through the zigzag-box resonator was demonstrated experimentally and numerically. Using FDTD simulations, we showed that the flat-wavefront and the size of the resonant self-collimated beam are preserved after passing through the box resonator. We have investigated multi-resonant transmission characteristics of two- and three-coupled zigzag-box resonators and analyzed the split frequencies by using the TB model. Slowing down the speed of self-collimated beams demonstrated by using a twelve coupled zigzag-box resonator could be useful in implementing devices to manipulate self-collimated beams in time domain.

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