Resolution of Riemann problem for weak shock in one space dimension

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We resolve a key part of the long-standing Riemann problem on stability of a shock wave arising in compressible fluid flow. More precisely, we prove the conjecture as follows: A weak shock of the compressible Euler equations in one space dimension is globally-in-time stable in the class of inviscid limits from the associated Navier-Stokes equations. To resolve it, we develop a groundbreaking methodology as a robust analytic tool applicable to a variety of PDE models.

1. Background (objectives)

Compressible Euler system, first formulated by Euler in 1752, is the system of nonlinear partial differential equations describing dynamics of compressible inviscid fluid, which is has been extensively used in a variety of fields. The most important feature of compressible Euler system is the formation of shock wave as a severe singularity due to jump discontinuity in space and irreversibility in time. The shock wave was first tackled by Riemann in 1860. The Riemann problem, first proposed by Riemann, is the issue for stability of physical perturbations of a self-similar wave starting from an initial datum composed of a jump discontinuity connecting two different constant states. This problem was reformulated as the following conjecture, based on the dramatic development of analysis and introduction of Navier-Stokes system as a variant of Euler system for the viscous effect of compressible fluid.

Conjecture: The Riemann problem is well-posed in the class of vanishing viscosity limits from Navier-Stokes system.

This conjecture remains totally open even in one space dimension.

2. Contents

In collaboration with Alexis Vasseur, the conjecture was first resolved for a single shock with small amplitude in the isentropic case of one space dimension. That is, we prove that a single self-similar shock with small amplitude to the Riemann problem is unique and orbitally stable in the class of vanishing viscosity limits of solutions to the associated Navier-Stokes system.

To construct the desired class, we prove uniform stability of any solutions to the Navier-Stokes system as perturbations of viscous shock (i.e., a traveling wave smooth solution corresponding to the shock). The uniform stability does not depend on the strength of viscosity. This provides good control on the vanishing viscosity limit process. More precisely, thanks to the uniformity, there exists a vanishing viscosity limit of the solution to the Navier-Stokes system. In addition, the vanishing viscosity limits satisfy some stability estimate measured by the relative entropy with respect to the shock. This implies the stability and uniqueness of the shock in the desired class.

3. Expected effect

This result becomes a very important cornerstone in studying on various PDE-based models, especially, for the hyperbolic system of conservation laws. Indeed, the mathematical hypotheses in the theory of the hyperbolic system of conservation laws have been developed through a generalization of properties of compressible Euler system. The hyperbolic system of conservation laws is the abstract representation of the Euler system and many other physical systems such as the system for dynamics of electromagnetism (so called Maxwell's equations); magnetohydrodynamics; hyperelastic materials (e.g. strings, membranes), etc. Recently, HSC is also used in modeling various phenomena arising in vehicular traffic flow; driven thin film flow; blood flow; tumor angiogenesis; oil recovery, etc.

Research outcomes

Paper
Moon-Jin Kang and Alexis Vasseur, Uniqueness and stability of entropy shocks to the isentropic Euler system in a class of inviscid limits from a large family of Navier-Stokes systems, Inventiones mathematicae, Vol. 224, pages 55-146, 2021.

Moon-Jin Kang and Alexis Vasseur, Contraction property for large perturbations of shocks of the barotropic Navier-Stokes system, Journal of the European Mathematical Society, Vol.23 pages 585-638, 2021.

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