Generalized analytic formulae for magneto-optical Kerr effects

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We have developed simplified analytic expressions for magneto-optical Kerr effects of both optically thick and ultrathin films in the general case, where a magnetic medium had an arbitrary direction of magnetization and a beam of light was obliquely incident to the medium. It was found that the simplified analytic formulae for the Kerr effects of \( p \) and \( s \) waves consisted of a product of two factors for both optically thick and ultrathin films: the prefactor dependent only on the optical parameters of the system and the main factor of the polar Kerr effect for a normal incidence case. We have also derived some useful relations among the Kerr effects in the polar and longitudinal configurations. We have demonstrated that the theoretical calculations using the present analytic formulae could well match the experimental polar and longitudinal Kerr rotation angles of magnetic films measured with varying incident angles. © 1998 American Institute of Physics.

I. INTRODUCTION

Recently, enormous studies on magnetic thin films have been carried out because of their many novel physical properties and diverse technological applications.\(^{1-4}\) In investigating these materials, the magneto-optical Kerr effect (MOKE) has been widely utilized as a tool in probing magnetism as well as a readout mechanism in magneto-optical recording, because it is very sensitive to magnetization. For instance, Bader et al. have used the MOKE for in situ investigation of magnetism in ultrathin Fe films.\(^5\) The MOKE, fundamentally related to the spin-polarized electronic band structure, is manifested itself by the change of polarization and/or intensity of incident polarized light when it is reflected from the surface of a magnetized medium.

The quantitative analysis of the MOKE for the special case, where the incident angle of light is normal and the direction of the magnetization is in plane or perpendicular to the film plane, has been extensively studied by many researchers.\(^6-10\) However, most experimental situations including in situ measurements of the MOKE in a vacuum chamber are the cases where the direction of the magnetization is arbitrary and the direction of the incident beam is not normal to the film plane. For this general case only a few studies have been reported, but the quantitative interpretation is not so simple because of the very complicated relations.\(^11-13\) Quite recently,\(^14\) we have reported simplified analytic formulae for the MOKEs of optically thick magnetic films, which could be greatly useful for quantitative analyses of the MOKEs. But, the formulae have some limitations for application to ultrathin magnetic films, in which most novel properties are observed.

In this paper, we have extended our work to develop simplified analytic formulae for the MOKEs of ultrathin magnetic films considering multiple reflections. This paper is organized as follows: In Sec. II, we describe the magneto-optical Fresnel reflection coefficients. In Sec. III, we derive simplified analytic formulae of the MOKEs for the general case and present some useful relations between \( p \) and \( s \) waves. For the completeness of this paper we include optically thick films as well as ultrathin films. The theoretical calculations with our formulae are compared with the experimental data in Sec. IV, and finally, some conclusions are drawn in Sec. V.

II. MAGNETO-OPTICAL FRESNEL REFLECTION COEFFICIENTS

As depicted in Fig. 1, there are various kinds of the MOKEs depending on the relative direction of the magnetization to the plane of incidence. Depending on the direction of the magnetization whether it is parallel to the surface normal, parallel to the surface and in the plane of incidence, or parallel to the surface and perpendicular to the plane of incidence, it is called the polar Kerr effect, the longitudinal Kerr effect, and the transverse (or equatorial) Kerr effect, respectively.\(^7\)

We first consider optically thick magnetic films where the multiple reflections could be ignored. When a beam of light is incident from a nonmagnetic medium 0 to a magnetic medium 1, having an arbitrary direction of magnetization as shown in Fig. 2, the dielectric tensor \( \epsilon \) can be generalized using Euler’s angle as follows:\(^{11,13,14}\)

\[
\epsilon = \epsilon_{xx} \begin{pmatrix}
1 & -iQm_z & iQm_y \\
 iQm_z & 1 & -iQm_x \\
- iQm_y & iQm_x & 1
\end{pmatrix}.
\]  

(1)
We assume $\varepsilon_{zz} = \varepsilon_{xx}$ for simplicity. The magneto-optical constant $Q$ is defined as

$$Q = i \frac{\varepsilon_{xy}}{\varepsilon_{xx}}.$$  \hspace{1cm} (2)

For generality, we treat all physical quantities as complex numbers. Here, we follow the same sign convention proposed by Atkinson and Lissberger\textsuperscript{15} for $Q$. It is the same as that proposed by Hunt\textsuperscript{11} and Yang and Scheinfein\textsuperscript{13} but opposite to that of Zak \textit{et al.}\textsuperscript{12} The magnetic permeability is considered to be equal to 1 in our treatment, since we are interested in the optical wavelength region.\textsuperscript{16}

In Eq. (1), $m_x$, $m_y$, and $m_z$ are the direction cosines of the magnetization vector $\mathbf{M}_s$. Solving Maxwell equations for the above dielectric tensor, magneto-optical Fresnel reflection matrix can be given as follows:

$$\mathbf{R} = \begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix},$$  \hspace{1cm} (3)

where $r_{ji}$, the ratio of the incident $j$ polarized electric field and reflected $i$ polarized electric field, is expressed by\textsuperscript{11–13}

$$r_{pp} = \frac{n_1 \cos \theta_0 - n_0 \cos \theta_1}{n_1 \cos \theta_0 + n_0 \cos \theta_1} + i \frac{2n_0 n_1 \cos \theta_0 \sin \theta_1 m_x Q}{n_1 \cos \theta_0 + n_0 \cos \theta_1},$$  \hspace{1cm} (4)

$$r_{sp} = \frac{-i n_0 n_1 \cos \theta_0 (m_z \cos \theta_1 + m_x \sin \theta_1) Q}{(n_1 \cos \theta_0 + n_0 \cos \theta_1)(n_0 \cos \theta_0 + n_1 \cos \theta_1) \cos \theta_1},$$  \hspace{1cm} (5)

$$r_{ss} = \frac{n_0 \cos \theta_0 - n_1 \cos \theta_1}{n_0 \cos \theta_0 + n_1 \cos \theta_1},$$  \hspace{1cm} (6)

$$r_{ps} = \frac{-i n_0 n_1 \cos \theta_0 (m_z \cos \theta_1 - m_x \sin \theta_1) Q}{(n_1 \cos \theta_0 + n_0 \cos \theta_1)(n_0 \cos \theta_0 + n_1 \cos \theta_1) \cos \theta_1}.$$  \hspace{1cm} (7)

In the above expressions, $\theta_0$, $n_0$, and $n_1$ are the angle of incidence, the refractive index of the nonmagnetic medium 0, and that of the magnetic medium 1, respectively. They are valid within the first-order approximation of the magneto-optical constant $Q$. In the above equations, the complex refractive angle $\theta_1$ in the magnetic medium 1 is determined by Snell’s law. The differences in the signs of $r_{sp}$ and $r_{ps}$ compared to those of Yang and Scheinfein\textsuperscript{13} are ascribed to the sign conventions as mentioned before and the different definitions of Euler’s angles.

Now, we consider ultrathin magnetic films where the multiple reflections should be taken into account. As depicted in Fig. 3, a beam of light passes from a nonmagnetic medium 0 to another nonmagnetic medium 2 through a magnetic medium 1 having an arbitrary direction of magnetization and thickness $d_1$. We have used the medium boundary matrices and medium propagation matrices\textsuperscript{12,17} to treat the multiple reflections. The medium boundary matrix $A_j$ for the $j$th layer having arbitrary direction of the magnetization can be expressed by

FIG. 1. Schematic configurations for the polar, longitudinal, and transverse (or equatorial) magneto-optical Kerr effects. The definition of the coordinate system is also shown.

FIG. 2. The coordinate system of the nonmagnetic medium 0 and the magnetic medium 1. The direction of the magnetization of medium 1 is arbitrary.

FIG. 3. The coordinate system of the nonmagnetic medium 0, the magnetic medium 1, and the nonmagnetic medium 2. The thickness of medium 1 is $d_1$. The magnetization direction of the medium 1 is arbitrary.
respectively. Here, $\alpha_j = \sin \theta_j$ and $\alpha_j = \cos \theta_j$. Here, the complex refractive angle $\theta_j$ is determined by Snell’s law. While, the medium propagation matrix $D_j$ for the $j$th layer can be expressed by

$$D_j = \begin{pmatrix}
U \cos \delta' & U \sin \delta' & 0 & 0 \\
-U \sin \delta' & U \cos \delta' & 0 & 0 \\
0 & 0 & U^{-1} \cos \delta' & U^{-1} \sin \delta' \\
0 & 0 & -U^{-1} \sin \delta' & U^{-1} \cos \delta'
\end{pmatrix},$$

where $U$, $\delta'$, $\delta''$, $g''$, and $g''$ are defined by

$$U = \exp \left( -i \frac{2 \pi}{\lambda} n_j \alpha_j d_j \right),$$

$$\delta' = -\frac{\pi n_j Q d_j g''}{\lambda \alpha_j},$$

$$\delta'' = -\frac{\pi n_j Q d_j g''}{\lambda \alpha_j},$$

$$g'' = m \alpha_j + m \alpha_j,$$

$$g'' = m \alpha_j - m \alpha_j,$$

respectively. Here, $d_j$ denotes the thickness of the $j$th layer. To obtain the magneto-optical Fresnel reflection coefficients, one should compute the matrix $M$ defined by

$$M = A_0^{-1} A_1 D_1 A_1^{-1} A_2.$$

The $4 \times 4$ matrix $M$ can be expressed in the form of $2 \times 2$ block matrices as follows:

$$G = \frac{1}{2} \begin{pmatrix} n_s \cos \theta_2 + i d \pi(n_0 n_s \cos \theta_0 \cos \theta_2 + n_1^2 \cos^2 \theta_1) & -n_1 d_1 \pi Q(m n_1 \cos \theta_2 + m n_s \sin \theta_1) \\
n_1 d_1 \pi Q(m n_1 \cos \theta_2 + m n_s \sin \theta_1) & \frac{n_s \cos \theta_2 - i d \pi(n_0 n_s \cos^2 \theta_1 + n_1^2 \cos \theta_0 \cos \theta_2)}{\lambda n_0 \cos \theta_0} \end{pmatrix},$$

$$I = \frac{1}{2} \begin{pmatrix} n_s \cos \theta_2 - i d \pi(n_0 n_s \cos \theta_0 \cos \theta_2 - n_1^2 \cos \theta_1) & n_1 d_1 \pi Q(m n_1 \cos \theta_2 + m n_s \sin \theta_1) \\
n_1 d_1 \pi Q(m n_1 \cos \theta_2 + m n_s \sin \theta_1) & \frac{n_s \cos \theta_2 + i d \pi(n_0 n_s \cos \theta_1 + n_1^2 \cos \theta_0 \cos \theta_2)}{\lambda n_0 \cos \theta_0} \end{pmatrix}. $$

In principle, one can obtain the analytic expressions for the magneto-optical Kerr effects from Eq. (17), even though these expressions are too complicated to provide any physical insight.
Here, \( n_0, n_1, n_s, \theta_0, \theta_1, \) and \( \theta_2 \) are the refractive indices and complex refractive angles of the nonmagnetic medium 0, the magnetic medium 1, and the nonmagnetic medium 2, respectively. Using Eqs. (17)–(19), one can obtain the expressions for \( r_{ij} \) as follows:

\[
r_{pp} = \frac{n_s \cos \theta_0 - n_0 \cos \theta_2}{n_s \cos \theta_0 + n_0 \cos \theta_2} + \frac{4 \pi i n_0 d_1 \cos \theta_0 (n_s^2 \cos^2 \theta_1 - n_1^2 \cos^2 \theta_2)}{\lambda (n_0 \cos \theta_2 + n_s \cos \theta_0)} + \frac{4 \pi n_0 n_1 Q d_1 \cos \theta_0 (n_s^2 \cos^2 \theta_1 - n_1^2 \cos^2 \theta_2)}{\lambda (n_0 \cos \theta_2 + n_s \cos \theta_0)^2},
\]

(20)

\[
r_{sp} = \frac{n_0 \cos \theta_0 - n_s \cos \theta_2}{n_0 \cos \theta_0 + n_s \cos \theta_2} + \frac{4 \pi i n_0 d_1 \cos \theta_0 (n_s^2 \cos^2 \theta_1 - n_1^2 \cos^2 \theta_2)}{\lambda (n_0 \cos \theta_2 + n_s \cos \theta_0)} + \frac{4 \pi n_0 n_1 Q d_1 \cos \theta_0 (m_s n_1 \cos \theta_2 - n_s \sin \theta_1)}{\lambda (n_0 \cos \theta_2 + n_s \cos \theta_0)},
\]

(21)

III. SIMPLIFIED FORMULAE FOR MOKES

We have thus expressed the magneto-optical Fresnel coefficients in the general case for the thick magnetic medium with Eqs. (4)–(7) and for the ultrathin magnetic medium with Eqs. (20)–(23). We define the complex Kerr effects as follows:

\[
\theta_{K}^{\text{pol.}} = \frac{r_{sp}}{r_{pp}},
\]

(24)

\[
\theta_{K}^{\text{pol.}} = \frac{r_{ps}}{r_{ss}}.
\]

(25)

Using these definitions, we first consider the simple cases of the polar and longitudinal configurations and then extend to the general case for both optically thick and ultrathin films.

A. Optically thick magnetic films

In the polar configuration, \( m_z = 1 \) and \( m_s = m_s = 0 \). For the \( p \)-polarized wave in this configuration, putting Eqs. (4) and (5) in Eq. (24), the following relation is easily obtained:

\[
\frac{r_{sp}^{\text{pol.}}}{r_{pp}^{\text{pol.}}} = \frac{i n_0 n_1 \cos \theta_0}{(n_0 \cos \theta_0 + n_1 \cos \theta_1)(n_1 \cos \theta_0 - n_0 \cos \theta_2)}.
\]

(26)

Then, \( \theta_{K}^{\text{pol.}} \) can be simplified as follows:

\[
\theta_{K}^{\text{pol.}} = \frac{r_{sp}^{\text{pol.}}}{r_{pp}^{\text{pol.}}} = \frac{\cos \theta_0}{\cos \theta_0 + \theta_1} \cdot \frac{i n_0 n_1 Q}{(n_2^2 - n_0^2)}.
\]

(27)

Details of the derivation have been reported elsewhere.\(^{14}\) In this expression, the second factor, \( i n_0 n_1 Q / (n_2^2 - n_0^2) \), is the well-known polar Kerr effect for normal incidence.\(^{3}\) For the oblique incident \( p \)-polarized wave, the Kerr effect, therefore, can be described by a product of two factors. The prefactor \( \cos \theta_0 / \cos (\theta_0 + \theta_1) \) is a simple function of the incident angle and the refractive angle determined by the refractive indices of the media, and the main factor contains information about the magneto-optical properties of medium 1.

One can get the following similar expression for the \( s \)-polarized wave from Eqs. (6) and (7):

\[
\theta_{K}^{s} = \frac{r_{ps}^{\text{pol.}}}{r_{ss}^{\text{pol.}}} = \frac{-\cos \theta_0}{\cos \theta_0 + \theta_1} \cdot \frac{i n_0 n_1 Q}{(n_1^2 - n_0^2)}.
\]

(28)

The only difference between Eqs. (27) and (28) is the sign of the argument of the cosine function in the denominator of the prefactor.

In the longitudinal configuration, \( m_z = 1 \) and \( m_s = m_s = 0 \), by similar mathematical treatment of Eqs. (4)–(7) as the polar configuration, the complex Kerr effects for the longitudinal configuration can be expressed by

\[
\theta_{K}^{\text{long.}} = \frac{r_{ps}^{\text{long.}}}{r_{pp}^{\text{long.}}} = \frac{\cos \theta_0 \tan \theta_1}{\cos \theta_0 + \theta_1} \cdot \frac{i n_0 n_1 Q}{(n_1^2 - n_0^2)},
\]

(29)

\[
\theta_{K}^{\text{long.}} = \frac{r_{ps}^{\text{long.}}}{r_{ss}^{\text{long.}}} = \frac{\cos \theta_0 \tan \theta_1}{\cos \theta_0 - \theta_2} \cdot \frac{i n_0 n_1 Q}{(n_1^2 - n_0^2)}.
\]

(30)

The expressions for the longitudinal Kerr effects are also similar to those of the polar Kerr effects and can be split into two factors. Then, the Kerr effects in the general case of the arbitrary magnetization direction and oblique incidence can be expressed from Eqs. (27), (28), (29), and (30) as follows:\(^{14}\)

\[
\theta_{K}^{\text{pol.}} = \frac{r_{sp}^{\text{pol.}}}{r_{pp}^{\text{pol.}}} = \frac{\cos \theta_0 (m_s \tan \theta_1 + m_z)}{\cos \theta_0 + \theta_1} \cdot \frac{i n_0 n_1 Q}{(n_1^2 - n_0^2)},
\]

(31)

\[
\theta_{K}^{\text{pol.}} = \frac{r_{ps}^{\text{pol.}}}{r_{ss}^{\text{pol.}}} = \frac{\cos \theta_0 (m_s \tan \theta_1 - m_z)}{\cos \theta_0 - \theta_2} \cdot \frac{i n_0 n_1 Q}{(n_1^2 - n_0^2)}.
\]

(32)

B. Ultrathin magnetic films

For the \( p \)-polarized wave in the polar configuration, where \( m_z = 1 \) and \( m_s = m_s = 0 \), the following relation could be obtained in the first-order approximation of \( 2 \pi n_1 d_1 / \lambda \) by substituting Eqs. (20) and (21) in Eq. (24):

\[
\frac{r_{sp}^{\text{pol.}}}{r_{pp}^{\text{pol.}}} = \frac{4 \pi n_0 n_1 Q d_1 \cos \theta_0 \mu_1 \cos \theta_2}{\lambda (n_0 \cos \theta_0 + n_s \cos \theta_2)(n_1 \cos \theta_0 - n_0 \cos \theta_2)}.
\]

(33)

In the above expression the denominator is similar to that in Eq. (26), except the fact that \( \lambda \) appears and \( n_1 \) is replaced by \( n_s \). Therefore, after a similar procedure, the denominator of Eq. (33) can be simplified as follows:

\[
\lambda (n_0 \cos \theta_0 + n_s \cos \theta_2)(n_1 \cos \theta_0 - n_0 \cos \theta_2) = \lambda (n_0^2 - n_0^2) \cos (\theta_0 + \theta_2).
\]

(34)

Substituting Eq. (34) into Eq. (33), \( \theta_{K}^{\text{pol.}} \) is given by

\[
\frac{\theta_{K}^{\text{pol.}}}{r_{pp}^{\text{pol.}}} = \frac{\cos \theta_0}{\cos \theta_0 + \theta_1} \cdot \frac{i n_0 n_1 Q}{(n_1^2 - n_0^2)}
\]

(35)
where $\Theta_n$ is the complex polar Kerr effect for normal incidence in the ultrathin film limit expressed by17–19
\[ \Theta_n = \frac{4\pi n_0 n_1^2 Q d}{\lambda (n_x^2 - n_y^2)}. \]  
(36)

For the $s$-wave case, using Eqs. (22) and (23),
\[ \Theta_{s}^{\text{pol}} = \frac{r_{ps}}{r_{ss}} \cos \theta_0 n_1 \cos \theta_2 = \frac{4\pi n_0 n_1 Q d_1 \cos \theta_0 n_1 \cos \theta_2}{\lambda (n_0 \cos \theta_2 + n_z \cos \theta_2)(n_0 \cos \theta_0 - n_z \cos \theta_2)} \]  
(37)

One can simplify Eqs. (39) and (40) as follows:
\[ \Theta_{k}^{\text{pol}} = \frac{\cos \theta_0}{\cos(\theta_0 + \theta_2)} \sin^2 \theta_1 \Theta_n, \]  
(41)
\[ \Theta_{k}^{\text{long}} = \frac{\cos \theta_0}{\cos(\theta_0 - \theta_2)} \sin^2 \theta_1 \Theta_n, \]  
(42)

It is worthwhile to note that the expressions for the magneto-optical Kerr effects of ultrathin magnetic films are also expressed as a product of two factors similar to those for optically thick films. The prefactor is a simple function of the optical parameters. The main factor $\Theta_n$ is the complex polar Kerr effect for normal incidence in the ultrathin film limit. And the first part, $\cos \theta_0/\cos(\theta_0 \pm \theta_2)$, is similar to the first part of the simplified expression for the optically thick film, except that $\theta_2$ in the argument of the cosine in the denominator for the optically thick film is replaced by $\theta_2$ of medium 2 in the ultrathin film limit, since the optical properties of the system are governed by those of the substrate for the ultrathin magnetic medium.

Since there is no contribution of the perpendicular component of the magnetization to the plane of incidence within the first order of $Q$, the contribution of the arbitrary magnetization comes from the component parallel to the plane of incidence. Therefore, one can ignore $m_z$ and consider only $m_x$ and $m_y$. Then, the Kerr effects in the general case of the arbitrary magnetization direction and oblique incidence can be expressed from Eqs. (35), (38), (41), and (42) for the ultrathin limit case as follows:
\[ \Theta_{k}^{\text{pol}} = \frac{\cos \theta_0}{\cos(\theta_0 + \theta_2)} \left( m_x \frac{\sin^2 \theta_1}{\sin \theta_2} + m_z \cos \theta_2 \right) \Theta_n, \]  
(43)
\[ \Theta_{k}^{\text{long}} = \frac{\cos \theta_0}{\cos(\theta_0 - \theta_2)} \left( m_x \frac{\sin^2 \theta_1}{\sin \theta_2} - m_z \cos \theta_2 \right) \Theta_n. \]  
(44)

Similar to the $p$-wave case,
\[ \Theta_{k}^{\text{pol}} = \frac{-\cos \theta_0}{\cos(\theta_0 - \theta_2)} \cos \theta_2 \Theta_n. \]  
(38)

It is interesting to point out that the difference in Eqs. (35) and (38) is just the sign of the argument of the cosine function in the denominator of the prefactor, exactly the same as in the optically thick films. In comparison with the optically thick films, $\theta_1$ is replaced by $\theta_2$ in the ultrathin films.

The complex Kerr effects of ultrathin magnetic films for the longitudinal configuration, where $m_y = 1$ and $m_z = m_x = 0$, can be expressed by

\[ \Theta_{k}^{\text{pol}} = \frac{-\cos \theta_0}{\cos(\theta_0 - \theta_2)} \cos \theta_2 \Theta_n. \]  
(39)
\[ \Theta_{k}^{\text{long}} = \frac{-\cos \theta_0}{\cos(\theta_0 - \theta_2)} \cos \theta_2 \Theta_n. \]  
(40)

C. Some useful relations

Here, we summarize some useful relations which might be helpful to reduce the number of measurements and/or confirm the experimental data. From Eqs. (4)–(7) and Eqs. (20)–(23), it can be noted that the magneto-optical Fresnel reflection coefficients $r_{sp}$ and $r_{ps}$ are related as follows: $r_{sp} = r_{ps}$ for the polar configuration and $r_{sp} = -r_{ps}$ for the longitudinal configuration.

From Eqs. (27), (28), (29), and (30) for the optically thick case, the relation of the Kerr effects between the $p$ and $s$ waves is given by
\[ \Theta_{k}^{\text{pol}} = \frac{\cos(\theta_0 - \theta_1)}{\cos(\theta_0 + \theta_1)}. \]  
(45)

Here, the negative and positive signs correspond to the polar and longitudinal configurations, respectively. For the ultrathin limit case, $\theta_1$ will be replaced by $\theta_2$ as can easily be seen from Eqs. (43) and (44). The simple relations between the polar and longitudinal Kerr effects are easily found to exist as
\[ \Theta_{k}^{\text{pol}} = \pm \tan \theta_1, \]  
(46)
for the optically thick case from Eqs. (31) and (32), and
\[ \Theta_{k}^{\text{pol}} = \pm \sin^2 \theta_1, \]  
(47)
for ultrathin limit case from Eqs. (43) and (44). Here, the positive and negative signs correspond to the $p$ and $s$ waves, respectively.
IV. COMPARISON TO THE EXPERIMENTAL DATA

The present simplified analytic formulae have been applied to fit the published experimental data on Co/Pd and Cu/Co multilayers by Deeter and Sarid,20 where they have measured the polar Kerr rotation angles of (1.8-Å Co/9-Å Pd)200 multilayers having perpendicular magnetic anisotropy and the longitudinal Kerr rotation angles of (50-Å Cu/55.8-Å Co)10 multilayers having in-plane magnetic anisotropy. The polar and longitudinal Kerr rotation angles of the p- and s-polarized waves were reported at wide incident angles ranging from 5° to 85° with an increment of 5°. The reflectivities for the p- and s-polarized waves were also reported at various incident angles. The complex refractive indices $n_1$ and the magneto-optical constants $Q$ of the samples at the wavelength of 6328 Å, determined using the least-square fitting method, were $n_1 = 1.58 + 3.58i$ and $Q = 0.0177 - 0.0063i$ for the Cu/Co multilayer, and $n_1 = 2.04 + 4.06i$ and $Q = 0.0038 - 0.00314i$ for the Co/Pd multilayer. These values were used in Eqs. (27), (28), (29), and (30) to calculate various Kerr rotation angles. The calculated results, together with the experimental data, are shown in Fig. 4. The open circles and rectangles represent the experimental results of Deeter, the solid and dashed lines represent the theoretical results obtained using Eqs. (27)–(30). As seen in Fig. 4, the experimental data are well explained by the present simplified formulae.

V. CONCLUSIONS

We have derived simplified analytic formulae for various MOKEs of optically thick and ultrathin magnetic films. Considering the multiple reflections in the ultrathin magnetic film, it was found that they could be described as a product of two factors, similar to those of the optically thick magnetic film case: The prefactor is a function of the optical parameters of the system and the main factor is the well-known complex polar Kerr effect for normal incidence for each case. And, we have shown that some useful relations existed among the various kinds of MOKEs. The validity of the simplified formulae was verified by fitting the experimental data of Cu/Co and Co/Pd multilayers with the present formulae. The derived simplified formulae and the relations between the various kinds of MOKEs will be useful in studying the MOKEs and the other magnetic properties of films having arbitrary magnetization directions.

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