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Editor's Introduction

In this column, the authors address the problem of finding the directivity for a uniform-amplitude planar phased array. By choosing suitable models for typical element antennas, they reduce the integral for the average pattern to summations. These eliminate the need to integrate the pattern over the many sidelobes, which would require many evaluations of the array summation. Since

their expressions include a progressive phase shift across the array, they can find the change in directivity as the array scans. The authors present a few curves to illustrate the trends, and supply the expressions needed to calculate your own cases. Our thanks to the authors for sharing their ideas and results.

Evaluation of Directivity for Planar Antenna Arrays

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Abstract

In this paper, the directivity, including phase shift factors, for several types of uniformly excited planar arrays is obtained. Four types of dipole arrays are considered: arrays of collinear short dipoles, and of parallel short dipoles; and broadside and end-fire arrays of crosses of short dipoles. Curves of directivity versus inter-element spacing and scan angle for planar arrays with these element power patterns are presented.

Keywords: Antenna arrays; phased arrays; planar arrays; dipole arrays; directivity; antenna radiation patterns

1. Introduction

In the design of many antenna arrays, the primary goal is the achievement of a specified directivity. Calculation of the directivity for variable spacings is then a useful task. Directivity,

including phase-shift factors, can also be used to calculate optimum tilt angles in radar antennas [1]. Until recently, the only results available for planar arrays were some approximations [2-4], and two exact calculations: in [5], a result for planar arrays with general spacing and excitation—both for isotropic [6] and for

Table 1. The coefficients used in computing the directivity.

	$ f(\theta, \phi) ^2$	a_0	a_1	a_2	b_1	b_2	c_1	c_2
Isotropic	1	1/2	$\frac{1}{A}$	0	$\frac{1}{B}$	0	$\frac{2}{C}$	0
Parallel Dipoles	$1 - \sin^2 \theta \cos^2 \phi$	1/3	$\frac{2}{A^3}$	$-\frac{2}{A^2}$	$\frac{1}{B} - \frac{1}{B^3}$	$\frac{1}{B^2}$	$\frac{2 \cos^2 \theta_t + 3 \sin^2 \theta_t - 1}{C} + \frac{3 \sin^2 \theta_t - 1}{C^3}$	$\frac{2 - 6 \sin^2 \theta_t}{C^2}$
Collinear Dipoles	$\sin^2 \theta$	1/3	$\frac{1}{A} - \frac{1}{A^3}$	$\frac{1}{A^2}$	$\frac{1}{B} - \frac{1}{B^3}$	$\frac{1}{B^2}$	$\frac{2}{C} - \frac{2}{C^3}$	$\frac{2}{C^2}$
Broadside Crosses	$\frac{1}{2}(1 + \sin^2 \theta \sin^2 \phi)$	1/3	$\frac{1}{2A} + \frac{1}{2A^3}$	$-\frac{1}{2A^2}$	$\frac{1}{B} - \frac{1}{B^3}$	$\frac{1}{B^2}$	$\frac{1 + \cos^2 \theta_t + 3 \sin^2 \theta_t - 2}{C} + \frac{3 \sin^2 \theta_t - 2}{C^3}$	$\frac{2 - 3 \sin^2 \theta_t}{C^2}$
End-fire Crosses	$\frac{1}{2}(1 + \cos^2 \theta)$	1/3	$\frac{1}{2A} + \frac{1}{2A^3}$	$-\frac{1}{2A^2}$	$\frac{1}{2B} + \frac{1}{2B^3}$	$-\frac{1}{2B^2}$	$\frac{1}{C} + \frac{1}{C^3}$	$-\frac{1}{C^2}$

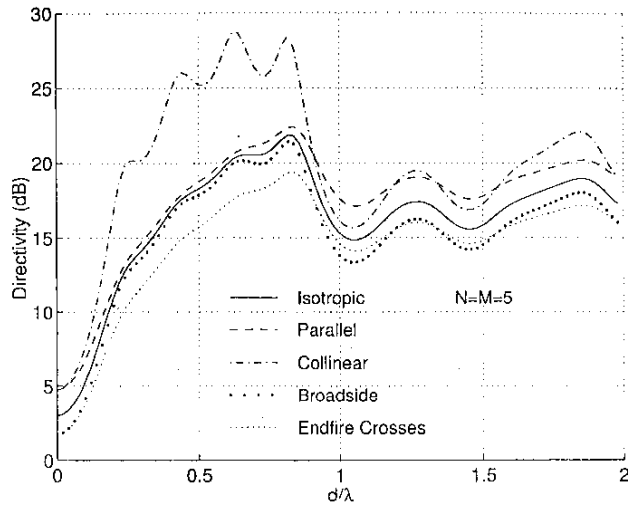


Figure 1. The directivity as a function of element spacing ($d_x = d_y$, with $\theta = 90^\circ$ and $\phi = 90^\circ$).

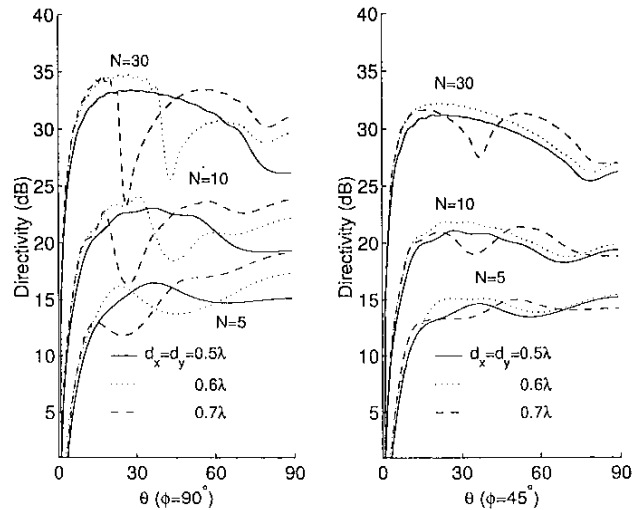


Figure 3. The directivity as a function of scan angle, θ , for collinear dipoles.

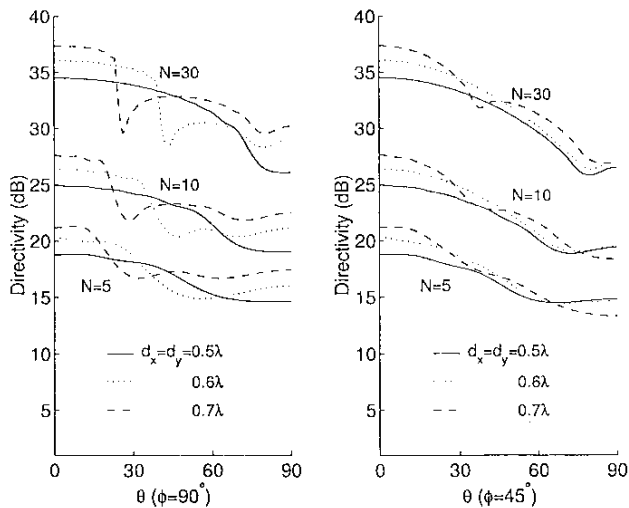


Figure 2. The directivity as a function of scan angle, θ , for parallel dipoles.

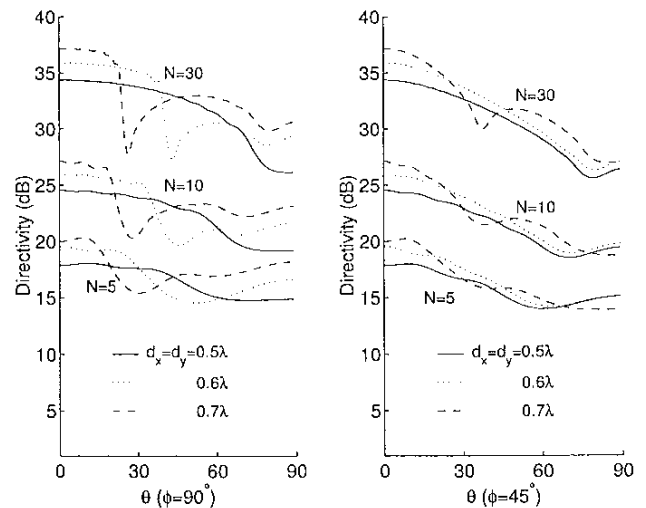


Figure 4. The directivity as a function of scan angle, θ , for broadside crosses.

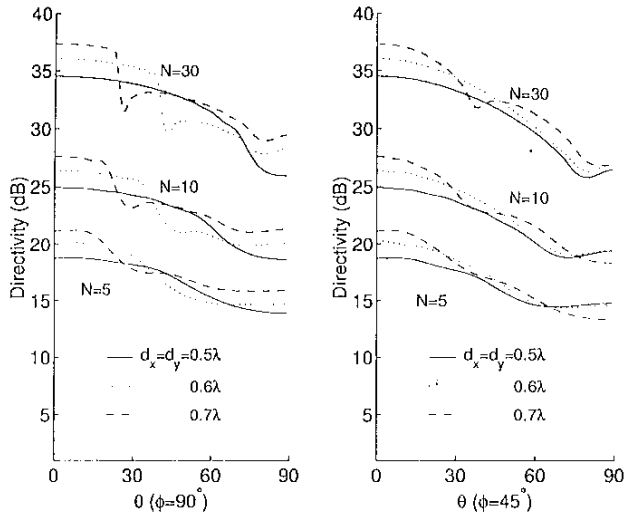


Figure 5. The directivity as a function of scan angle, θ , for end-fire crosses.

$\cos^{n+0.5}(\theta)$ conical-element patterns is derived, which has been extended to parallel short dipoles in [7].

In this paper, the directivity, including phase-shift factors for several types of uniformly excited planar arrays, is obtained. Four types of dipole arrays are considered: arrays of collinear short dipoles, and of parallel short dipoles; and broadside and end-fire arrays of crosses of short dipoles. Curves of directivity versus inter-element spacing and scan angle, for planar arrays with these element power patterns, are also presented.

2. Directivity Calculation

It is assumed that the elements are phased to give a beam in the direction (θ_0, ϕ_0) , and that radiation occurs only into the half-space above the array. Assume that the array elements, depicted as in [8], are located in the x, y plane. Then, the array directivity can then be expressed as

$$D(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{\int_0^{\pi/2} \int_0^{2\pi} U(\theta, \phi) d\phi \sin \theta d\theta}, \quad (1)$$

where the intensity $U(\theta, \phi) = |g_a(\theta, \phi)|^2 |f(\theta, \phi)|^2$, $f(\theta, \phi)$ is the element factor, and the array factor, g_a , for the rectangular M -by- N array is given by

$$g_a(\theta, \phi) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} e^{-jn(\alpha - \alpha_0) - jm(\beta - \beta_0)}. \quad (2)$$

In this paper, the angles θ and ϕ are the same as those defined in, for example, [7]. In Equation (2),

$$\alpha = \frac{2\pi}{\lambda} d_x \sin \theta \cos \phi, \quad (3)$$

$$\beta = \frac{2\pi}{\lambda} d_y \sin \theta \sin \phi, \quad (4)$$

$$\alpha_0 = \frac{2\pi}{\lambda} d_x \sin \theta_0 \cos \phi_0, \quad (5)$$

and

$$\beta_0 = \frac{2\pi}{\lambda} d_y \sin \theta_0 \sin \phi_0, \quad (6)$$

with λ being the wavelength. Since

$$\begin{aligned} \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} e^{-j(n-n')(\alpha - \alpha_0)} &= N + 2 \sum_{n=1}^{N-1} \sum_{r=1}^n \cos[r(\alpha - \alpha_0)] \\ &= N + 2 \sum_{r=1}^{N-1} (N-r) \cos[r(\alpha - \alpha_0)], \end{aligned} \quad (7)$$

the intensity, $U(\theta, \phi)$, becomes

$$\begin{aligned} U(\theta, \phi) &= MN |f(\theta, \phi)|^2 \left[1 + 2 \sum_{r=1}^{N-1} \left(1 - \frac{r}{N}\right) \cos[r(\alpha - \alpha_0)] \right. \\ &\quad \left. + 2 \sum_{q=1}^{M-1} \left(1 - \frac{q}{M}\right) \cos[q(\beta - \beta_0)] \right. \\ &\quad \left. + 4 \sum_{r=1}^{N-1} \sum_{q=1}^{M-1} \left(1 - \frac{r}{N}\right) \left(1 - \frac{q}{M}\right) \cos[r(\alpha - \alpha_0)] \cos[q(\beta - \beta_0)] \right]. \end{aligned} \quad (8)$$

With some manipulations, we have

$$\int_0^{\pi/2} \int_0^{2\pi} U(\theta, \phi) d\phi \sin \theta d\theta = 4\pi MNT, \quad (9)$$

where

$$\begin{aligned} T &= \alpha_0 + \sum_{r=1}^{N-1} \cos(r\alpha_0) \left(1 - \frac{r}{N}\right) (a_1 \sin A + a_2 \cos A) \\ &\quad + \sum_{q=1}^{M-1} \cos(q\beta_0) \left(1 - \frac{q}{M}\right) (b_1 \sin B + b_2 \cos B) \\ &\quad + \sum_{r=1}^{N-1} \sum_{q=1}^{M-1} \cos(r\alpha_0 + q\beta_0) \left(1 - \frac{r}{N}\right) \left(1 - \frac{q}{M}\right) (c_1 \sin C + c_2 \cos C), \end{aligned} \quad (10)$$

$A = r \frac{2\pi}{\lambda} d_x$, $B = q \frac{2\pi}{\lambda} d_y$, $C = \sqrt{A^2 + B^2}$, $\phi_r = \tan^{-1} \left(\frac{rd_x}{qd_y} \right)$, and

the coefficients in the five cases are listed in Table 1. The directivity is thus obtained as

$$D(\theta, \phi) = \frac{|g_a(\theta, \phi)|^2 |f(\theta, \phi)|^2}{MNT}. \quad (11)$$

3. Numerical Results

The curves of Figure 1 have a dip at a spacing near one wavelength. This is caused by the emergence of grating lobes into

the visible region. The periodic effect of grating lobes for larger spacings is also shown. From this result, it is shown that a collinear dipole array has the highest directivity among the five types of dipoles, for spacing less than λ . For spacings greater than 0.5λ , the effect of rising grating lobes with increasing scan angle can be seen in Figures 2-5, for the four types of dipole arrays with $\phi = 90^\circ$ and $\phi = 45^\circ$.

4. Concluding Remark

In this paper, the directivity, including phase-shift factors, for several types of uniformly excited planar arrays is obtained. For the determination of the minimum number of elements appropriate to a specific design, the results derived in this paper may be helpful. In addition, the directivity obtained in this paper can be used for calculating the optimum tilt angles for radar systems with the types of antenna arrays considered in this paper.

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6. References

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Ideas for Antenna Designer's Notebook

Ideas are needed for future issues of the Antenna Designer's Notebook. Please send your suggestions to Tom Milligan, and they will be considered for publication as quickly as possible. Topics can include antenna design tips, equations, nomographs, or shortcuts, as well as ideas to improve or facilitate measurements. ☺

