A Markovian approach for modeling IS-856 reverse link rate control

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Abstract—The IS-856 standard provides bandwidth-efficient and high-speed wireless internet services to mobile subscribers. It can provide a maximum data rate of 2.4 Mbps on the forward link and 153.6 kbps on the reverse link by adopting advanced techniques. On the reverse link, there is a distributed rate control algorithm, which is based on a probabilistic model. Each mobile determines its data rate at the beginning of a new frame using a reverse activity bit and persistence probabilities. In this paper, the performance of the rate control is analyzed by modeling it as a Markov process. The results show that there is a trade-off between the reverse link throughput and the outage probability. In addition, the performance of the rate control is highly dependent on system parameters of IS-856.

I. INTRODUCTION

The TIA/EIA IS-856 standard [1], also known as cdma2000 1xEV-DO, provides bandwidth-efficient and high-speed wireless internet services to mobile subscribers. The International Telecommunication Union (ITU) approved the standard as a member of IMT-2000 family in 2001. The standard is considered as an evolution of cdma2000, and is suitable for packet data services which are characterized by asymmetric data rates. It achieves forward link data rates up to 2.4 Mbps and reverse link data rates up to 153.6 kbps in 1.25 MHz bandwidth. To improve the system capacity, IS-856 adopted advanced techniques such as adaptive modulation and coding, Type-II hybrid ARQ, fast channel feedback information, and multiuser diversity. A time slot is a basic transmission unit, and is 1.666... ms long (2048 chips). A group of 16 slots is referred to as a frame.

The forward link of IS-856 consists of a pilot channel, a medium access control (MAC) channel, and a control or traffic channel. All channels on the forward link are time-division multiplexed, and they are transmitted at the same power level. The pilot channel is used in system acquisition, tracking and demodulation, active set management, and channel estimation/prediction. The MAC channel consists of two sub-channels: a reverse power control (RPC) channel and a reverse activity (RA) channel. The RPC channel carries a power control bit for a specific mobile at a 600 Hz update rate. The RA channel carries a reverse activity bit, which is critical information for the reverse link rate control. The control channel carries control messages broadcast to all mobiles, and the traffic channel carries user data packets. The forward traffic channel is shared by all mobiles in a cell (or sector) and supports data rates from 38.4 kbps to 2.4 Mbps, depending on channel conditions. A mobile receives data packets from one base station at a time even though it is connected to several base stations in an ‘active set’. It can re-select its serving base station and adjust the forward data rate adaptively in time-varying channel conditions.

The reverse link of IS-856 consists of an access channel and a reverse traffic channel. The access channel is used to send signaling messages to the access network in the idle state. The reverse traffic channel is assigned to each mobile in the connected state, and consists of a pilot channel, a reverse rate indicator (RRI) channel, a data rate control (DRC) channel, an acknowledgement (ACK) channel, and a data channel. Each channel in the reverse traffic channel is spread by an appropriate orthogonal Walsh function. The pilot channel is used for channel estimation and coherent detection. The RRI channel is used to indicate the data rate of the associated data channel, and is time-multiplexed with the pilot channel. The DRC and ACK channels are related to the forward link operation. The DRC channel informs the access network of the supported data rate on the forward traffic channel and the best serving base station for the forward link. A DRC message is repeated over DRCLength, which takes 1, 2, 4, or 8 slots. The ACK channel informs the access network whether a data packet transmitted on the forward traffic channel has been received successfully. The data channel carries user data packets and supports data rates from 9.6 kbps to 153.6 kbps. A mobile has a rate control algorithm that determines the data rate on the reverse link based on a probabilistic model.

The capacity of the IS-856 reverse link has been discussed intensively in the literature [2], [3]. However, there has been insufficient research on the performance of the reverse link rate control. Most results were obtained only through computer simulation [4], [5]. In this paper, we analyze the IS-856 reverse link rate control by modeling it as a Markov process. Assuming perfect power control, we first derive an expression for the noise rise, which is used as the traffic load on the reverse link in the rate control. Then, we propose a system model for the rate control, and obtain the reverse link throughput and the outage probability as two performance metrics.
II. REVERSE LINK RATE CONTROL

In this section, the IS-856 rate control algorithm on the reverse link is explained briefly. Data rates of 9.6, 19.2, 38.4, 76.8, and 153.6 kbps are available on the reverse data channel, and they are denoted by $R_1$, $R_2$, $R_3$, $R_4$, and $R_5$, respectively. Before the beginning of a new frame, each mobile determines its data rate according to the rate control algorithm. To assist the operation, a base station broadcasts a reverse activity bit (RAB), which indicates whether or not the traffic load on the reverse link is above a certain threshold. A typical example of the traffic load is the noise rise, which represents the amount of interference at the receiver. If the noise rise at the base station exceeds the threshold, the RAB is set to 1. Otherwise, it is set to 0. Fig. 1 illustrates the basic concept of the rate control, assuming a single-cell environment where a mobile receives only one RAB at a time. Each mobile utilizes the latest RAB value and persistence probabilities ($p_i$ and $q_i$) for the rate control. If a mobile with a data rate of $R_i$, receives an RAB of 0, it increases the data rate to $R_{i+1}$ with probability $p_i$, and stays at the current rate with probability $1 - p_i$. If the mobile receives an RAB of 1, it reduces the data rate to $R_{i-1}$ with probability $q_i$, and stays at the current rate with probability $1 - q_i$.

In a multi-cell environment, a mobile may receive several RAB values simultaneously from all base stations in the active set. Suppose that a mobile increases its data rate according to a single RAB from a specific base station. Since this can have a serious influence on other base stations, the mobile should take into account the combined reverse activity bit (CRAB). The value of CRAB is obtained by the logical OR operation on the latest RAB values received from all base stations in the active set. Therefore, if the last received RAB is set to 1 from any base station in the active set, the CRAB is 1. Otherwise, it is 0. The unique difference compared with a single-cell environment is that the mobile determines its data rate by using the CRAB instead of a single RAB from a specific base station. So, ‘RAB’ in Fig. 1 is replaced by ‘CRAB’ in the multi-cell environment.

Every mobile has an initial data rate of 9.6 kbps. If a mobile has no data to transmit, the next arriving packet is transmitted with the initial data rate. The RateLimit, which is given by the access network, is the highest data rate of the mobile on the reverse link, and it can be lower than 153.6 kbps. More detailed and additional requirements on the rate control can be found in [1].

![Data rate transition diagram (single-cell case).](image)

III. PERFORMANCE ANALYSIS

We analyze the performance of IS-856 reverse link rate control with a Markovian approach. A single-cell environment is assumed, and the cell is not sectored. Since there is only one base station in the active set, a single RAB is considered in the rate control. In this paper, the traffic load on the reverse link is measured by the noise rise at the base station. We first derive an expression of the noise rise assuming perfect power control. Then, a Markov model for IS-856 reverse link rate control is introduced, and the performance analysis is performed.

A. Noise rise calculation

In IS-856, reverse link power control is applied to the pilot channel only. The power allocated to data, DRC, and ACK channels is adjusted by a fixed gain relative to the pilot channel to guarantee the desired performance. The data channel gain relative to the pilot channel has a value determined by the data rate. The default data channel gain is defined in [1] and reproduced in Table I. We denote the data rate of mobile $j$ by $r_j$ and the corresponding data channel gain by $G_{data}(r_j)$, where $r_j \in \{R_i, i = 1, \cdots, 5\}$. In addition, we denote the DRC channel gain relative to the pilot channel by $G_{DRC}$. The recommended value of $G_{DRC}$ is a function of $DRCLength$ [3]. The effect of the ACK channel on the total received power is negligible because only one mobile, at most, occupies the ACK channel at a time. Therefore, the received power of mobile $j$ at the base station, $P_j$, is expressed as

$$P_j = c(r_j) \cdot P_j,$$

where $c(r_j) \triangleq 1 + 10^{G_{data}(r_j)/10} + 10^{G_{DRC}/10}$, and $P_j$ is the received pilot channel power of mobile $j$.

In this paper, we assume perfect power control where the transmission power of each mobile immediately converges to the optimal value after mobiles change their data rates. The noise rise $\eta$ is defined as the ratio of the total received power $P_T$ to the thermal noise power $P_N$ (i.e., $\eta \triangleq P_T/P_N$). The reverse link power control of IS-856 maintains the signal quality of the pilot channel at a certain target level. The signal quality is usually measured by $E_c/N_0$ (ratio of pilot energy per chip to noise power spectral density), which is closely related to the packet error rate [2]. The $E_c/N_0$ of mobile $j$, $(E_c/N_0)_j$, is expressed as

$$\frac{(E_c/N_0)_j}{P_j} = \frac{P_j}{P_T - P_j},$$

using the orthogonal property of Walsh functions. Since the target value of $(E_c/N_0)_j$ depends on the data rate of mobile

<table>
<thead>
<tr>
<th>Data rate (kbps)</th>
<th>Symbol</th>
<th>Data channel gain (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.6</td>
<td>$R_1$</td>
<td>3.75</td>
</tr>
<tr>
<td>19.2</td>
<td>$R_2$</td>
<td>6.75</td>
</tr>
<tr>
<td>38.4</td>
<td>$R_3$</td>
<td>9.75</td>
</tr>
<tr>
<td>76.8</td>
<td>$R_4$</td>
<td>13.25</td>
</tr>
<tr>
<td>153.6</td>
<td>$R_5$</td>
<td>18.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table I</th>
<th>DEFAULT DATA CHANNEL GAINS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data rate (kbps)</td>
<td>Symbol</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------</td>
</tr>
<tr>
<td>9.6</td>
<td>$R_1$</td>
</tr>
<tr>
<td>19.2</td>
<td>$R_2$</td>
</tr>
<tr>
<td>38.4</td>
<td>$R_3$</td>
</tr>
<tr>
<td>76.8</td>
<td>$R_4$</td>
</tr>
<tr>
<td>153.6</td>
<td>$R_5$</td>
</tr>
</tbody>
</table>
We denote the target value by $\tau(r_j)$. Then, after the transmission power of each mobile converges to the optimal value, $P_j$ can be expressed as

$$P_j = \frac{P_T}{c(r_j) + 1/\tau(r_j)}.$$  

(3)

Let $\Gamma$ be the number of mobiles. From $P_N = P_T - \sum_{j=1}^{\Gamma} P_j$, the noise rise is given by

$$\eta = \left(1 - \sum_{j=1}^{\Gamma} \frac{c(r_j)}{c(r_j) + 1/\tau(r_j)}\right)^{-1}.$$  

(4)

In (4), $\eta$ can take a negative value depending on the data rate distribution of mobiles. This means that, even at the maximum power of a mobile, it is difficult to satisfy the required $E_c/N_0$. Therefore, if $\sum_{j=1}^{\Gamma} c(r_j)/(c(r_j) + 1/\tau(r_j)) > 1$, the value of $\eta$ is regarded as $\infty$.

**B. System model**

We assume that the reverse link frame boundaries of all mobiles are synchronized. Thus, all mobiles change their data rates simultaneously at the beginning of a new frame. The RAB updating period is short enough for mobiles to utilize the latest RAB value. Mobiles always have data to transmit, and they have a common RateLimit value of 153.6 kbps. Each mobile has an RAB decoding error probability $z$ that the received RAB is decoded as the opposite value. For example, if the base station transmits an RAB of 0, a mobile decodes the value as 1 with probability $z$, and as 0 with probability $1-z$. Throughout the analysis, random variables and their samples are represented by capital letters and the corresponding lowercase letters, respectively, unless otherwise stated. Vectors are denoted by boldface letters. Each subscript indicates the index of the corresponding data rate.

Let $S_i(t)$ denote the number of mobiles with a data rate of $R_i$ at the $t$-th frame. We define a state as

$$S(t) = (S_1(t), S_2(t), S_3(t), S_4(t)).$$  

(5)

Because $S_5(t)$ can be deduced from $S_4(t) = \Gamma - \sum_{i=1}^{4} S_i(t)$, it is not required in the definition of $S(t)$. In this model, the number of valid states that satisfy $0 \leq \sum_{i=1}^{4} S_i(t) \leq \Gamma$ is $\left(\frac{\Gamma + 4}{5}\right)$. The noise rise calculated by (4) is related to the data rate distribution of mobiles. Therefore, the noise rise for a state $x = (x_1, x_2, x_3, x_4)$, $\eta(x)$, can be expressed as

$$\eta(x) = \left(1 - \sum_{i=1}^{4} \frac{x_i c(R_i)}{c(R_i) + 1/\tau(R_i)}\right)^{-1},$$  

(6)

where $x_5 = \Gamma - \sum_{i=1}^{4} x_i$. Let $\eta_{TH}$ be the noise rise threshold of the rate control. If $\eta(x) \geq \eta_{TH}$, the RAB is set to 1. Otherwise, it is set to 0. Because $\eta(x)$ and the resulting RAB depend only on the current state, this system model can be considered as a first-order discrete Markov process.

The interesting characteristic of the rate control is that the rate transition is restricted to neighboring data rates. Let $x$ and $y = (y_1, y_2, y_3, y_4)$ be the samples of $S(t)$ and $S(t + 1)$, respectively. Then, the valid range of $y$ should satisfy the following inequalities:

$$0 \leq y_1 \leq x_1 + x_2$$  

(7)

and

$$\sum_{k=1}^{i-1} y_k \leq \sum_{k=1}^{i} x_k, \quad i = 2, 3, 4.$$  

(8)

If $y$ is not in the valid range, the state transition probability $p_{xy} \triangleq \Pr(S(t + 1) = y | S(t) = x)$ will be 0. To obtain the $p_{xy}$ for $y$ in the valid range, we assume that $U_i$ and $D_i$ mobiles increase and reduce their rates from a data rate of $R_i$, respectively, at the beginning of the $(t + 1)$-th frame. If we define $e_i \triangleq \sum_{j=1}^{i}(y_j - x_j)$, the $p_{xy}$ can be expressed as

$$p_{xy} = \Pr(D - U = e | S(t) = x),$$  

(9)

where $D = (D_2, D_3, D_4, D_5)$, $U = (U_1, U_2, U_3, U_4)$, and $e = (e_1, e_2, e_3, e_4)$. Let $N_i$ be the number of mobiles that decode the RAB as 0 at a data rate of $R_i$. By conditioning on $N_i$ and $U_i$, the $p_{xy}$ in (9) can be expressed as

$$p_{xy} = \sum_n \Pr(D = u + e | u, n, x) \times \Pr(U = u | n, x) \times \Pr(N = n | x),$$  

(10)

where $N = (N_1, N_2, N_3, N_4, N_5)$. Each probability in (10) is given as follows:

$$\Pr(D = u + e | u, n, x) = \prod_{i=1}^{4} \left(\frac{x_i + 1 - n_i + 1}{u_i + e_i}\right) q_{i+1}^{u_i+e_i} \times (1-q_{i+1})^{x_i+n_i-u_i-e_i}.$$  

(11)

$$\Pr(U = u | n, x) = \prod_{i=1}^{n} \left(\frac{n_i}{u_i}\right) p_{i+1}^{u_i} (1-p_i)^{n_i-u_i}.$$  

(12)

$$\Pr(N = n | x) = \prod_{i=1}^{5} \left(\frac{x_i}{n_i}\right)^{n_i} (1-z)^{x_i-n_i}, \quad x \in \{x_o\},$$  

$$\prod_{i=1}^{5} \left(\frac{x_i}{n_i}\right)^{n_i} (1-z)^{x_i-n_i}, \quad x \in \{x_o\}.$$  

(13)

In (13), $\{x_o\}$ is a set of $x$ that satisfies $\eta(x) < \eta_{TH}$, and $\{x_e\}$ is a set of $x$ that satisfies $\eta(x) \geq \eta_{TH}$. The ranges of $n$ and $u$ are $\{0 \leq n_i \leq x_i, i = 1, 2, 3, 4\}$ and $\{u \max(0, -e_i) \leq u_i \leq \min(n_i, x_i+1-n_i+1-e_i), i = 1, 2, 3, 4\}$, respectively.

Because the Markov model is irreducible, aperiodic, and positive recurrent, there will be a unique steady-state probability $\pi_x$ for a state $x$. $\{\pi_x\}$ can be obtained by solving a set of linear equations $\pi_y = \sum_{x} \pi_x p_{xy} \pi_x$ and $\sum_{x} \pi_x = 1$ [6]. Using the obtained steady-state probabilities, various performance metrics can be derived.
IV. RESULTS AND DISCUSSIONS

We define two performance metrics for the IS-856 reverse link rate control: 1) Reverse link throughput, and 2) Outage probability. The reverse link throughput indicates the total average data rate on the reverse link. Therefore,

\[
\text{Reverse link throughput} = \sum_{\forall x} \pi_x \left( \sum_{i=1}^{5} R_i x_i \right). \quad (14)
\]

The outage probability is defined as the probability that a mobile cannot satisfy the required $\frac{E_c}{N_0}$ even at the maximum transmission power. Therefore, if \[\sum_{i=1}^{5} x_i c(R_i)/(c(R_i) + 1/\tau(R_i)) \geq 1\] in (6), the outage situation occurs because the corresponding noise rise goes to infinity. Letting $\{x_{\text{ outage}}\}$ be a set of $x$ that causes the outage situation, the outage probability is given by

\[
\text{Outage probability} = \sum_{x \in \{x_{\text{ outage}}\}} \pi_x. \quad (15)
\]

In this analysis, $G_{DRC}$ is set to -1.5 dB assuming DRC Length of 2 slots [3]. $\tau(\cdot)$ should be determined by a link level simulation that considers various wireless conditions. A value of $\tau(\cdot)$ between -25 and -22 dB achieves 1% packet error rate (PER) in a single-cell environment with Rayleigh fading channels [2]. We set $\tau(\cdot)$ to -22 dB for all data rates to guarantee a PER of less than 1% in a conservative manner. In addition, $\tau = 5\%$ is assumed for all mobiles. In the following sub-sections, we examine the effect of the noise rise threshold ($\eta_{TH}$) and persistence probabilities ($p_i$ and $q_i$) on the performance of IS-856 reverse link rate control.

A. Effect of the noise rise threshold ($\eta_{TH}$)

We fix the persistence probability as $(p_1, p_2, p_3, p_4) = (0.2, 0.1, 0.05, 0.025)$ and $(q_2, q_3, q_4, q_5) = (0.1, 0.2, 0.4, 0.8)$ for all mobiles. Fig. 2 illustrates the reverse link throughput and the outage probability for various values of $\eta_{TH}$. In this figure, both the reverse link throughput and the outage probability increases as $\eta_{TH}$ increases. Therefore, it is not a good choice to increase $\eta_{TH}$ continually in order to improve the reverse link throughput because the resulting outage probability increases as well. There is a trade-off between the reverse link throughput and the outage probability.

In Fig. 2(b), it is observed that the outage probability achieves a maximum value and then decreases as the number of mobiles increases. This decrease is due to non-proportional data channel gains on the reverse link. As the number of mobiles increases, the average data rate of each mobile decreases according to the rate control. Therefore, the proportion of mobiles having a low data rate increases, whereas the proportion having a high data rate decreases. However, the data channel gain is not entirely proportional to the data rate (see Table I). For example, one mobile with a data rate of $R_5$ causes a higher noise rise than do two mobiles with a data rate of $R_4$, because $G_{\text{data}}(R_5)$ is 5.25 dB (not 3 dB !) higher than $G_{\text{data}}(R_4)$.

Generally, $p_1 > p_2 > p_3 > p_4$ and $q_2 < q_3 < q_4 < q_5$ are selected for stable data rate control.

\begin{table}[h]
\centering
\caption{Samples of Persistence Probabilities}
\begin{tabular}{|c|c|c|}
\hline
 & $(p_1, p_2, p_3, p_4)$ & $(q_2, q_3, q_4, q_5)$ \\
\hline
Set A & (0.4, 0.2, 0.1, 0.05) & (0.05, 0.1, 0.2, 0.4) \\
Set B & (0.2, 0.1, 0.05, 0.025) & (0.05, 0.1, 0.2, 0.4) \\
Set C & (0.2, 0.1, 0.05, 0.025) & (0.1, 0.2, 0.4, 0.8) \\
Set D & (0.1, 0.05, 0.025, 0.0125) & (0.05, 0.1, 0.2, 0.4) \\
Set E & (0.05, 0.025, 0.0125, 0.00625) & (0.025, 0.05, 0.1, 0.2) \\
\hline
\end{tabular}
\end{table}

B. Effect of persistence probabilities ($p_i$ and $q_i$)

We fix $\eta_{TH}$ to 6.8 dB. To illustrate the effect of persistence probabilities, five different sets of persistence probabilities are selected as shown in Table II. Fig. 3 shows the reverse link throughput and the outage probability when $\{p_i\}$ and $\{q_i\}$ are adjusted separately (Set A, B, and C). Each $p_i$ in Set B is lower than that in Set A, and each $q_i$ in Set C is higher than that in Set B. From this figure, we can see that a higher value of $\{p_i\}$ produces a higher throughput and a higher outage probability. The reason is that the mobile can increase its data rate more easily. Similar explanation can be applied to the effect of $\{q_i\}$. Since the mobile can reduce its data rate more easily with a higher value of $\{q_i\}$, a lower throughput and a lower outage probability are produced.
V. CONCLUSIONS

In this paper, the performance of IS-856 reverse link rate control has been analyzed. By assuming perfect power control, we have derived an expression for the noise rise, which is used as the traffic load on the reverse link in the rate control. Then, we have modeled the rate control as a first-order discrete Markov process. After obtaining steady-state probabilities of the system model, the reverse link throughput and the outage probability have been derived as two performance metrics. From the results, we can see that there is a trade-off between the reverse link throughput and the outage probability, and the system performance is highly dependent on the noise rise threshold ($\eta_{TH}$) and the persistence probabilities ($p_i$ and $q_i$). Therefore, we can conclude that the system operator of IS-856 should configure the system parameters very carefully for reasonable operation.

REFERENCES