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Routing and Wavelength Assignment in Survivable WDM Networks without Wavelength Conversion*

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ABSTRACT

In this paper, we consider the routing and wavelength assignment problem in survivable WDM transport network without wavelength conversion. We assume the single-link failure and a path protection scheme in optical layer. When a physical network and a set of working paths are given, the problem is to select a link-disjoint protection path for each working path and assign a wavelength for each working and protection path. We give an integer programming formulation of the problem and propose an algorithm to solve it. Though the formulation has exponentially many variables, we solve the linear programming relaxation of it by using column generation technique. We devise a branch-and-price algorithm to solve the column generation problem. After solving the linear programming relaxation, we apply a variable fixing procedure combined with the column generation to get an integral solution. We test the proposed algorithm on some randomly generated data and test results show that the algorithm gives very good solutions.

Keywords : WDM, RWA, Path Protection, Column Generation

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1. INTRODUCTION

Wavelength division multiplexing (WDM) technology is used to accommodate several wavelength channels on a fiber. All-optical networks based on WDM technique are prime candidates for wide-area backbone networks. In WDM network, an optical path (*lightpath*) with a dedicated wavelength is established for each required connection and no two paths using the same wavelength pass through the same link to avoid wavelength collision. The routing and wavelength assignment (RWA) problem is to select suitable paths and to assign wavelengths for required connections without collision. RWA is important to increase the efficiency of WDM networks. Thus, many studies on RWA have been performed [6, 7, 8, 16].

Survivability is an ability to recover the traffic when a network component fails and it has been an important issue in designing the fiber-optic based telecommunication network. In WDM network, several lightpaths pass a link and the failure of a network component such as a fiber link can lead to the failure of all those lightpaths. Moreover, each lightpath is expected to operate at a rate of several Gbps, such a failure can lead to a severe disruption in the network's traffic.

A layered transmission network such as WDM network, several layers (such as SONET, ATM and IP) may have their own recovery procedures. However, the recovery time for higher layers (such as ATM and IP) is still significantly large (on the order of seconds), whereas we expect that restoration times at the optical layer will be on the order of milliseconds to minimize data losses [3]. Furthermore it is beneficial to consider protection mechanisms in the optical layer for the following reasons : (a) the optical layer can efficiently multiplex protection resources among several higher-layer network applications, and (b) survivability at the optical layer provides protection to higher-layer protocols that may not have built-in protection [13]. Gerstel and Ramaswami [4] gives more detailed needs of optical layer protection and its limitations.

There are several protection schemes in optical layer when a failure occurs. Among them, the path protection is to set up the backup paths for the failed lightpaths and to switch data to them. The scheme is being implemented in novel OXC (Optical Cross-Connect) which has the capability to change the routing patterns at a node [14]. In wavelength assignment for protection paths, there are two possible methods [10, 11]. One method assigns the same wavelength to the protection paths as its corresponding working path (method-I). The other method assigns an arbitrary wavelength for each protection path (method-II). In method-I because of the more strict limitation on wavelength reuse throughout the network the inefficient use of network resources such as wavelengths and optical fibers may be caused. Method-II entails the cooperation of the electrical level path cross-connect because the failed working path of a certain wavelength must be replaced by the protection path with another wavelength [11]. Thus, in method-II, the higher the bit rate of lightpath, the more significantly the protection performance is affected by the processing capability of the electrical level switching and method-II may face some difficulties when rapid protection is required.

Many studies on RWA considering survivability have been performed. Nagatsu *et al.* [11] decompose the problem into two subproblems and solve them sequentially. One is to construct working paths and the other is to construct protection paths. They proposed a heuristic procedure for each subproblem. Miyao and Saito [9] proposed an integer programming formulation considering a path protection when full wavelength conversion is permitted in every nodes. Then, the problem is free from wavelength assignment for each path. They assume that the set of possible pairs of a working path and corresponding protection path is given and solve the formulation by CPLEX. Narula-Tam *et al.* [12] considers the RWA on WDM ring networks. They calculated the lower bound on required wavelengths and proposed routing and wavelength assignment algorithms. Kennington *et al.* [5] consider RWA problem under a path protection scheme. They proposed a mathematical formulation but they proposed a heuristic algorithm to solve the problem. Zang *et al.* [17] also consider the RWA under a path protection scheme. They consider a problem to route a working path and a link-disjoint protection path and to assign wavelength for the paths. They proposed a mathematical formulation with flow variables and developed a heuristic algorithm to solve the problem. They solve the problem by separating routing and wavelength assignment.

In this paper, we consider the routing and wavelength assignment (RWA) under a path protection scheme when the working paths are given. We assume the single-link failure scenario and method-II for wavelength assignment. Single-link failure means that at most one link can be failed at a time and it is commonly used in survivability studies. To minimize the number of wavelengths, it is efficient to decide the routing and wavelength assignment of working and corresponding protection path simultaneously. But, the problem is very hard. In this paper, we consider an intermediate problem which is to determine the routing of protection paths and the wavelength assignment of working paths and protection paths when working paths are given. When a physical network and a set of working paths are given, we must select a link-disjoint protection path for each given working path. When a link fails, it is needed to use the predetermined protection paths without wavelength collision instead of the working paths passing the failed link. Thus, we must assign a wavelength for each working and protection path without wavelength collision. In other words, the working paths are given and we decide the routing of protection paths and the wavelength assignment. Constraints in wavelength assignment are described in the following section. The objective is to minimize required wavelengths and it means to maximize the wavelength reuse.

We call the problem survivable routing and wavelength assignment (SRWA) problem. We give an integer programming (IP) formulation of SRWA. The formulation has exponentially many variables but we can solve the linear programming (LP) relaxation of the formulation by column generation technique [1, 15]. The column generation problem is also an NP-hard problem. We formulate it as an IP and we propose a branch-and-price algorithm to solve it. After solving the LP relaxation, we propose a variable fixing procedure combined with column generation to obtain an integral solution.

This paper is organized as follows. In section 2, we describe SRWA and give an integer programming formulation. We give an algorithm to solve the column generation problem in section 3 and we present the overview of our algorithm for SRWA in section 4. In section 5, we show computational results and conclusions are given in section 6.

2. PROBLEM FORMULATION

As mentioned in the previous section, we consider the routing and wavelength assignment problem in survivable WDM network. We assume the single-link failure and the following path protection scheme against the link failure. When a working path is established, a protection path for the working path is also determined and the protection path is activated when the working path fails. The predetermined protection path for each working path is independent to the location of the failure. In other words, a working path and the corresponding protection path are link-disjoint. Thus, we assume that each working path has at least one link-disjoint path.

In wavelength assignment, two working paths or a working path and a protection path which share links cannot use the same wavelength. But, the constraint between two protection paths can be relaxed. Two protection paths sharing links can use the same wavelength if one protection path does not share any link with the working path corresponding to the other protection path. Consider an example in Figure 1. Two protection paths r_1 and r_2 sharing link (1, 2) can use the same wavelength because r_1 and r_2 don't be activated at the same time.

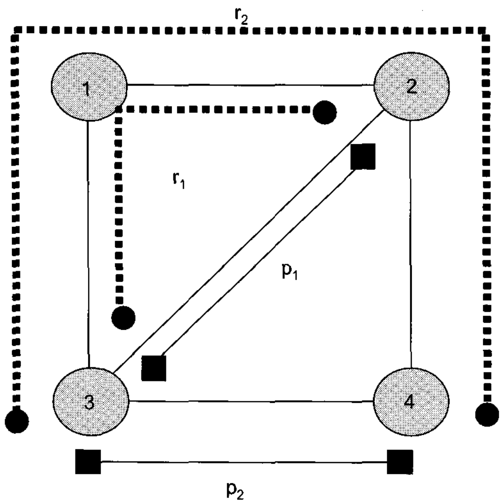

Figure 1. An example for wavelength assignment constraint

We introduce the concept of *identical survivable independent routing configuration* (ISIRC) which is a set of paths such that all contained paths can be established using one wavelength under above protection scheme. If working path p is contained in an ISIRC, then a corresponding protection path r must be contained in it because p and r must use the same wavelength. A walk in graph is a finite non-null sequence $v_0e_1v_1e_2v_2\cdots e_kv_k$ whose terms are alternatively vertices and edges such that the ends of e_i are v_{i-1} and v_i for $1 \le i \le k$. A walk is closed if its origin node and terminal node are the same and a closed trail of a graph is a closed walk that traverses each link at most once [2]. Because p and r are link-disjoint, p and r form a closed trail. In other words, an ISIRC consists of several closed trails and each closed trail is divided into a working path and a corresponding protection path.

To formulate SRWA, we introduce some notation.

$G = (V,E)$: physical network
P : set of given working paths
o_p, d_p : two end nodes of working path $p \in P$

$R(p)$: set of all possible protection paths for working path $p \in P$

$R = \bigcup_{p \in P} R(p)$

$E(p)$: set of links used by path p

$H(p)$: set of all possible closed trails on G which contain working path $p \in P$

$H = \bigcup_{p \in P} H(p)$

$E(h)$: set of links used by closed trail $h \in H$

C_I : set of all *ISIRC*'s.

Note that a closed trail in $H(p)$ consists of a working path p and a protection path $r \in R(p)$. In other words, $E(h) = E(p) \cup E(r)$ and $|H(p)| = |R(p)|$. Then, an ISIRC can be represented by a binary vector $a_c \in B^{|P|}$. The p^{th} element of a_c, denoted as $a_{pc} = 1$ if a closed trail in $H(p)$ is contained in ISIRC c, otherwise, $a_{pc} = 0$.

With above notation, we can formulate SRWA as the following integer program.

$$
\begin{aligned}
\text{(SRWAP)} \quad & \min \quad \sum_{c \in C_I} z_c \\
& s.t. \quad \sum_{c \in C_I} a_{pc} z_c \geq 1, \qquad \textit{for all} \;\; p \in P \qquad (1) \\
& z_c \in \{0,1\} \qquad \textit{for all} \;\; c \in C_I \qquad (2)
\end{aligned}
$$

Each decision variable $z_c = 1$, $c \in C_I$ if ISIRC c is established, otherwise, $z_c = 0$. Constraints (1) ensure that at least one closed trail for each working path must be selected. It also means that a protection path for each working path should be established. We can easily obtain the wavelength assignment by assigning the same wavelength to paths contained in an ISIRC. Then, a working path and corresponding protection path use the same wavelength.

Let SRWALP be the LP relaxation of SRWAP, i.e. the problem obtained after changing constraints (2) to $z_c \geq 0$ in SRWAP. We can omit $z_c \leq 1$ because that $a_{pc} \in \{0,1\}$ and the objective function is to minimize $\sum_{c \in C_I} z_c$. Then the optimal objective value of SRWALP provides a lower bound on the optimal objective value of SRWAP since SRWALP is a relaxation of SRWAP. Note that SRWALP has exponentially many variables. However, we can solve SRWALP by using the column generation technique. In the column generation technique, first, we assume that a subset C_I' of C_I is given. Then we construct a restricted LP relaxation SRWARLP, whose solution is suboptimal to SRWALP, by replacing C_I by C_I' in SRWALP. Let α_p be the dual variables corresponding to the p^{th} constraint in (1). We can solve SRWARLP and let z^* be an optimal solution to SRWARLP and α^* be the values of the dual variables in SRWARLP. Then, z^* is an optimal solution to SRWALP if $\sum_{p \in P} \alpha^* a_{pc} \leq 1$ for all $c \in C_I \setminus C_I'$ because the reduced costs of all variables are nonnegative [1]. In other words, α^* is a feasible solution to the dual of SRWALP. Therefore, the column generation problem can be formulated as follows and we can check whether the current solution z^* is optimal to SRWALP or not by solving the problem.

$$\begin{array}{lll} \text{(CGP)} & \max & \sum_{p \in P} \alpha^* a_{pc} \\ & s.t. & c \in C_I \end{array} \tag{3}$$

Note that CGP is the problem to find a maximum weight ISIRC, where closed trail $h \in H(p)$ has weight α_p^*. If the optimal objective value of SP is greater than 1, the obtained ISIRC is added to SRWARLP as a new entering column and the updated SRWARLP is solved to optimality again. Otherwise, z^* is an optimal solution to SRWALP because that the above optimality condition is satisfied.

3. ALGORITHM FOR COLUMN GENERATION PROBLEM

As noted in the previous section, column generation problem is to find a maximum weight ISIRC, where closed trail h for p has weight α_p^*. In an ISIRC, note that no two protection paths are activated at the same time because the corresponding working paths must be in the ISIRC and they don't share any link. Thus a link can be used either by only one working path or the link can be used arbitrary number of protection paths or the link can be shared by protection paths in an ISIRC. Then, column generation problem can be formulated as follows.

$$\text{(SP1)} \quad \max \quad \sum_{p \in P} \sum_{h \in H(p)} \alpha_p^* x_h$$

$$\text{s.t.} \quad \sum_{p \in P} \sum_{h \in H(p)} d_{eh}^r x_h - |P|(1 - y_e) \le 0 \qquad \textit{for all } e \in E \tag{4}$$

$$\sum_{p \in P} \sum_{h \in H(p)} d_{eh}^p x_h - y_e \le 0 \qquad \textit{for all } e \in E \tag{5}$$

$$x_h, y_e \in \{0,1\} \qquad \textit{for all } h \in H \text{ and } e \in E$$

A closed trail consists of working path p and a corresponding protection path $r \in R(p)$. The coefficient $d_{eh}^r = 1$ if the protection path r in closed trail h passes edge e, otherwise $d_{eh}^r = 0$. Similarly, $d_{eh}^h = 1$ if working path p in closed trail h passes edge e, otherwise $d_{eh}^h = 0$. Each decision variable $x_h = 1$ if closed trail $h \in H$ is selected, otherwise, $x_h = 0$. Decision variable $y_e = 1$ if edge $e \in E$ can be used by a working path , otherwise, $y_e = 0$. Constraints (4) ensure that link e may be used by protection paths if the link is not used by any working path. Constraints (5) ensure that at most one working path can pass on a link. Constraints (4) and (5) satisfy the restriction for an ISIRC described in the previous section, so a feasible solution to SP1 is an ISIRC.

The optimal objective value of the LP relaxation of SP1 gives an upper bound on the optimal objective value of SP1. In constraint (4), $|P|$, the coefficient of y_e, can be changed any other sufficient large number such that all possible protection paths can pass the link. But, when the coefficient is large, the LP bound might be bad. To get a better LP bound, consider the following another formulation.

$$\text{(SP)} \quad \max \quad \sum_{p \in P} \sum_{h \in H(p)} \alpha_p^* x_h$$

$$\text{s.t.} \quad \sum_{h \in H(p)} d_{eh}^r x_h - (1 - y_e) \le 0 \qquad \textit{for all } p \in P,\ e \in E \tag{6}$$

$$\sum_{p \in P} \sum_{h \in H(p)} d_{eh}^p x_h - y_e \le 0 \qquad \textit{for all } e \in E \tag{7}$$

$$x_h, y_e \in \{0,1\} \qquad \textit{for all } h \in H,\ e \in E$$

Constraints (6) mean that link e can be used by a protection path for each working path if the link is not used by any working path, so SP is also a valid formulation. SP has more constraints than SP1. But, constraints (6) dominate constraints (4) because the summation of (6) for each link gives constraints (4). Thus, the LP relaxation of SP may give tighter bound than that of SP1 and we use SP for column generation.

The number of variables x_h in SP is exponentially many. But, we can solve the LP relaxation of SP by the column generation technique. After solving the LP relaxation, we develop a branch-and-price procedure to get an optimal solution to SP.

3.1 Column generation procedure for SP

Now, we explain the column generation procedure to solve the LP relaxation of SP. Let SLP be the LP relaxation of SP and let RSLP be a restricted LP relaxation obtained by replacing H(*p*) in SLP by $H'(p) \subseteq H(p)$, for all $p \in P$. Let β_{pe}, for all $p \in P$ and $e \in E$ and ρ_e, for all $e \in E$, be the nonnegative dual variables associated with constraints (6) and (7), respectively. We can solve RSLP and let (x^*, y^*) be the obtained optimal solution to RSLP and let $(\bar{\beta}, \bar{\rho})$ be the corresponding optimal dual solution. Then, the reduced cost of each closed trail $h \in H(p)$, denoted c_h is as follows.

$$c_h = \alpha_p^* - \sum_{e \in E(r)} \bar{\beta}_{pe} - \sum_{e \in E(p)} \bar{\rho}_e$$

If there is no closed trail whose reduced cost is positive, then current solution (x^*, y^*) is optimal. Thus, column generation problem is to find a closed trail whose reduced cost is maximum. For a working path *p*, the problem is to find a closed trail containing *p* whose reduced cost is maximum. Note that α_p^* and $\sum_{e \in E(p)} \bar{\rho}_e$ are determined values for a working path *p* because *E*(*p*) is known. Thus, we only find a protection path between o_p and d_p which has maximum weight where link *e* has weight $\bar{\beta}_{pe}$ and the path must be link-disjoint with *p*. Then, the column generation problem for $p \in P$ is to find a shortest path between o_p and d_p on a given network $G = (V, E \setminus E(p))$ with the link weight $\bar{\beta}_{pe}$. Because the link weights are nonnegative, we can solve the problem in polynomial time. Denote the obtained shortest path as r_p^* and the weight of r_p^* as l_{pr^*}. If $\alpha_p^* - l_{pr^*} - \sum_{e \in E(p)} \bar{\rho}_e \leq 0$ for all $p \in P$, then $(\bar{\beta}, \bar{\rho})$ is an optimal dual solution to the dual of SLP and (x^*, y^*) is an optimal solution to SLP. Otherwise, we add $h \in H(p)$ such that $E(h) = E(p) \cup E(r^*)$ to RSLP for each $p \in P$ with $\alpha_p^* - l_{pr^*} -$

$\sum_{e \in E(p)} \bar{\rho}_e > 0$ and solve RSLP again. We can solve SLP by repeating the above column generation procedure until no more columns is generated.

3.2 Branch-and-price procedure for SP

If the obtained optimal solution to SLP is integral then we solved SP. Otherwise, we perform branch-and-price procedure to get an optimal solution to SP. Branch-and-price procedure is the same as the branch-and-bound procedure except that the column generation procedure is used to solve SLP at every node in the branch-and-bound tree. In the branch-and-price procedure, a branching rule is required such that the column generation is possible after branching. Devising such a branching rule is an important key in the branch-and-price procedure [1, 15]. We devise a branching rule such that the column generation problem does not be changed after branching.

Suppose that an optimal solution (x^*, y^*) to SLP is obtained. First we check whether $y_e^* \in \{0,1\}$ for all $e \in E$. Denote $U(x^*, y^*) \subseteq E$ as the set of links such that corresponding y^* has fractional value. If $|U(x^*, y^*)| > 0$, we branch on y_{e^*} such that $e^* = \arg\max_{e \in U(x^*, y^*)} y_{e^*}$. We make two new branches, one of them would force y_{e^*} to be 1, the other would force y_{e^*} be 0. In the branch such that $y_{e^*} = 1$, the column generation problem is not changed. In the other branch, link e^* cannot be used by any working path. So, column generation for the working paths passing link e^* does not be needed. For a working path which does not pass link e^*, we can generate columns by solving above shortest path problem.

If $|U(x^*, y^*)| = 0$, then either (x^*, y^*) is integral or x^* is not integral. Suppose that x^* is not integral. We define $P(x^*, y^*; e) \subset P$ for all $e \in E$ such that working path $p \in P(x^*, y^*; e)$ if and only if there exists a closed trail $h \in H(p)$ with $x_h^* > 0$ such that $e \in E(p)$. Note that it is determined whether each link is used by a working path or not because $|U(x^*, y^*)| = 0$. Then, we can derive following proposition.

Proposition 1. Suppose (x^*, y^*) is an optimal solution to RSLP. If $|U(x^*, y^*)| = 0$ and $|P(x^*, y^*; e)| \leq 1$ for all $e \in E$ then $\sum_{h \in H(p)} x_h^* = 0$ or 1 for all $p \in P$.

Proof. Suppose that there exists p^* such that $0 < \sum_{h \in H(p^*)} x_h^* < 1$, then there exist a closed trail $h \in H(p)$ such that $x_{h^*}^* > 0$. Consider another solution $(\bar{x}, \bar{y})$ such that $\bar{y} = y^*$, $\bar{x}_h = x_h^*$ for all $h \neq h^*$, and $\bar{x}_{h^*} = 1 - \sum_{h \in H(p) \setminus \{h^*\}} x_h^*$. It is easily known that $(\bar{x}, \bar{y})$ is feasible solution to RSLP because $|U(x^*, y^*)| = 0$ and $|P(x^*, y^*; e)| \leq 1$ for all $e \in E$. Moreover, the objective value of $(\bar{x}, \bar{y})$ is greater than or equal to that of (x^*, y^*) because the objective coefficient of x_{h^*}, $\alpha_{p^*}^*$, is greater than or equal to 0. First, if $\alpha_{p^*}^* > 0$, then it is a contradiction since (x^*, y^*) cannot be an optimal solution to RSLP. Second, if $\alpha_{p^*}^* = 0$, then no closed trail for p^* has positive reduced cost and no closed trail in $H(p^*)$ can be generated. Thus, it is also a contradiction because $\sum_{h \in H(p^*)} x_h^* = 0$.

If (x^*, y^*) is an integral solution, then $|U(x^*, y^*)| = 0$ and $|P(x^*, y^*; e)| \leq 1$. Converse is not always true. However, we can derive following positive results from proposition 1.

Proposition 2. Suppose an optimal solution (x^*, y^*) to RSLP is obtained by the simplex method. If $|U(x^*, y^*)| = 0$, if $|P(x^*, y^*; e)| \leq 1$ for all $e \in E$ then (x^*, y^*) is integral.

Proof. Clearly, y^* is integral. Suppose that x^* is not integral, then there exists p^* such that there exist x_h^*'s whose value is fractional and $\sum_{h \in H(p^*)} x_h^* = 1$ by proposition 1. Define $H(p^*; x^*) \subseteq H(p^*)$ such that $h \in H(p^*; x^*)$ if and only if $x_h^* > 0$ and $h \in H(p^*)$. Consider a solution $(\bar{x}, \bar{y})^j$, $j \in H(p^*; x^*)$ which is obtained as follows. We let $\bar{y} = y^*$, $\bar{x}_h = x_h^*$ if $h \notin H(p^*; x^*)$, $\bar{x}_h = 0$ if $h \in H(p^*; x^*) \setminus \{j\}$, and $\bar{x}_h = 1$ if $h = j$. It is easily shown that $(\bar{x}, \bar{y})^j$ is a feasible solution to RSLP. Moreover, $(x^*, y^*) = \sum_{h \in H(p^*; x^*)} x_h^* \cdot (\bar{x}, \bar{y})$ and $\sum_{h \in H(p^*; x^*)} x_h^* = \sum_{h \in H(p^*)} x_h^* = 1$. In other words, (x^*, y^*) is represented by a convex combination of other feasible solutions to RSLP. It is a contradiction since the simplex method always finds an extreme point optimal solution. Thus, (x^*, y^*) is integral.

By the above propositions, when y^* is integral ($|U(x^*,y^*)|=0$), we only check if $|P(x^*,y^*;e)|\le 1$ for all $e \in E$ instead of checking the integrality of (x^*,y^*). For a given solution (x^*,y^*) such that $|U(x^*,y^*)|=0$, we get $P(x^*,y^*;e)$ for all $e \in E$. Then, we choose e^* such that $e^* = \arg\max_{e\in E} |P(x^*,y^*;e)|$. If $|P(x^*,y^*;e)|=1$, we get an integral solution by proposition 2. Otherwise, we divide $P(x^*,y^*;e^*)$ into two disjoint sets $P_1(x^*,y^*;e^*)$ and $P_2(x^*,y^*;e^*)$, such that $P_1(x^*,y^*;e^*)=\{p^*\}$ and $P_2(x^*,y^*;e^*)=P(x^*,y^*;e^*)\setminus P_1(x^*,y^*;e^*)$, where $p^* = \arg\max_{p\in P(x^*,y^*;e^*)} \sum_{h\in H(p)} x_h^*$. We choose the lowest index p^* if ties occur. Then we make two branches in the branch-and-bound tree such that any closed trail for $p \in P_1(x^*,y^*;e^*)$ can not be selected in the first node and any closed trail for $p \in P_2(x^*,y^*;e^*)$ can not be selected in the second node. That is, for the first node, we require $x_h = 0$, for all $h \in H(p)$ such that $p \in P_1(x^*,y^*;e^*)$ and for the second node, we require $x_h = 0$, for all $h \in H(p)$ such that $p \in P_2(x^*,y^*;e^*)$.

To satisfy the above requirements, for each variable x_h which is already generated, we set the upper bound of the variable to 0 if $h \in H(p)$ and $p \in P_1(x^*,y^*;e^*)$ in the first node, i.e. we set $x_h = 0$. Further, for each $p \in P_1(x^*,y^*;e^*)$, we don't need to generate column in the first node. Thus, the column generation is still possible in the branch. We perform similar bound setting and column generation in the second node.

We use the above branching rule by giving priority to y in branching. We branch on y first if there exists any one which has fractional value and we branch on x if y is integral. We can get an optimal solution to SP by branch-and-price procedure using above branching rule.

4. OVERVIEW OF THE ALGORITHM FOR SRWA

In this section, we present the algorithm to solve SRWAP. First, we construct a restricted LP relaxation, SRWARLP, which has a subset of the variables at first stage and then add other variables when they are needed. In the previous section, we explained the algorithm to solve the column generation problem SP which gives an ISIRC to be added. After solving SRWALP, if the obtained optimal solution to SRWALP is not integral, we perform a variable fixing procedure to get an integral solution to SRWA. In the procedure, we select a variable which has fractional value in final SRWARLP and fix the value of the variable to be 1. We solve the SRWARLP with the fixed variable and we generate more columns. We repeat the variable fixing and column generation until we get an integral solution.

4.1 Column generation procedure for SRWA

Now, we explain the column generation procedure to solve SRWALP. First, we construct an initial SRWARLP with a subset of C_I. As shown in the previous section, SP is to find a maximum weight ISIRC. Thus, we can get an ISIRC by solving SP with weight $\alpha^* = \mathbf{1}$, where $\mathbf{1}$ is a vector with all of its components 1. We remove the paths in the obtained ISIRC from requirements and solve SP again. We can construct initial SRWARLP with the ISIRC's obtained by repeating above steps. In addition to the obtained ISIRC's, we add a dummy column consisting of all ones with a large objective value to SRWARLP and solve it to optimality. The dummy column ensures that a feasible solution to the SRWARLP exists. This dummy column will be kept in the variable fixing procedure described in the next subsection for the same reason. Let z^* be the obtained optimal solution to SRWARLP and α^* be the optimal dual solution. Then, we solve SP with the weight vector α^*. If the optimal objective value of SP is less than or equal to 1, then z^* satisfies the optimality condition and we have found an optimal solution to SRWALP. Otherwise, we add the obtained ISIRC by solving SP to SRWARLP and solve it again. We repeat the same procedure until no more column is generated. Then, we get an optimal solution to SRWALP.

If the obtained optimal solution to SRWALP is integral then the solution is an optimal solution to SRWAP. Otherwise, we perform variable fixing procedure described in the following subsection.

4.2 Variable fixing procedure for SRWA

After solving SRWALP, we must use the branch-and-price procedure to obtain an optimal solution to SRWA. But the procedure has some difficulty in column regeneration after branching as mentioned in section 3. So, we use a variable fixing procedure only consider the case that z_c is fixed to 1. To overcome the difficulty, we devise a branching rule to solve column generation problem in section 3. However, we can't devise any method to overcome the difficulty on SRWAP and we consider only the case that variable z_c is fixed to 1 and generate more columns after fixing the variable.

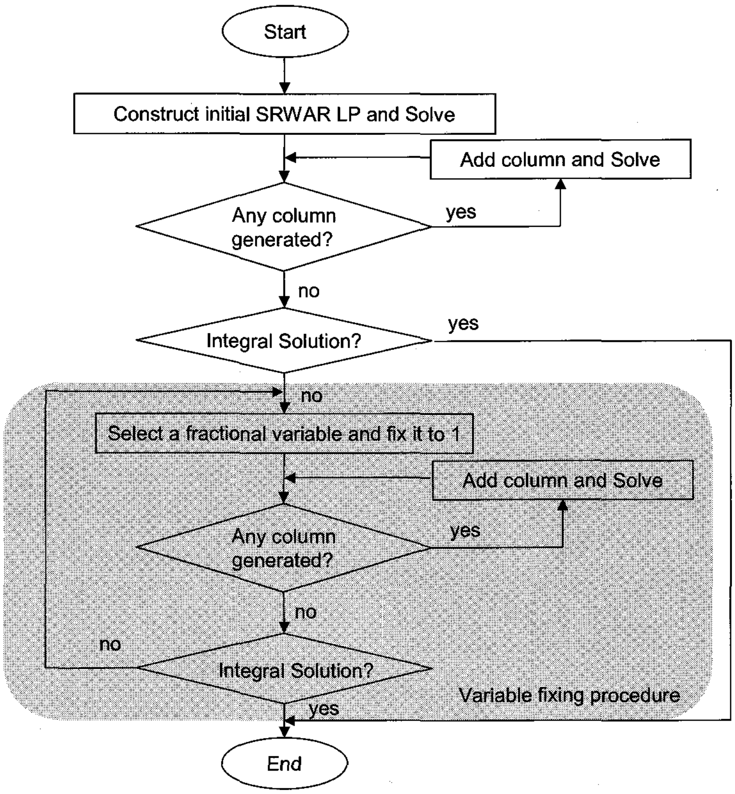

Figure 2. Flow chart of algorithm

First, we select a variable which has maximum value among the variables having fractional value in the last SRWARLP and then fixed the value of the variable to 1. After fixing, we solve SRWARLP and we perform the column generation procedure until no more columns is generated. If the obtained solution is integral then we have found an integral solution to SRWA. Otherwise, we select another variable which has fractional value and fixed it to 1 and then generate columns again. We repeat above steps until we get an integral solution. The procedure does not guarantee to find an optimal solution to SRWA. But the last SRWARLP may contain many columns that are part of the optimal solution because the most profitable columns are generated and we generate more columns in the variable fixing procedure. Thus, we can expect to find a good solution. We can check the quality of our solution by comparing it with the lower bound obtained from the optimal value of SRWALP. Computational results in the next section show that our solution is very good. The flow chart of overall algorithm is given in Figure 2.

5. COMPUTATIONAL RESULTS

We tested our algorithm on two networks. One is the NSFNET(National Science Fundamental Network) and the other is EON(European Optical Network) which are shown in Figure 3 and 4. For the test, we randomly generated 20 randomly generated problem instances for each network. In each problem instance, we generated 1 or 2 working paths for all possible pairs of nodes with the same probability. We use the shortest path between two nodes as the working path. The tests were run on a Pentium PC (2.0GHz) and we used CPLEX 9.0 callable mixed integer library as an LP solver.

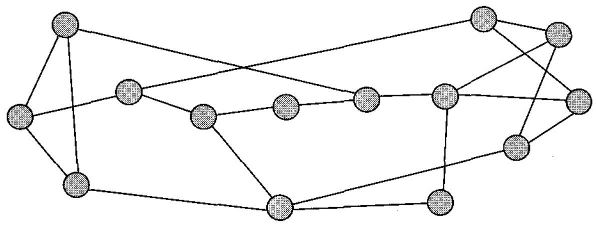

Figure 3. National Science Fundamental Network

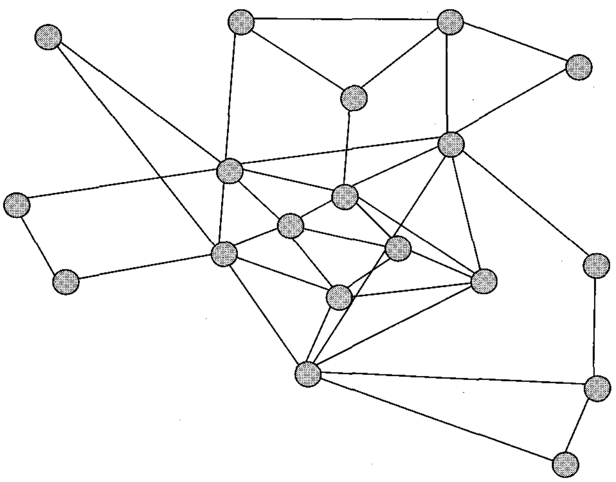

Figure 4. 19-node European Optical Network

Test results are summarized in Table 4.1 and 4.2. In the tables, the heading # of col1 and # of col2 refer to the number of generated columns until SRWALP is solved to optimality and the number of generated columns in the variable fixing procedure after solving SRWALP, respectively. The heading # of fix refers to the number of fixed variables in the variable fixing procedure. $\lceil Z_{SRWALP} \rceil$ refers to the value obtained by rounding up the optimal objective value of SRWALP which provides a lower bound on the optimal objective value of SRWA. Z refers to the objective value obtained by our algorithm. Dif is defined as Dif = Z-$\lceil Z_{SRWALP} \rceil$ and it gives an upper bound on the difference between the optimal solution value and obtained solution value. We give the objective value obtained by branch-and-bound procedure without more column generation under the heading of Z_{bnb}. The time to solve the problem by our algorithm is reported under the heading of Time.

Table 1. Computational results on NSFNET

	# of col1	# of col2	# of fix	$\lceil Z_{SRWALP} \rceil$	Z_{bnb}	Z	Dif	Time
1	132	32	6	28	29	28	0	95.7
2	235	39	12	25	26	25	0	1226.3
3	200	45	10	27	28	27	0	529.9
4	336	99	6	23	24	23	0	2093.4
5	172	114	8	29	30	29	0	981.6
6	176	62	5	25	26	25	0	131.0
7	247	70	15	29	30	29	0	1010.5
8	199	97	7	25	26	25	0	480.4
9	165	31	5	26	26	26	0	345.0
10	221	77	6	21	22	21	0	929.1
11	143	16	1	25	25	25	0	310.0
12	255	30	4	29	30	29	0	1151.3
13	179	14	2	24	24	24	0	268.7
14	177	125	12	24	26	24	0	1088.7
15	254	51	5	25	26	25	0	758.3
16	201	51	5	28	29	28	0	629.6
17	100	20	9	29	30	29	0	19.2
18	179	27	1	24	24	24	0	362.0
19	172	19	2	23	23	23	0	327.2
20	157	55	13	26	27	26	0	435.3

Test results show that our algorithm gives optimal solutions to all test problem instances. SRWALP gives a very tight lower bound on the optimal objective value of SRWA. As shown in the Table 1, # of col2 is much less than and # of col1 because most of the columns contained in an optimal solution are generated before variable fixing procedure on NSFN. Z_{bnb} shows that branch-and-bound procedure without additional column generation gives comparatively good solutions on NSFN. But, the # of col2 is relatively large on EON and Z_{bnb} is not good. This means that the column generation after variable fixing is effective and our variable fixing procedure gives an optimal solution to all test problem instances.

Table 2. Computational results on EON

	# of col1	# of col2	# of fix	$\lceil Z_{SRWALP} \rceil$	Z_{bnb}	Z	Dif	Time
1	109	32	14	53	54	53	0	36.3
2	109	84	29	52	55	52	0	129.6
3	118	49	16	57	60	57	0	60.6
4	92	79	23	56	58	56	0	82.7
5	127	159	27	49	54	49	0	131.4
6	97	29	17	57	58	57	0	53.6
7	116	138	31	48	53	48	0	210.9
8	117	195	27	51	55	51	0	208.8
9	102	43	17	52	55	52	0	52.5
10	113	155	27	49	54	49	0	108.7
11	98	30	15	52	55	52	0	85.9
12	114	74	25	55	58	55	0	59.7
13	89	45	28	56	58	56	0	44.7
14	110	77	20	52	56	52	0	62.5
15	112	60	18	52	55	52	0	49.6
16	115	117	24	51	54	51	0	133.6
17	106	42	18	55	58	55	0	45.6
18	92	47	25	57	60	57	0	82.9
19	87	38	12	57	58	57	0	59.0
20	104	72	26	54	57	54	0	220.0

6. CONCLUDING REMARKS

In this paper, we consider the routing and wavelength assignment problem on survivable WDM network under the single-link failure. We assume that a working path and corresponding protection path use the same wavelength. We proposed an integer programming formulation and an algorithm to solve it based on the column generation technique. To solve the column generation problem, we developed a branch-and-price algorithm. In the algorithm, we devise a branching rule not destroying the structure of the column generation. After solving the LP relaxation, we applied a variable fixing procedure combined with column generation to obtain an integral solution. We tested our algorithm on randomly generated data and test results showed that our algorithm provided very good solutions

In this paper, we considered the path protection scheme. Considering other protection schemes could be good research works. We consider the case that working path are given. Then, the routing of working path may affect the results. To check the influence of the routing of working paths using the model in this paper under various routing strategies of working path can be a meaningful work. Moreover, the problem to decide the routing of working paths and corresponding protection paths and wavelength assignment together may be a good research. Researches on that case have been performed [5, 17] but they are concentrated on developing heuristic algorithms.

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