Multi-User Resource Allocation Scheme Based on Mutual Information in OFDMA Downlink Systems

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Abstract
Recent cellular systems apply a single codeword, rather than multiple codewords, to resource units that are assigned to a single user. In this paper, we propose a heuristic scheme for allocating resources when single codeword transmission is performed in a multiple user environment.

1. Introduction
Some channel coding schemes that have been used in recent OFDMA systems generate a long codeword which requires a great amount of radio resource to be transmitted. For this reason, single codeword(SCW) transmission that a single code rate is applied over all the bits transmitted to a user within a frame is performed in many system. However, most previous works assume multiple codeword(MCW) transmission, which makes allocating resources easier than when using SCW transmission. The objective of this paper is to propose a resource allocation scheme to achieve maximal performance in OFDMA cellular systems.

2. System Model
Let resource unit be rectangular resource region for minimal unit of link adaptation and resource allocation, which is composed of one subchannel and one time slot. Let C be the set of rates of available coding schemes. The Adaptive Modulation and Coding(AMC) operation chooses modulation and channel coding scheme such that PER is less than a target PER. It is a function of the SINR and the modulation level. The system performs SCW transmission in which the modulation level can be determined adaptively for each resource unit and a single code rate is applied to each user. Mutual information is a metric for the link quality of a subcarrier that is used for estimating PER. It is a function of the SINR and the modulation level. The detailed definition and derivation of mutual information is in [2].

Let \(N\) be the number of users in the cell, and \(N, S\) and \(T\) be the set of indices of users, subchannels, and time slot, respectively. Then, the resource allocation problem can be formulated as following

\[
P_1: \max \sum_{i \in N} U_i \left( \sum_{j \in S} d_{ij} b_{ij} c_i \right)
\]

\[
\sum_{j \in S} b_{ij} d_{ij} I_{ij} \geq \alpha(c_i) \sum_{j \in S} b_{ij} d_{ij}, \quad \forall i \in N
\]

\[
\sum_{i \in N} b_{ij} \leq N_s, \quad \forall j \in S
\]

\[
b_{ij} \geq 0, c_i \in C, \quad \forall i \in N, j \in S.
\]

Here, \(U_i(x)\) is a utility function of user \(i\). \(b_{ij}\) and \(c_i\) are the variables, where \(b_{ij}\) indicates the number of allocated resource units in subchannel \(j\) and time slot \(k\) to user \(i\) and \(c_i\) is the channel coding level for user \(i\).

3. Resource Allocation Scheme
We approximate \(P_1\) into linear form. Firstly, \(\alpha(c_i)\) can be approximated as a function of \(c_i\). From the information given in Table I, we can make linear approximation as following,

\[
\gamma_p \simeq \gamma_q, \quad \beta_p \simeq c_i + 0.04 \quad \forall p, q \in C.
\]

Based on (3), \(\alpha(c_i)\) can be expressed as following,

\[
\alpha(c_i) = \sqrt{2er f^{-1}(1-2P_t)\gamma_i} + \beta_i \simeq A + c_i
\]

where \(A = \sqrt{2er f^{-1}(1-2P_t)\gamma_1} + 0.04\).

Secondly, the nonlinear term \(c_i b_{ij}\) can be converted to a positive real variable \(y_{ij}\). Letting \(cr_1\) and \(cr_C\) be the minimum and maximum code rates in \(C\), respectively, \(P_1\) can be converted as following.

\[
P_2: \max \sum_{i \in N} U_i \left( \sum_{j \in S} d_{ij} y_{ij} \right)
\]

\[
\sum_{j \in S} [y_{ij} + (A - I_{ij}) b_{ij}] d_{ij} \leq 0, \quad \forall i \in N
\]

\[
\sum_{i \in N} b_{ij} \leq N_s, \quad \forall j \in S
\]

\[
b_{ij} \geq 0, cr_1 b_{ij} \leq y_{ij} \leq cr_C b_{ij}, \quad \forall i \in N, j \in S.
\]

The solution of \(P_2\) can be an upper bound of the performance, because the optimal solution of the original problem satisfies the constraints in \(P_2\) and can be an element of the feasible set of \(P_2\).

3.1. Heuristic Algorithm
To get the intuition, we observe KKT conditions of \(P_1\), which can be expressed as following

\[-U'_i(R_i)d_{ij}c_i + \mu_i \frac{\partial g(M_i, c_i)}{\partial b_{ij}} \sum_{j \in S} d_{ij} b_{ij} + \lambda_j - \kappa_{ij} = 0, \quad \forall i \in N, j \in S\]

\[-U'_i(R_i) \sum_{j \in S} b_{ij} d_{ij} + \mu_i \frac{\partial g(M_i, c_i)}{\partial c_{ij}} + \epsilon_i - \rho_i = 0, \forall i \in N\]

\[
\lambda_j \left( \sum_{i \in N} b_{ij} - N_s \right) = \kappa_{ij} b_{ij} = 0, \quad \forall j \in S
\]

\[
\mu_i (g(M_i, c_i) - p_i) = \rho_i c_i = \epsilon_i (cr_C - c_i) = 0, \forall i \in N.
\]
Using Proposition 1 in [3], we can obtain a necessary condition of the optimal solution as following.

**Lemma 1:** The necessary condition for the assignment of subchannel $j$ to user $i^\ast$ to be the optimal solution is

$$i^\ast = \arg \max_{i \in \mathbb{N}} \left\{ U_i^\prime(R_i) d_i j c_i - \mu_i \frac{\partial g(M_i, c_i)}{\partial b_i j} \frac{d_j (I_j - M_i)}{\sum_{j \in S} d_j b_j} \right\}$$

(14)

where $M_i$ is the average mutual information of user $i$.

**Proof:** The proof of this lemma is similar to that for Proposition 1 in [3]. Removing $\kappa_{ij}$ from (13) and (15), we obtain the following equation

$$b_{ij} (-U_i^\prime(R_i) d_i j c_i + \mu_i \frac{\partial g(M_i, c_i)}{\partial b_i j} \frac{d_j (I_j - M_i)}{\sum_{j \in S} d_j b_j} + \lambda_j) = 0.$$  

(15)

Since $\kappa_{ij} \geq 0$, the following inequality will be satisfied

$$\lambda_j \geq U_i^\prime(R_i) d_i j c_i - \mu_i \frac{\partial g(M_i, c_i)}{\partial b_i j} \frac{d_j (I_j - M_i)}{\sum_{j \in S} d_j b_j}.$$  

(16)

If $b_{ij} > 0$, the equality holds in (19) which yields an upper bound $\lambda_j$. Thus, Lemma 1 follows.

In addition, the following lemma can be derived:

**Lemma 2:** $g$ satisfies the following equation

$$\frac{\partial g(M_i, c_i)}{\partial c_i} \simeq - \frac{\partial g(M_i, c_i)}{\partial b_i j}.$$  

(17)

**Proof:** Using (8), (3) becomes as following,

$$g(M_i, c_i) \simeq \frac{1}{2} \left[ 1 - erf \left( \frac{M_i - c_i - 0.04}{\sqrt{2} \gamma} \right) \right].$$  

(18)

Taking the partial derivative with respect to $c_i$,

$$\frac{\partial g(M_i, c_i)}{\partial c_i} = \frac{-1}{\sqrt{\pi}} e^{(M_i - c_i - 0.04)^2} = - \frac{\partial g(M_i, c_i)}{\partial b_i j}.$$  

(19)

Using the above lemmas, the following can be derived:

**Proposition 1:** The optimal solution can be achieved if subchannel $j$ is assigned to user $i^\ast$ based on the following criterion,

$$i^\ast = \arg \max_{i \in \mathbb{N}} \left\{ U_i^\prime(R_i) d_i j c_i + I_j - M_i \right\}.$$  

(20)

**Proof:** Using lemma 2, (14) can be converted as following

$$-\mu_i \frac{\partial g(M_i, c_i)}{\partial b_i j} = U_i^\prime(R_i) \sum_{j \in S} b_{ij} d_{ij} - \epsilon_i + \rho_i.$$  

(21)

Here, $\rho_i = 0$ because the assignment of subchannel $j$ to user $i$ implies that $c_i > 0$. In addition, we can set $\epsilon_i$ to 0, because the code rate is usually less than $c_r C$. Using (24), (17) becomes as following

$$U_i^\prime(R_i) d_i j c_i - \mu_i \frac{\partial g(M_i, c_i)}{\partial b_i j} \frac{d_j (I_j - M_i)}{\sum_{j \in S} d_j b_j}.$$  

(22)

$$= U_i^\prime(R_i) d_i j c_i + U_i^\prime(R_i) \sum_{j \in S} d_j b_j \frac{d_j (I_j - M_i)}{\sum_{j \in S} d_j b_j}.$$  

(23)

$$= U_i^\prime(R_i) d_i [c_i + I_j - M_i].$$  

(24)

This implies that Lemma 1 is equivalent to Proposition 1. Based on above proposition, we propose a resource allocation algorithm. The detailed operations are described below.

### 4. Results and discussion

We set the simulation parameters based on the IEEE 802.16e system. For the channel model, we used Pedestrian B model in ITU-R. The conventional scheme, which is denoted as single-user, allocates all the resource units in a frame to a single user, and decides the modulation levels and code rate of the selected user. [1] We used max C/I and proportional fair scheduler.

Fig. 1 shows that the average link capacity of the scheduled users increases with the number of users when the average SINR is 7dB. The performance of the proposed scheme is improved by 53%-73% over that of conventional single-user scheme due to multi-user diversity. In addition, the gap between the performance of the proposed scheme and the upper bound is rather small for all the cases. Note that the link capacity is saturated when it reaches 5 bit/s/Hz, because it is the maximum data rate that can be achieved with the set of MCS levels used in the simulation.

### 5. References

