



Kramers-Kronig receiver operable without digital upsampling

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Abstract: The Kramers-Kronig (KK) receiver is capable of retrieving the phase information of optical single-sideband (SSB) signal from the optical intensity when the optical signal satisfies the minimum phase condition. Thus, it is possible to direct-detect the optical SSB signal without suffering from the signal-signal beat interference and linear transmission impairments. However, due to the spectral broadening induced by nonlinear operations in the conventional KK algorithm, it is necessary to employ the digital upsampling at the beginning of the digital signal processing (DSP). The increased number of samples at the DSP would hinder the real-time implementation of this attractive receiver. Hence, we propose a new DSP algorithm for KK receiver operable at 2 samples per symbol. We adopt a couple of mathematical approximations to avoid the use of nonlinear operations such as logarithm and exponential functions. By using the proposed algorithm, we demonstrate the transmission of 112-Gb/s SSB orthogonal frequency-division-multiplexed signal over an 80-km fiber link. The results show that the proposed algorithm operating at 2 samples per symbol exhibits similar performance to the conventional KK one operating at 6 samples per symbol. We also present the error analysis of the proposed algorithm for KK receiver in comparison with the conventional one.

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1. Introduction

There is an abiding interest in the direct-detection (DD) receiver for high-speed optical transmission system, due to its intrinsically simple structure and cost-effectiveness [1]. However, the square-law detection of optical double-sideband signal always suffers from the dispersion-induced RF power fading. Since the phase information of light is lost upon DD, this fading limits the maximum transmission distance over dispersive single-mode fiber when no optical dispersion compensation is employed. In this regard, the optical single-sideband (SSB) transmission has recently drawn a great deal of attention since it not only overcomes the dispersion-induced RF power fading but it can also double the spectral efficiency (SE). However, due to the signal-signal beat interference (SSBI) inherent in the direct detection of optical SSB signal, it was necessary to sacrifice either the SE by inserting a frequency gap between the carrier and SSB signal [2] or the receiver sensitivity by increasing the carrier-to-signal power ratio (CSPR) [3]. Recently, the Kramers-Kronig (KK) receiver has been proposed to overcome these problems [4]. In this receiver, the phase information (lost upon DD) can be retrieved from the intensity information by using digital signal processing (DSP) if the optical signal satisfies the minimum phase condition. Thus, this receiver allows us to cancel the SSBI components without having a large CSPR (in comparison with other SSBI cancellation techniques [5]). It is also possible to compensate for linear transmission impairments (such as fiber's chromatic dispersion) using the electrical equalization technique at the receiver [6]. This DD receiver can be used to reconstruct the electric fields of polarization-division-multiplexed signals when a carrier is provided at the receiver [7,8].

One of technical challenges associated with the implementation of the KK receiver is that the DSP needs to operate a couple of times faster than the Nyquist sampling rate. This is because some nonlinear operations (such as logarithm and exponential functions) required to perform the KK algorithm broaden the signal's spectrum significantly. To accommodate such spectral broadening, the digital upsampling is typically employed at the beginning of DSP. As a result, a faster processing speed and a larger memory would be required in the DSP chip. For example, it is expected that the KK receiver should run at as fast as 192 Gsample/s (i.e., three times the Nyquist sampling rate) when it is used in the detection of the 32-Gbaud signal. This high sampling rate of DSP would be a major obstacle to the realization of the KK receiver. Even though the parallel implementation of DSP could lower the actual processing speed inside the DSP chip, the digital upsampling would increase the complexity and power consumption considerably. An alternative DSP algorithm for KK receiver which avoids the digital upsampling was proposed in [4]. Due to multiple iterations, however, it is expected that this algorithm would incur a significant latency and complexity when implemented.

We have recently proposed the non-iterative DSP algorithm for KK receiver operable without the digital upsampling [10]. Thus, it is possible to operate this algorithm at the Nyquist sampling rate [i.e., 2 samples per symbol (SPS)]. The key idea is to utilize appropriate mathematical approximations to replace the nonlinear operations which broaden the spectrum significantly in DSP [10]. In this paper, we provide the detailed derivations of our approximations and carry out the error analysis. Also presented in this paper is the impact of the sampling rate on the performance of the proposed algorithm. We evaluate the performance of the proposed algorithm in a single-channel 112-Gb/s SSB orthogonal frequency-division-multiplexing (OFDM) transmission system. We show that the proposed algorithm (SPS = 2) exhibits similar performance to the conventional one (SPS = 6).

2. Principles

An optical SSB signal impinging upon the DD receiver can be expressed as

$$E(t) = E_0 + s(t) + j\hat{s}(t) \quad (1)$$

where E_0 is the optical carrier, and $s(t)$ and $\hat{s}(t)$ are the Hilbert transform pairs for the signal $s(t)$. It should be noted that $s(t)$ contains all the information to be delivered, while the redundant information in $\hat{s}(t)$ makes $E(t)$ to be an SSB signal. In general, the magnitude of the optical carrier should be large enough to satisfy the minimum phase condition [4]. At the receiver, we obtain the electrical signal $I(t)$ after the square-law detection:

$$I(t) = |E(t)|^2 = \eta |E_0 + s(t) + j\hat{s}(t)|^2 \quad (2)$$

where η is the receiver responsivity and we assume $\eta = 1$ without loss of generality.

2.1 The conventional DSP algorithm for KK receiver

In the conventional KK algorithm, the phase information can be retrieved from the intensity information through the Hilbert transform as

$$\phi(t) = H[\ln \sqrt{I(t)}] \quad (3)$$

where $\ln(\cdot)$ is the natural logarithm operation and $H[\cdot]$ is the Hilbert transform operator. Then, the electrical field of the optical signal can be recovered by

$$E(t) = \sqrt{I(t)} \exp[j\phi(t)] \quad (4)$$

A block diagram of the conventional KK algorithm is illustrated in Fig. 1(a). The digital upsampling precedes the DSP blocks for this KK algorithm to accommodate the spectral broadening due to the nonlinear operations including logarithm and exponential functions. Recent experimental demonstrations show that typical values of SPS range from 4 to 6 [5]-[9]. The digital downsampling is placed at the end of the block diagram for the subsequent DSP such as the electrical equalization and demodulation.

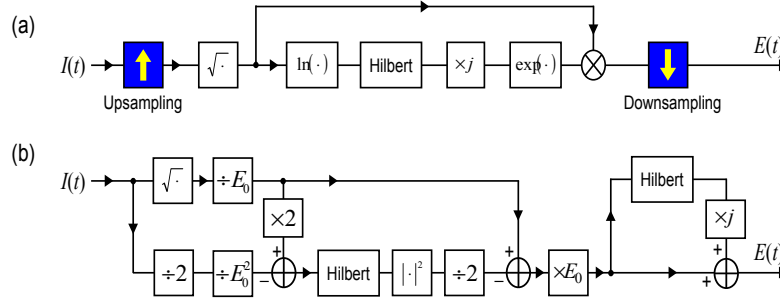


Fig. 1. Block diagrams of (a) the conventional KK receiver [5] and (b) proposed KK receiver.

2.2 The proposed DSP algorithm for KK receiver

Here we propose the use of appropriate mathematical approximations to avoid the use of some nonlinear operations found in the conventional KK algorithm. First, the square-root of $I(t)$ can be approximated to the second-order binomial expansion as [11]

$$\begin{aligned}
\sqrt{I(t)} &= E_0 \sqrt{1 + \frac{2s(t)}{E_0} + \frac{s^2(t) + \hat{s}^2(t)}{E_0^2}} \\
&= E_0 \left\{ 1 + \frac{1}{2} \left[\frac{2s(t)}{E_0} + \frac{s^2(t) + \hat{s}^2(t)}{E_0^2} \right] - \frac{1}{8} \left[\frac{2s(t)}{E_0} + \frac{s^2(t) + \hat{s}^2(t)}{E_0^2} \right]^2 + O(E_0^{-5}) \right\} \quad (5) \\
&= E_0 + s(t) + \frac{\hat{s}^2(t)}{2E_0} + O(E_0^{-2})
\end{aligned}$$

where $O(\cdot)$ denotes the order of approximation. In the last equation of Eq. (5), the 3rd term represents the SSBI component.

In our derivations, we assume that the magnitude of optical carrier is much larger than the magnitude of signal, i.e., $E_0 \gg |s(t)|$. Fortunately, this assumption is satisfied in most cases due to the minimum phase condition. Thus, the signal's phase can be approximated as

$$\phi(t) = \tan^{-1} \frac{\hat{s}(t)}{E_0 + s(t)} = \frac{\hat{s}(t)}{E_0} + O(E_0^{-2}) \quad (6)$$

From Eqs. (3) and (6), we then obtain

$$\frac{\hat{s}(t)}{E_0} \cong H \left[\ln \sqrt{I(t)} \right] \quad (7)$$

Then, we can express $\ln \sqrt{I(t)}$ by using the second-order Taylor's expansion at $I(t) = E_0^2$ as

$$\ln \sqrt{I(t)} = \ln \left\{ E_0 \left[\left(\frac{\sqrt{I(t)}}{E_0} - 1 \right) + 1 \right] \right\} = 2 \frac{\sqrt{I(t)}}{E_0} - \frac{1}{2} \frac{I(t)}{E_0^2} + \ln E_0 - \frac{3}{2} + O(E_0^{-3}) \quad (8)$$

In Eq. (8), we can ignore the third and fourth constant terms on the right hand side since these DC components are removed by the subsequent Hilbert transform. Finally, combining three Eqs. (5), (7) and (8) yields the real part of the optical field, $E_0 + s(t)$, as follows:

$$E_0 + s(t) \cong \sqrt{I(t)} - \frac{E_0}{2} \left\{ H \left[2 \frac{\sqrt{I(t)}}{E_0} - \frac{1}{2} \frac{I(t)}{E_0^2} \right] \right\}^2 \quad (9)$$

The imaginary part of the optical field can be obtained by taking the Hilbert transform of the real part. Figure 1(b) shows the block diagram of the proposed KK algorithm.

It is worth emphasizing that we do not simply replace some of the nonlinear operations with their approximations. As shown in Fig. 1(b), we express the electric field of the optical signal, $E(t)$, as the sum of real and imaginary parts, rather than their phasor form (i.e., magnitude multiplied by the exponential phase). As a result, the nonlinear operations such as logarithm and exponential functions are now removed in this newly proposed algorithm. Although the square-root operation [found in (4)] is still used in (9), the performance improvement achieved by using the digital upsampling is not significant (in comparison with the conventional algorithm) since this nonlinear operation slightly broadens the signal's spectrum.

3. Experiment

We carry out transmission experiments to evaluate the performance of the proposed DSP algorithm for KK receiver operable at the Nyquist sampling rate without digital upsampling. Figure 2(a) shows the experimental setup. We first generate a 112-Gb/s SSB OFDM signal. A pseudo-random bit sequence with the length of $2^{15}-1$ is mapped onto 16-quadrature amplitude modulation (QAM) symbols and then loaded into 448 subcarriers out of 1024 subcarriers. The unmodulated subcarriers are padded to zeros. A 1024-point inverse fast Fourier transform is then performed to generate an OFDM signal. Cyclic prefix and preamble symbols are then appended to this OFDM signal. The net data rate of the signal is 103.4 Gb/s, including the overhead of forward error correction. After parallel-to-serial conversion, the real and imaginary parts of the signal are sent to an arbitrary waveform generator (AWG) operating at 64 Gsample/s. An optical signal from an external-cavity laser (@1550 nm) is then modulated by using an in-phase and quadrature (IQ) modulator. The CSRR is adjusted by giving a bias offset from the quadrature point of the IQ-modulator and measured by using an optical spectrum analyzer [5]. We employ an optical booster amplifier (noise figure = 5 dB) operating in the constant power mode at the output of the IQ modulator. The fiber launch power into standard single-mode fiber (SSMF) is set to be 7 dBm to avoid provoking the stimulated Brillouin scattering. After the transmission, the optical signal is detected by using an optically pre-amplified receiver (noise figure = 3.3 dB, optical gain = 42 dB). The received optical power is measured before the pre-amplifier. An optical bandpass filter (bandwidth = 0.4 nm) is used to reject the amplified spontaneous emission noise outside of the signal spectrum. Figure 2(b) shows the optical spectrum of the SSB OFDM signal at the receiver. The upper sideband is suppressed by >30 dB. After DD, the signal is digitized by using a real-time oscilloscope operating at 80 Gsample/s. The offline DSP is composed of resampling, KK algorithm, synchronization based on symbol correlation, fast Fourier transform, and one-tap channel equalization. The loss of DC-term in our AC-coupled receiver is easily recovered by sweeping the magnitude of the DC component in our offline DSP. Finally, the bit-error rate (BER) is calculated by using direct error counting of 1 million bits.

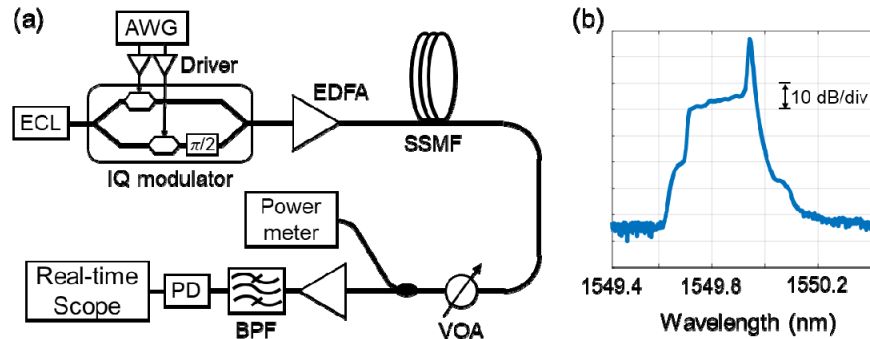


Fig. 2. (a). Experiment setup. ECL: external-cavity laser, AWG: arbitrary waveform generator, EDFA: Er-doped fiber amplifier, BPF: band-pass filter, PD: photodetector. (b) Optical spectrum measured before the PD.

We first find the magnitude of optical carrier, E_0 , by sweeping it while measuring the BER performance due to its loss by the AC-coupled receiver. It should be noted that the same procedure is required for the conventional KK algorithm when the AC-coupled receiver is used. This is because $I(t)$ should be always positive as shown in Eq. (3) and its value is dependent upon E_0 . Figure 3 shows the measured BER performance after 80-km transmission as a function of E_0 when the CSRR is 10 dB and the received optical power is -19 dBm. The results show that the BER performance is sensitive to the estimated E_0 value when it is smaller than the optimum E_0 value. However, this sensitivity is ameliorated beyond the

optimum value. From this measurement, we find out that the optimum E_0 value is 0.1 in this case. It should be noted that in the case where a DC-coupled receiver is employed, we can estimate E_0 by using the following equation.

$$E_0 = \sqrt{DC \times \left(1 - \frac{1}{1 + CSPP}\right)} \quad (10)$$

Here, DC is the DC component at the receiver. This equation can be approximated as $DC^{1/2}$ when the CSPP is large. Equation (10) also implies that the E_0 value remains constant as long as the received optical power and the CSPP are unchanged.

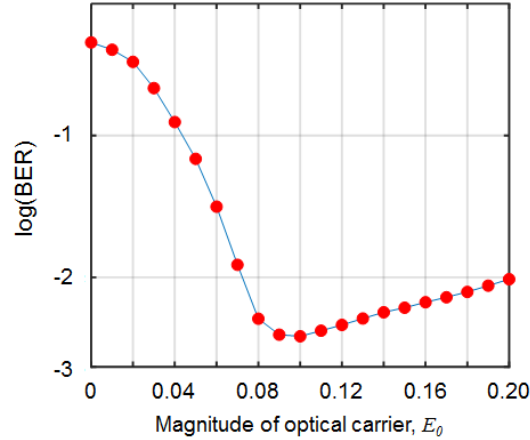


Fig. 3. BER performance as a function of the magnitude of optical carrier.

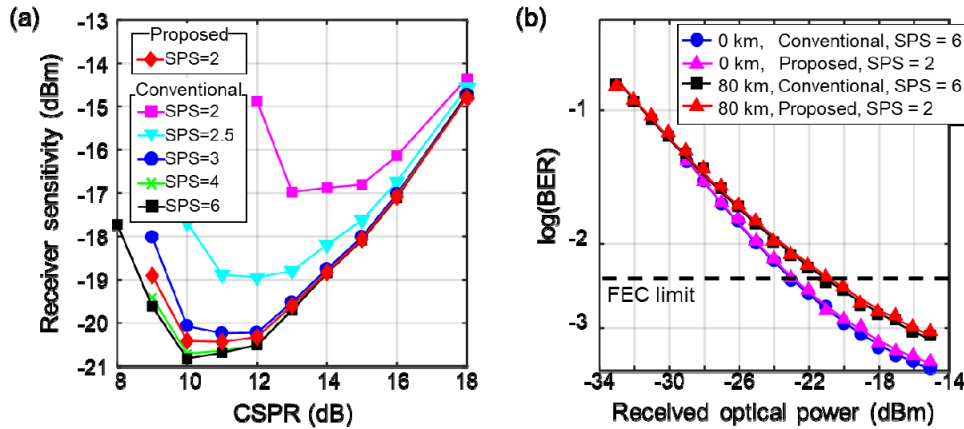


Fig. 4. Experimental results. (a) Measured receiver sensitivity (@BER = 3.8×10^{-3}) versus CSPP after 80-km transmission, (b) BER curves.

Figure 4(a) shows the measured receiver sensitivity (@BER = 3.8×10^{-3}) as a function of CSPP after transmission over 80-km long SSF. The detected signal is resampled at 2 SPSs and no digital upsampling is performed in the proposed algorithm. The result shows that we achieve the best receiver sensitivity of -20.4 dBm when the CSPP is 10 dB. Also shown in this plot for comparison is the receiver sensitivities obtained by using the conventional algorithm for various SPSs. In this case, the receiver sensitivity is improved with the number of SPSs. The CSPP which gives us the best receiver sensitivity is also reduced as we increase the number of SPSs. For example, the receiver sensitivity is measured to be -17.0 dBm when

the SPS is 2 and the CSPR is 13 dB. However, the best receiver sensitivity of -20.8 dBm is achieved when the SPS and CSPR are 6 and 10 dB, respectively. This is because as 2 SPS is not sufficient enough to accommodate the bandwidth expansion induced by nonlinear operations (e.g., logarithm and exponential functions) in the conventional KK algorithm. On the other hand, the proposed algorithm (SPS = 2) incurs a 0.4-dB penalty with respect to the conventional one (SPS = 6). It has been recently shown that the KK receiver requires the least CSPR value for the best receiver sensitivity when compared with SSBI cancellation techniques [5]. The results show that the proposed algorithm provides the same optimum CSPR value for the receiver sensitivity as the conventional algorithm. It is worth noting that the optimum CSPR value for the receiver sensitivity is smaller than the CSPR value which satisfies the minimum phase condition completely. This is because the receiver sensitivity would not deteriorate much by a slight violation of minimum phase condition when the signal has a wide range of amplitude variation.

Figure 4(b) shows the BER performance after 0- and 80-km transmissions for the proposed (SPS = 2) and conventional (SPS = 6) DSP algorithms for KK receiver. The CSPR is set to be 10 dB, which is found to be the optimum in Fig. 4(a). We can see that the proposed algorithm exhibits similar performance to the conventional one after the transmissions.

4. Discussions

4.1 Error analysis of the proposed algorithm

The proposed DSP algorithm for KK receiver utilizes three mathematical approximations to recover the electric field of the detected optical SSB signal, without employing logarithm and exponential functions. Those approximations are (i) the second-order binomial approximation to estimate the square root of the DD signal [in Eq. (5)], (ii) the first-order Taylor approximation of $\tan^{-1}(\cdot)$ function to estimate the phase φ [in Eq. (6)], and (iii) the second-order Taylor approximation of logarithm function [in Eq. (8)]. Altogether, Eq. (9) estimates the real part of the electric field by using all the approximations mentioned above. Therefore, there exist a slight discrepancy between the actual waveform of the electric field and the waveform recovered from the proposed algorithm. To investigate this discrepancy, we carry out a computer simulation where a random waveform in the form of optical SSB modulation is generated and then the mean squared errors between the original and the recovered waveforms are estimated. The random waveform is sampled at the Nyquist sampling rate and the total number of samples is 10^7 . The normalized mean squared error (NMSE) in decibel is calculated by using [12]

$$NMSE = 10 \cdot \log_{10} \frac{E\{[x(n) - \tilde{x}(n)]^2\}}{E\{x(n)\}E\{\tilde{x}(n)\}} \quad (11)$$

where n is the time index, $x(n)$ is the sampled value of the original waveform at time n , $\tilde{x}(n)$ is the recovered value obtained from the approximation, and $E[\cdot]$ is the expectation notation. Figure 5 shows the NMSEs of Eqs. (5) and (9) as a function of CSPR. Since Eq. (5) employs the second-order binomial approximation only, it exhibits a lower NMSE than Eq. (9). Thus, the NMSE difference between Eqs. (9) and (5) should be attributed to the other two approximations [i.e., first- and second-order Taylor approximations in Eqs. (6) and (8)]. The result shows that the NMSE difference between these two approximations is negligible when the CSPR is larger than 4 dB. This implies that the accuracy of our proposed DSP algorithm is mainly limited by the second-order binomial approximation shown in Eq. (5). Also shown in the figure is that the NMSE of the proposed algorithm decreases rapidly with the CSPR at a rate of -20 dB/decade. This is because the accuracy of the second-order binomial approximation is improved inversely proportional to the square of carrier magnitude [see Eq.

(5)]. Therefore, the NMSE of Eq. (9) becomes lower than -28 dB when the CSPR of the detected signal is larger than 10 dB. According to the previously reported results, the typical values of CSPR for the best BER performance range from 6 to 14 dB [5]–[9]. The error analysis results depicted in Fig. 5 show that the NMSE of our proposed algorithm would be in the range of -20 to -36 dB for those CSPR values. This implies that our proposed algorithm would cause merely 1% mean squared error in the estimation of the actual waveform even when the CSPR of the signal is as low as 6 dB.

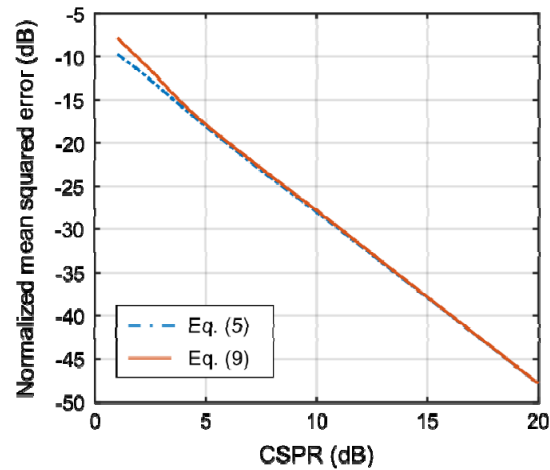


Fig. 5. The normalized mean square error between the original and approximated waveforms.

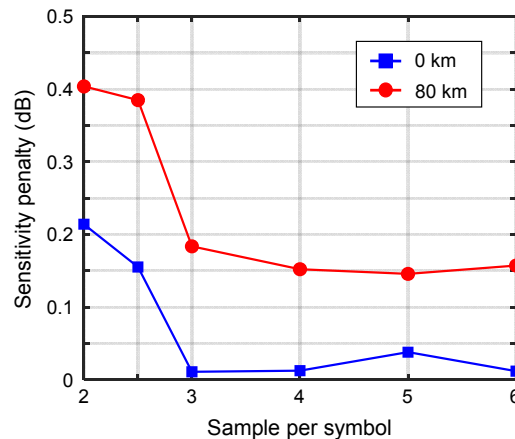


Fig. 6. Effect of the sampling rate on the performance of the proposed algorithm. The sensitivity penalty is measured with respect to the conventional KK algorithm operating at 6 SPSs.

4.2 Impact of the sampling rate on the performance of the proposed algorithm

Our proposed algorithm avoids the use of logarithm and exponential functions, but it retains some nonlinear functions such as square root and square. Thus, the performance of the proposed algorithm could be improved as we increase the sampling rate since these nonlinear functions also broaden the spectrum. To investigate the impact of the digital upsampling on the performance of the proposed algorithm, we utilize the experimental data described in Section 3. In this case, we insert the digital upsampling and downsampling blocks in the beginning and end of our proposed algorithm, respectively, and then measure the receiver

sensitivity. Figure 6 shows the sensitivity penalty (with respect to the conventional algorithm operating at $\text{SPS} = 6$) as a function of SPS. The CSRR of the signal is 10 dB. The results show that the receiver sensitivity is improved as we increase the sampling rate. For example, the sensitivity penalty after 80-km transmission is reduced by 0.2 dB when the SPS is increased from 2 to 3. However, the penalty levels off when we further increase the sampling rate. For example, at 6 SPSs, the sensitivity penalties with respect to the conventional algorithm are measured to be 0.02 and 0.15 dB after 0- and 80-km transmissions, respectively. These penalties should be ascribed to the approximations of the proposed algorithm. Nonetheless, the proposed algorithm is capable of performing similar to the conventional one even though it requires fewer samples per symbol.

5. Conclusions

We have proposed a non-iterative DSP algorithm for KK receiver operable without digital upsampling. We adopt a couple of mathematical approximations about the nonlinear functions required in the conventional KK algorithm. Thus, it is possible to run this algorithm at 2 samples per symbol without incurring significant performance degradation in comparison with the conventional algorithm. The experimental verification performed with 112-Gb/s SSB-OFDM signal shows that our proposed algorithm operating at 2 samples per symbol exhibits merely 0.2 and 0.4-dB sensitivity penalties with respect to the conventional one operating at 6 samples per symbol after transmissions over 0- and 80-km long SSMF, respectively. We have also carried out the error analysis to evaluate the performance of the proposed algorithm. The results confirm that the proposed algorithm (operating at 2 samples per symbol) has the same performance as the conventional one (operating at 6 samples per symbol) as the CSRR and the sampling rate increases. Nevertheless, the proposed DSP algorithm for KK receiver is capable of recovering the electric field of the directly-detected signal accurately even when the CSRR is as low as 6 dB and it runs at 2 samples per symbol. Thus, we believe that the proposed DSP algorithm could be used to implement the real-time KK receiver cost-effectively.

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