

# Parameter study on the quasi-statically axial crush of frusta with small semi-apical angles using finite element method

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**Abstract:** This paper deals with collapse modes and energy absorption responses of thin-walled frusta under quasi-statically axial compression using finite element method. Experiments of cylindrical tubes are carried out. The results of numerical programs match well with that of experiments. The numerical analysis is designed using design of experiments (DOE). Various sets of numerical simulations are carried out using ABAQUS/Standard. The different energy absorbing characteristics are approximated using response surface methodology (RSM). To investigate the influences of flow stress of the material on the deformation mode, specific energy absorption, the maximum load, total efficiency and effective crush distance, numerical simulations were carried out using two different materials, SPRC340S and TRIP80.

**Key words:** Axial compression, Thin-walled frusta, DOE, RSM, semi-apical angles

## Nomenclature

$h$  : thickness of the frusta  
 $D_t$  : diameter of the top end  
 $D_b$  : diameter of the bottom end  
 $D$  : mean diameter of the frusta,  $D = (D_t + D_b)/2$   
 $L$  : length of the frusta  
 $d$  : actual crush distance  
 $d_{\max}$  : maximum stroke length  
 $\alpha$  : semi-apical angle  
 $E$  : absorbed energy  
 $M$  : mass of the tube  
 $F_{\max}$  : maximum force  
 $F_{\text{avg}}$  : average force  
 $A_e$  : crush force efficiency,  $A_e = F_{\text{avg}}/F_{\max}$   
 $D_c$  : deformation capability  
 $S_e$  : stroke efficiency,  $S_e = D_c(d_{\max})$   
 $T_e$  : total efficiency  
 $SEA$  : specific energy absorption

## 1. INTRODUCTION

Energy absorbers are devices that convert kinetic energy to other forms of energy, such as plastic deformation energy in deformable solids. The process of conversion for plastic deformation depends mainly on the magnitude and method of application of loads, transmission rates, deformation displacement patterns and material properties such as the flow stress of the material.

Thin-walled structures such as circular tubes,<sup>1-3)</sup> square tubes,<sup>4-5)</sup> and frusta<sup>6)</sup> are commonly used as energy absorbing elements in crashworthiness application. As a relatively new type of energy absorber, the thin-walled frusta commonly known as frusta have been considered preferable to straight or cylindrical tubes since they are more likely to provide a desirable constant mean crush load-deflection response under axial loading.<sup>6)</sup> Also, they are capable of withstanding oblique impact loads as effectively as axial loads, making them suitable for applications in which the direction of the impact load varies.

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Furthermore, tapered tubes are less likely to fail by global buckling, which is an undesirable deformation mode since it reduces the energy absorption capacity significantly.

Postlethwaite and Mills performed the axial crushing tests on frusta of uniform wall thickness and semi-apical angles of 5-20° and studied their energy absorption capacity.<sup>7)</sup> Mamalis and Johnson performed experiments to study the crumbling of aluminum cylinders and frusta under quasi-static axial loading.<sup>8)</sup> They distinguished the collapse modes into concertina and diamond modes and also developed empirical relationships for both of these two deformation modes. Mamalis *et al.* elevated the compressing velocity to investigate the effect of strain rate and classified the deformation modes of frusta as concertina, concertina-diamond, and diamond.<sup>9)</sup> Mamalis *et al.*<sup>10)</sup> modified the model of Postlethwaite and Mills<sup>7)</sup> and validated the new model experimentally. Gupta and Abbas developed a mathematical model for the axisymmetric axial crushing of thin frusta which considered the variation of circumferential strain and found that the new model matched well with experiments.<sup>11)</sup>

Since the semi-apical angles of frusta studied by Postlethwaite, Mamalis Gupta *et al.* are no more than 20°, Gupta N.K. conducted experiments wherein aluminum conical frusta of semi-apical angles varying over a wide range from 16.5° to 65° and different slenderness ratios and proposed an analytical model for prediction of load-deformation and energy-compression curves.<sup>12)</sup> Gupta P. K. studied the collapse mode of varying wall thickness metallic frusta experimentally and numerically and the mechanism of the collapse is discussed on the basis of the variations of the computed strains and stresses.<sup>13)</sup>

In the previous studies, whether the semi-apical angle is small or large, the density of the semi-apical angle is low, such as 5°, 10° and 15°. Since the sparseness of the selected semi-apical angles, the studies before may not reflect the detailed influence of semi-apical angles on the deformation behavior of frusta.

This paper deals with the numerical analysis of the energy absorption characteristics of frusta under axial compression. A finite element model of axisymmetric collapse model, using ABAQUS/Standard, was developed and validated. A series of axial crush simulations of frusta

with semi-apical angles from 6° to 10°, thickness between 1mm and 3mm and length from 80mm to 120mm were designed using D-optimal design of design of experiments. The modes of deformation and the associated energy absorbing parameters, such as maximum load, specific energy absorption, total efficiency and effective crush distance, were studied. Two different materials SPRC340S and TRIP80 were selected in the study to investigate the influence of flow stress on the deformation modes and energy absorption capacity.

## 2. Finite element modeling

### 2.1 Description of the finite element model

A model of the conical frusta was developed using the FE code ABAQUS/standard version 6.5. According to Andrews *et al.*<sup>1)</sup> and Guillow *et al.*<sup>2)</sup>, the ratio of  $h/D_i$  is constraint between 0.01 and 0.03 and the ratio of  $L/D_i$  between 0.8 and 1.2 in order to make sure that the deformation modes are axisymmetric. Also due to the axisymmetry of the geometry and loading as well as the deformation, one kind of axisymmetric, 4-node bilinear, incompatible reduced-integration elements, CAX4I is used in the model. Since each frustum deformed via the desired symmetric collapse mode, no initial imperfections were required to perturb the mesh.

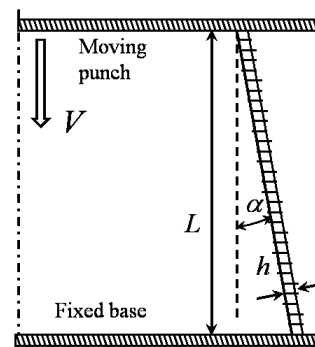


Fig. 1 Geometry and loading arrangement used in the FE model

Fig. 1 shows the geometry and loading arrangement used in the FE model. The mean diameter of the top end of frusta was 100mm, while the length of each tube varied from 80mm to 120mm. The semi-apical angle  $\alpha$  was obtained by varying the diameter of the bottom end. The base of each

tube was fully fixed to the stationary rigid surface while the top end was free. The punch, which was also modeled as a rigid surface, compress the frusta with a constant velocity of 6mm/min. Contact between the punch and the tube walls and self-contact between the tube walls during collapse were modeled by exponential contact pressure-over-closure relationship with friction of 0.1.

## 2.2 Model validation

The tube response predicted by the FE model was compared with that of experiments for straight tubes under quasi-static compression. Fig. 3 compared the quasi-static load-deflection curves of the FE model with that of experiments for the straight tube with wall thickness of 1.8mm, length of 120mm and mean diameter of 46.8mm. The load-deflection curves of experimental and numerical results of cylindrical tubes matched well even though the numerical results underestimated the initial buckling force. The comparison of load-stroke between experimental and numerical results indicates that the FE model was trustworthy.

## 2.3 Design of the tests

The tests are designed using D-optimal design of

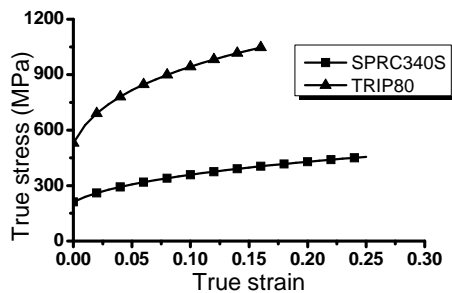


Fig. 2 Stress-strain curves of SPRC340S and TRIP80

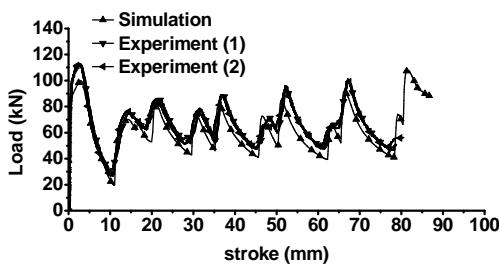


Fig. 3 Comparison of load-stroke curve of finite element model with that of experiments

Table 1 Dimensions of conical frusta

thickness (mm)	length (mm)	semi-apical angle (°)
1, 2, 3	80, 100, 120	6, 8, 10

experiments to reduce the number of tests. The final simulation number is 16 for each material, as shown in table 1. The responses, such as the specific energy absorption, the maximum force, force efficiency, stroke efficiency and the total efficiency, are modeled by RSM method. The advantage of RSM is that it offers a large amount of information from a small number of experiments and it is possible to observe the interaction effect of the independent parameters on the responses.

## 3. DISCUSSIONS

The deformation modes of axial crush of frusta with the semi-apical angles between  $6^\circ$  and  $10^\circ$  is analyzed and a new deformation mode, radial concertina mode, is observed. Different responses are approximated using RSM methods and the effects of different factors are investigated.

### 3.1 Collapse mode analysis

It is observed that initially the shell behaves elastically until the load reaches the initial peak load. After that, instability of the structural occurs with the formation of folds at the small end. For the frusta with semi-apical angles of  $10^\circ$ , the fold bended inward after it was developed and finally the radial concertina mode was formed while the developed folds piled up in the axial direction and the axial concertina mode came into being when the semi-apical angle is  $6^\circ$  and  $8^\circ$ . Fig.4 (a) and (c) show the axial concertina mode and Fig.4 (b) and (d) show the radial concertina mode.

Fig. 5 and Fig. 6 compared the collapse modes of two different materials: SPRC340 which stands for mild steels and TRIP80, one kind of high strength steels. We can conclude that these two different kinds of deformation modes are independent of flow stress. Also we can see that the radial concertina mode occurs when the semi-apical angle is large while the axial concertina mode exists when the semi-apical angle is no more than  $8^\circ$ . At the critical semi-apical angle, frusta with small ratio of thickness to

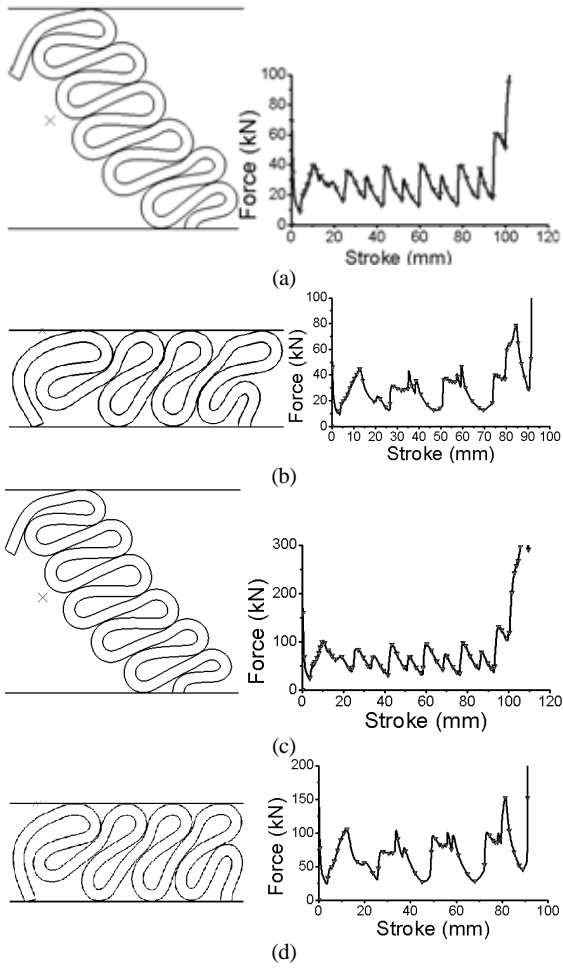


Fig. 4 Deformation shapes and force-stroke curves: (a)  $h=1\text{mm}$ ,  $L=120\text{mm}$ ,  $\alpha=6^\circ$ , SPRC340S; (b)  $h=1\text{mm}$ ,  $L=100\text{mm}$ ,  $\alpha=10^\circ$ , SPRC340S; (c)  $h=1\text{mm}$ ,  $L=120\text{mm}$ ,  $\alpha=6^\circ$ , TRIP80; (d)  $h=1\text{mm}$ ,  $L=100\text{mm}$ ,  $\alpha=10^\circ$ , TRIP80.

diameter and large ratio of length to diameter tend to collapse radially.

### 3.2 Parametric analysis

Of interest in this study was the effect of the various geometry parameters on the responses of frusta under quasi-static axial loading. Such knowledge provides a guide as to the design of such structures in applications of energy absorbing systems. The concerned geometry parameters were the wall thickness, length, and semi-apical angle. The baseline model of the tubes used for the parametric study are shown in Fig. 1. The interested responses in the study include the specific energy absorption, the maximum load, the force efficiency, the

stroke efficiency, and the total efficiency.

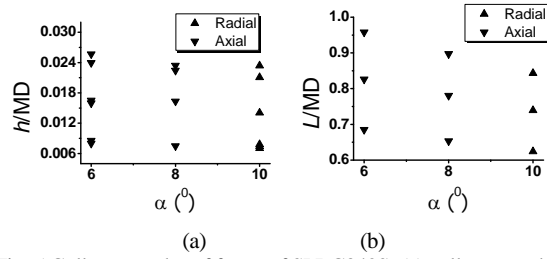


Fig. 5 Collapse modes of frusta of SPRC340S: (a) collapse mode at different  $h/MD$  ratio; (b) collapse mode at different  $L/MD$  ratio.

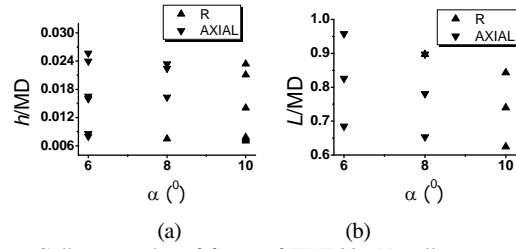


Fig. 6 Collapse modes of frusta of TRIP80: (a) collapse mode at different  $h/MD$  ratio; (b) collapse mode at different  $L/MD$  ratio.

#### 3.2.1 Specific energy absorption

Specific energy absorption, the absorbed energy per unit mass, is a useful measure for comparing the energy absorption capacity of structures when weight is an important consideration, such as in the automotive industry. Higher value of SEA indicates lighter weight of energy absorbers. As shown in Fig. 7, SEA of high strength steel TRIP80 is much higher than that of mild steel SPRC340S due to the flow stress of TRIP80 is greater than that of SPRC340S. Also it increases as the thickness increases and the length and the semi-apical angle have little effect on it. Accordingly, using high strength steels instead of mild ones and increasing the wall thickness are two effective methods to obtain high specific energy absorption.

#### 3.2.2 Maximum force

Large force means high acceleration, which may cause severe damage to human body especially for the semi-fluid organs such as brains. Therefore, while we make great efforts to increase the specific energy absorption, we hope that the maximum force is as small as possible.

From the main effects of Fig. 7, the maximum force increases with the flow stress of material and the wall thickness but it is not sensitive to the length and the semi-

apical angle.

### 3.2.3 Force efficiency

The shape of the force-stroke curve of an ideal absorber is rectangular and the force efficiency, the ratio of the average force to the maximum force, is a measure of how close the force-stroke curve of an absorber approaches the ideal one.

From Fig. 7, it seems that the force efficiency is only a function of the frusta's geometry but not a function the material. It increases rapidly as the wall thickness increases but slightly decreases with the length. The effect of the semi-apical angle is not obvious.

### 3.2.4 Stroke efficiency

The ratio of the maximum permissible stroke to the entire length of the absorber, usually termed as the stroke efficiency, is a measure of how closely the performance of a component approaches the best possible performance.

As shown in Fig. 7, the stroke efficiency decreases with the wall thickness but increases slightly as the length and the semi-apical angle increases.

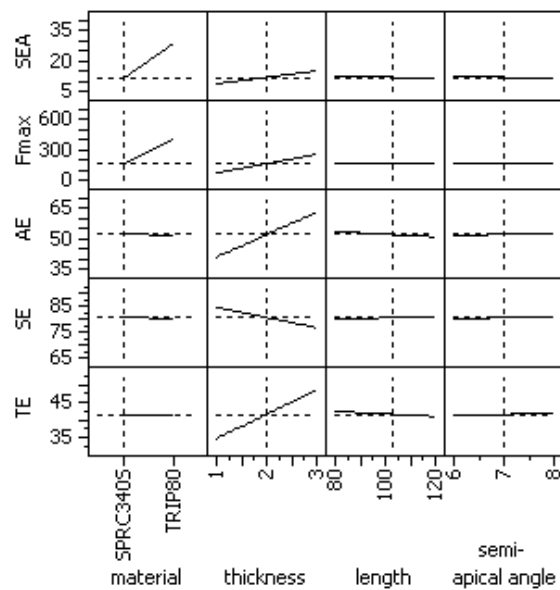


Fig. 7 The effect of different factors on responses

### 3.2.5 Total efficiency

Total efficiency is an integrative measure of the energy absorption capacity of both force efficiency and stroke efficiency. It is the multiplication of force efficiency and

stroke efficiency.

From Fig. 7, the total efficiency increases as the wall thickness and the semi-apical angle increases and it is more sensitive to the wall thickness than semi-apical angle. The total efficiency is a negative function of the tube length.

## 4. CONCLUSION

This paper investigated the response of frusta under quasi-static axial compression. The studied parameters were the tube wall thickness, semi-apical angle, length and the material properties, while the responses were the deformation mode, specific energy absorption, maximum load, force efficiency, effective crush distance and total efficiency. The main conclusions are summarized as follows:

- 1) The collapse mode of frusta largely depends on the semi-apical angle. At low value of semi-apical angles (up to  $8^\circ$ ), the frusta collapse in axial concertina mode while radial concertina mode occurs with higher semi-apical angles.
- 2) The specific energy absorption and the maximum force are larger for higher strength material and frusta with thicker wall but it is not sensitive to the length and the semi-apical angle.
- 3) The maximum force increases as the strength of the material and the thickness increases but the length and the semi-apical angle has little influence on it.
- 4) The force efficiency is only a function of frusta's shape and it increases as the wall thickness and the length increases but it is not sensitive to the length and semi-apical angle.
- 5) The stroke efficiency rapidly decreases with the wall thickness. It increases slightly as the length and the semi-apical angle increases.
- 6) The total efficiency increases with the wall thickness and the semi-apical angle but it depends more on the wall thickness. However, the effect of the length is negative on the total efficiency.

## References

[1] K. R. F. Andrews, G. L. England and E. Ghani, "Classification of the axial collapse of circular tubes

- under quasi-static loading”, *Int. J. Mech. Sci.*, Vol. 25, pp. 687-96, 1983.
- [2] S. R. Guillow, G. Lu and R. H. Grzebieta, “Quasi-static compression of thin-walled circular aluminum tubes”, *Int. J. Mech. Sci.*, Vol. 43, pp. 2103-23, 2001.
- [3] W. Abramowicz and N. Jones, “Dynamic axial crushing of circular tubes”, *Int. J. Impact Eng.*, Vol. 2, pp. 263-61, 1984.
- [4] M. Langseth and O. S. Hopperstand, “Static and dynamic axial crushing of square thin-walled aluminum extrusions”, *Int. J. Impact Eng.*, Vol. 18(7/8), pp. 949-68, 1996.
- [5] M. Langseth, O. S. Hopperstand and T. Berstad, “Crashworthiness of aluminum extrusions: validation of numerical simulation, effect of mass ratio and impact velocity”, *Int. J. Impact Eng.*, Vol. 22(8), pp. 829-54, 1999.
- [6] S. R. Reid and T. Y. Reddy, “Static and dynamic crushing of tapered sheet metal tubes of rectangular cross-section”, *Int. J. Mech. Sci.* Vol. 28(9), pp. 623-37, 1986.
- [7] H. E. Postlethwaite and B. Mills, “Use of collapsible structural elements as impact isolators with special reference to automotive applications”, *J. Strain Anal.*, Vol. 5, pp. 57-73, 1970.
- [8] A. G. Mamalis and W. Johnson, “The quasi-static crumpling of thin-walled circular cylinders and frusta under axial compression”, *Int. J. Mech. Sci.*, Vol. 25, pp. 713-32, 1983.
- [9] A. G. Mamalis, W. Johnson and G. L. Viegelaahn, “The crumpling of steel thin-walled tubes and frusta under axial compression at elevated strain-rates: some experimental results”, *Int. J. Mech. Sci.*, Vol. 26, pp. 537-47, 1984.
- [10] A. G. Mamalis, D. E. Manolakos, S. Saigal, G. L. Viegelaahn and W. Johnson, “Extensible plastic collapse of thin-wall frusta as energy absorbers”, *Int. J. Mech. Sci.* Vol. 28, pp. 219-29, 1986.
- [11] N. K. Gupta and H. Abbas, “Axisymmetric axial crushing of thin frusta”, *Thin-Walled Struct.*, Vol. 36, pp. 169-79, 2000.
- [12] N. K. Gupta, G. L. Easwara Prasad and S. K. Gupta, “Plastic collapse of metallic conical frusta of large semi-apical angles”, *Int. J. Crash*, Vol. 2(4), pp. 349-66, 1997.
- [13] P. K. Gupta, “A study on mode of collapse of varying wall thickness metallic frusta subjected to axial compression”, *Thin-Walled Struct.*, Vol. 46, pp. 561-71, 2008.