System Reliability in the Presence of Common-Cause Failures

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Key Words—Common-cause failure, System reliability, Multivariate exponential distribution.

Reader Aids—  
Purpose: Widens the state-of-art  
Special math needed for explanations: Elementary probability  
Special math needed to use results: None  
Results useful to: Reliability analysts and theoreticians.

Summary & Conclusions—This paper presents a method for calculating the reliability of a system depicted by a reliability block diagram, with identically distributed components, in the presence of common-cause failures. To represent common-cause failures, we use the Marshall & Olkin formulation of the multivariate exponential distribution. That is, the components are subject to failure by Poisson failure processes that govern simultaneous failure of a specific subset of the components. The method for calculating system reliability requires that a procedure exist for determining system reliability from component reliabilities under the statistically-independent-component assumption. The paper includes several examples to illustrate the method and compares the reliability of a system with common-cause failures to a system with statistically-independent components.

The examples clearly show that common-cause failure processes as modeled in this paper materially affect system reliability. However, inclusion of common-cause failure processes in the system analysis introduces the problem of estimating the rates of simultaneous failure for multiple components in addition to their individual failure rates.

1. INTRODUCTION

Common-cause failures are the failure of multiple components due to a single occurrence or condition. For example, contaminated fluid causes two pumps to fail that are operated in “parallel”. In this event, the reliability of the “parallel” configuration with redundancy is less than a similar configuration with s-independent components.

This paper presents a method for calculating the reliability of a system depicted by a reliability block diagram, with identically distributed components in the presence of common-cause failures. The assumption of identically distributed components implies that the reliability of each component is identical. We assume the Marshall & Olkin [1] formulation of the multivariate exponential distribution (MVED). Our assumptions are:

1. The system is composed of components and the components are subject to failure by a number of different failure processes.
2. Each failure process governs simultaneous failures of a specific subset (excluding the null set) of system components.
3. Each failure process is s-independent and has a Poisson distribution; thus, the time between any two successive events of a failure process is exponentially distributed.
4. The rate of failure for each failure process depends on the number of components that fail, but this rate of failure is the same regardless of the particular components involved.

For example, for a 3-component system, let there be 7 failure processes. Failure processes 1-3 govern the occurrence of failures by individual components. Failure process 4 represents simultaneous failures of components one and two. Failure processes 5-6 are similarly defined for the other two possible combinations of two components. Failure process 7 specifies simultaneous failure of all three components. Failure processes 1-3 have the same rate of occurrence; and the failure rates of failure processes 4-6 are the same.


We derive an expression for the reliability of a specified system configuration in three steps:

1. Find an expression for the reliability of a specified component.
2. Find the probability a specific group of m components out of the n component system are all good.
3. Construct an expression giving the reliability of an arbitrary system configuration, using the previous results.

2. NOTATION

\[ n^C_r \] number of combinations of \( r \) items out of a possible \( n \) items
\[ FIC(t) \] 1-RICC(t)
\[ FII(t) = 1-RIIC(t) \]
\[ \lambda_r \] rate parameter for the failure process \( Z_r \)
\[ P(t) \] reliability of a single component at time \( t \)
\[ P_n^{(m)}(t) \] probability that all components of a specific
3. RELIABILITY OF A SPECIFIED COMPONENT

A specific component can fail due to the occurrence of several different failure processes. 1. There is the 1-component process $Z_1$ for $s$-independent failure of the specified component. 2. There are 2-component processes that include the specified component. There are a total of $\binom{s}{2}$ i.i.d. $Z_2$ failure processes but only $\binom{s-1}{1}$ of these processes include the specified component. In general, there are $\binom{s}{C_r}$ i.i.d. $Z_r$ failure processes with parameter $\lambda_r$ governing the simultaneous failure of $r$ components. Of these $\binom{s}{C_r}$ failure processes, $\binom{s-1}{r-1}$ of them include the specified component.

The $P_n(t)$ is the probability that the specific component is operating at time $t$, viz, the probability that none of the processes governing the simultaneous failure of $r$ components, $r = 1, 2, ..., n$, includes the specific component. Based on $s$-independence of the Poisson processes—

$$
P_n^{(i)}(t) = \prod_{r=1}^{n} \left[ \exp(-\lambda_r) \right]^{s-C_r-1} \cdot \exp(-\sum_{r=1}^{s-1} C_{r-1} \lambda_r)
$$

(1)

4. PROBABILITY A GROUP OF $m$ COMPONENTS ARE ALL GOOD

The probability that a specific group of $m$ components out of the $n$-component system are all good is:

$$
P_n^{(m)}(t) = \Pr\{S_1 \cap S_2 \cap S_3 \cap ... \cap S_m; t\}.
$$

The probability that both components $S_1$ and $S_2$ are good at time $t$ is:

$$
\Pr\{S_1 \cap S_2; t\} = \Pr\{S_1; t\} \cdot \Pr\{S_2; S_1; t\}
$$

$$
\Pr\{S_2; S_1; t\} = \text{probability that component 2 is good at time } t \text{ given no event of any common-cause failure processes associated with the failure of component 1 has occurred} = \text{probability that component 2 is good at time } t \text{ for an } n-1 \text{ component system, which is the original system with component 1 excluded} = P_n^{(m-1)}(t)
$$

since the components are identically distributed. In general—

$$
P_n^{(m)}(t) = \Pr\{S_1; t\} \cdot \Pr\{S_2|S_1; t\} \cdot \Pr\{S_3|S_1S_2; t\} \cdot \ldots \cdot \Pr\{S_m|S_1, S_2, ..., S_{m-1}; t\}
$$

$$
= P_1^{(1)}(t) P_2^{(1)}(t) P_3^{(1)}(t) \ldots P_{n-m+1}^{(1)}(t)
$$

$$
= \prod_{k=m}^{n} P_k^{(1)}(t)
$$

(2)

5. RELIABILITY OF A SYSTEM CONFIGURATION

This section specifies the method for calculating the reliability of a system configuration with identically distributed common-cause failures. To illustrate the approach, let us review an approach for calculating the reliability of a system with i.i.d. components (no common-cause failures).

Consider, for example, the Venn diagram for a 1-out-of-3-G system. The circles A, B, C in figure 1 represent successful operation of components A, B, C. The union of A, B, C represents successful operation of the system.

![Venn diagram for 3 components](image)
can express the reliability for any system configuration having identically distributed components with common-cause failures as a linear function of $P_n^{(m)}(t), m = 1, 2, ..., n$. This is true because a Venn diagram can represent the reliability for any system configuration. Moreover, this linear function is the same as that for i.i.d. components. For i.i.d. components $P_n^{(m)}(t) = P^m(t)$. One can obtain this linear function using a reference such as Shooman [10] or by direct calculation. If the MVED can represent the occurrence of common-cause failures, then use (1) and (2) for $P_n^{(m)}(t)$.

6. EXAMPLES

To illustrate the calculation of system reliability, we use three system configurations:

1. A 1-out-of-3:G system
3. A 4-component "series parallel" system whose reliability block diagram appears in figure 2.

![Fig. 2. 4-Component series-parallel configuration](image)

The failure rates are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of Simultaneous Failures</th>
<th>Failure Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>1</td>
<td>0.002</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>2</td>
<td>0.001</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>3</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>4</td>
<td>0.00025</td>
</tr>
</tbody>
</table>

1-out-of-3:G System

The reliability for this system is:

$$RICC(t) = 3P_3^{(1)}(t) - 3P_3^{(2)}(t) + P_3^{(3)}(t)$$

After substituting (1)—

$$RICC(t) = 3\exp[- (\lambda_1 + 2\lambda_2 + \lambda_3)t] - 3\exp[- (2\lambda_1 + 3\lambda_2 + \lambda_3)t] + \exp[- (3\lambda_1 + 3\lambda_2 + \lambda_3)t]$$

After substituting the selected failure rates —

$RICC(10) = 0.99413, FICC(10) = 0.587\%$.

For comparison purposes, the 1-component reliability remains at the value for a component in the 3-component common-cause system, but the system consists of i.i.d. components. That is, calculate $RIC(t)$ when $P(t) = P_3^{(1)}(t)$. The resulting reliability neglects the system effects of common-cause failures and represents the prediction of a practitioner assessing all failure causes against a component but assuming an "s-independence" model. In that case,

$$P(t) = \exp(- (\lambda_1 + 2\lambda_2 + \lambda_3)t),$$

$P(10) = 0.955997$,

$RICC(10) = 3P(10) - 3P^2(10) + P^3(10) = 0.9999148$,

$FIIC(10) = 0.00852\%$.

The system unreliability assuming i.i.d. components is appreciably lower (by a factor of 100) than that calculated assuming common-cause failures.

1-out-of-3:F System

The reliability for a 1-out-of-3:F system with i.i.d. components is:

$$RICC(t) = P^1(t)$$

The reliability of one component in a 3-component system at time 10 is

$P(10) = 0.955997$;

$RICC(10) = 0.87372, FIIC(10) = 12.628\%$

For this system the common-cause reliability is:

$$RICC(t) = P_3^{(1)}(t) = P_1^{(1)}(t)P_2^{(1)}(t)P_3^{(1)}(t)$$

$$= \exp(- (3\lambda_1 + 3\lambda_2 + \lambda_3)t)$$

$RICC(10) = 0.90937, FICC(10) = 9.063\%$

In this example, the system reliability assuming s-independence is appreciably lower than that considering common-cause failures. This is true because the 2-component and 3-component processes are not applied independently in calculating the common-cause reliability. For example, the 3-component failure process either causes all components to fail simultaneously or it does not occur, and application of the 3-component process individually to each component unnecessarily degrades the system reliability.
The reliability for the series-parallel system shown in figure 2 with i.i.d. components is:

\[
RIC(t) = (1 - (1 - P(t))^2)^2 = 4P^2(t) - 4P^3(t) + P^4(t).
\]

The reliability of a single component in a 4-component system given by (1) is:

\[
P_4^{(1)}(t) = \exp(-\lambda_1 + 3\lambda_2 + 3\lambda_3 + \lambda_4)t).
\]

At time 10, the single component reliability is:

\[
P_4^{(1)}(10) = 0.93473
\]

Substituting the above value of \(P_4^{(1)}(10)\) for \(P(t)\) in the equation for \(RIC(t)\), the i.i.d. system reliability becomes:

\[
RIC(10) = 0.99150, \quad FIIC(10) = 0.850\%.
\]

For the common-cause reliability, we have, from (2):

\[
RIC(t) = 4P_4^{(2)}(t) - 4P_4^{(3)}(t) + P_4^{(4)}(t)
\]

\[
= 4P_3^{(1)}(t)P_4^{(1)}(t) - 4P_2^{(1)}(t)P_3^{(1)}(t)P_4^{(1)}(t) + P_1^{(1)}(t)P_3^{(1)}(t)P_2^{(1)}(t)P_4^{(1)}(t).
\]

\[
RIC(10) = 0.95566, \quad FIIC(10) = 4.434\%.
\]

Thus, for this case the system reliability assuming common-cause failures is appreciably lower than the i.i.d. system reliability.

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There are 4 corrections for [1].

1. Page 257, col 1: The city for all authors should be Kawasaki (not Tokyo).

2. Page 257, col 2, Notation: The definition of \(A\) should be:

\[
A = \ln(\text{error factor})/1.645
\]

3. Page 259, col 1, eq. (15): Eq. (15) should be:

\[
\sigma = \ln(\text{error factor})/1.645
\]

4. Pages 260, col 1, eq. (18): The matrix element in row 7, col 3 should be \(\lambda_7\), not \(\lambda_8\); all elements in that sub-diagonal are the same.

**REFERENCES**