III. CONCLUSIONS

A VSC scheme has been proposed for active nutation damping in momentum biased stabilized spacecraft with multiple torque inputs. It was found that a natural hierarchy of controls is imposed by the spatial arrangement of the torque effectors. This facilitates the off-line resolution of possible conflicts among input objectives in their quest for individual sliding surface reachability. Once the sliding surface is reached, and sliding conditions are satisfied, the controlled motions are asymptotically exponentially stable according to prespecified qualitative characteristics. The robustness of VSC with respect to plant parameter variations and external perturbations is studied in [11], following [12].

Nutation as a desired spacecraft maneuver has been little investigated in the literature, except for [5] and [10]. The ability of VSC laws to render controlled behavior with characteristics not present in any of the intervening closed-loop structures makes them attractive for applications where geometric performance constraints are to be imposed. Induced nutation via VSC as a means of periodic scanning maneuvers is a possible area for further research. The outstanding robustness properties of VSC also remain to be explored in the control of flexible spacecraft and energy dissipation control strategies for large space stations.

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Direct Adaptive Control with Integral Action for Nonminimum Phase Systems

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Abstract—This note presents a direct adaptive control scheme for nonminimum phase systems where controller parameters are estimated from the recursive least-squares algorithm. Some additional auxiliary parameters are obtained from the proposed polynomial identity, and a

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local convergence is guaranteed without any extra conditions. Integral action is incorporated into the adaptive controller to eliminate the steady-state error, and to satisfy a condition of the unique solution for the polynomial identity as well.

I. INTRODUCTION

Recently, significant progress has been made on the problem of direct adaptive control for nonminimum phase systems. In [1], this control scheme needs the polynomial factorization or a nonlinear identification procedure. With a standard linear parameter estimation Elliott [2] resolved the above problems. However, this scheme arises in arbitrary pole placement and requires the estimation of more parameters than those effectively needed for control. These extra parameters are those of a partial state predictor. Allidina and Hughes [3] resolved the problems of [1] and [2] with a self-tuning control structure. Their scheme merely requires the solution of a polynomial identity. The problem of this scheme is the introduction of an unknown polynomial. And also, Praly [4] resolved the problems of [1] and [2] with a bilinear estimation. However, the proposed bilinear parameter estimation problem leads to a computational burden. This note presents a direct or implicit adaptive control structure for single-input single-output nonminimum phase systems. Here we introduce a polynomial identity which together with the Bezout identity resolves the above problems. The above polynomial identity is derived from the pole placement equation and the Bezout identity. By this proposed identity, the open-loop zeros are retained. Hence, it is not assumed that the system is either stable or stably invertible. To estimate the controller parameters a recursive least-squares algorithm [5], [6] is used. And some additional auxiliary parameters stemmed from the Bezout identity are obtained from the relatively straightforward solution of the polynomial identity using the estimated controller parameters such that the estimated closed-loop system is given predetermined poles. An integrator is incorporated into the adaptive controller in a straightforward fashion to satisfy a condition of the solution for the polynomial identity in addition to eliminating the steady-state error in the output sequence. This proposed direct pole placement scheme guarantees local convergence without any assumptions about parameter convergence or the nature of the external input.

II. DESIGN OF DIRECT ADAPTIVE CONTROL STRUCTURE

We will consider here a causal feedback control law with the integral action

\[ S(q^{-1})U(k) = R(q^{-1})e(k) \]  

with

\[ S(q^{-1}) = (1 - q^{-1})(1 + s_1 q^{-1} + \cdots + s_m q^{-m}) \]

\[ R(q^{-1}) = 1 + r_0 q^{-\alpha} + \cdots + r_m q^{-m\alpha} \]

\[ e(k) = U(k) - Y(k) \]

where \( q^{-1} \) is the delay operator, \( Y(k) \) is the system output, \( U(k) \) the controller output and the system input. \( U(k) \) the arbitrary bounded set point sequence to be followed, and \( e(k) \) the tracking error. Consider a single-input single-output, discrete, linear time-invariant system described by

\[ A(q^{-1})Y(k) = q^{-d}B(q^{-1})U(k); \quad d > 0 \]

with

\[ A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_m q^{-m\alpha} \]

\[ B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_n q^{-n\alpha} \]

where \( q^{-d} \) represents the system time delay.

We will use the following assumptions.

A.1: \( n_a, n_b, \) and \( d \) are known.

A.2: \( A(q^{-1}) \) and \( B(q^{-1}) \) are coprime (but having unknown coefficients).

Following Elliott [2] the system can be written in the form:

\[ A(q^{-1})Z(k) = U(k) \]

\[ Y(k) = q^{-d}B(q^{-1})Z(k). \]

A. Nonadaptive Control Structure

Let us first design a nonadaptive controller which can assign the poles of (2) arbitrarily when \( A(q^{-1}) \) and \( B(q^{-1}) \) are known. This will then be converted to an adaptive controller assuming \( A(q^{-1}) \) and \( B(q^{-1}) \) to be unknown. Applying the control law to (3) results in the following closed-loop system:

\[ [A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})]Z(k) = R(q^{-1})Um(k) \]

\[ Y(k) = q^{-d}B(q^{-1})Z(k). \]

(4)

Let \( C(q^{-1}) \) be a monic asymptotically stable polynomial of degree \( ne \) whose zeros represent the desired closed-loop pole locations for (4). These can be assigned provided \( S(q^{-1}) \) and \( R(q^{-1}) \) satisfy

\[ A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = C(q^{-1}) \]

(5)

where

\[ C(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_n q^{-nc}. \]

(5a)

Since \( A(q^{-1}) \) and \( B(q^{-1}) \) are coprime, a unique solution for \( S(q^{-1}) \) and \( R(q^{-1}) \) always exists under the condition of \( ns = nb + d - 1, \) \( nr = \max(na, nc - nb - d) \) [7]. In this case, it is clear from Assumption A.2 that \( B(q^{-1}) \) must have no root at \( q = 1. \) When (5) holds, (4) simplifies to

\[ C(q^{-1})Z(k) = R(q^{-1})Um(k) \]

\[ Y(k) = q^{-d}B(q^{-1})Z(k). \]

(6)

Thus, after cancellation the closed-loop transfer function relating \( Y(k) \) and \( Um(k) \) becomes

\[ Y(k) = \frac{q^{-d}B(q^{-1})R(q^{-1})}{C(q^{-1})}. \]

(7)

This scheme can be posed as a standard linear parameter estimation problem by introducing the following Bezout identity:

\[ A(q^{-1})K(q^{-1}) + q^{-d}B(q^{-1})H(q^{-1}) = k_0. \]

(8)

Since \( A(q^{-1}) \) and \( B(q^{-1}) \) are assumed coprime polynomials, there exists a unique pair of polynomials \( H_0(q^{-1}) \) and \( K_0(q^{-1}) \) of degrees \( nh = na - 1 \) and \( nk = nb + d - 1, \) and also polynomials \( H(q^{-1}) \) and \( K(q^{-1}) \) which are \( k_0 \) multiples of polynomials \( H_0(q^{-1}) \) and \( K_0(q^{-1}) \), respectively.

\[ H(q^{-1}) = k_0H_0(q^{-1}) = h_0 + h_1 q^{-1} + \cdots + h_m q^{-mh} \]

(9)

\[ K(q^{-1}) = k_0K_0(q^{-1}) = k_0 + k_1 q^{-1} + \cdots + k_m q^{-mk}. \]

(10)

The coefficient \( k_0 \) in (8) is an arbitrary constant which is not equal to zero. When (8) holds, (5) can be written as

\[ A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R'(q^{-1}) + q^{-d}B(q^{-1}) \]

\[ = (1/k_0)q^{-d}B(q^{-1})H(q^{-1})C(q^{-1}) + (1/k_0)A(q^{-1})K(q^{-1})C(q^{-1}) \]

(11)

where

\[ R'(q^{-1}) = R(q^{-1}) - 1. \]

(11a)

Multiplying (11) by \( Z(k) \) and using (3) yields

\[ Y(k) = -S(q^{-1})U(k) - R'(q^{-1})Y(k) \]

\[ + (1/k_0)H(q^{-1})C(q^{-1})Y(k) + (1/k_0)K(q^{-1})C(q^{-1})U(k). \]

(12)
Then (12) can be written more compactly as
\[ Y(k) = P^T \phi(k) \] (13)
with
\[ \phi(k)^T = [\phi_1(k)^T; \phi_2(k)^T] \] (13a)
and
\[ P^T = [P_1^T; P_2^T] \] (13b)
where
\[ \phi_1(k)^T = [(q^{-1} - 1) U(k - 1), \ldots, (q^{-1} - 1) U(k - n_2), -Y(k), \ldots, -Y(k - nr)] \] (13c)
and
\[ \phi_2(k)^T = [C(q^{-1}) Y(k), \ldots, C(q^{-1}) Y(k - nh), C(q^{-1}) U(k - 1), \ldots, C(q^{-1}) U(k - nk), (C(q^{-1}) - 1 + q^{-1}) U(k)] \] (13d)
and
\[ P_1^T = [s_{1}, \ldots, s_{na}, t_{0}, \ldots, r_{nr}] \] (13e)
and
\[ P_2^T = [h_{1}', \ldots, h_{nh}', k_{1}', \ldots, k_{nk}'] \] (13f)

Then (12) can be written more compactly as
\[ Y(k) = P^T \phi(k) \] (13)
with
\[ \phi(k)^T = [\phi_1(k)^T; \phi_2(k)^T] \] (13a)
and
\[ P^T = [P_1^T; P_2^T] \] (13b)
where
\[ \phi_1(k)^T = [(q^{-1} - 1) U(k - 1), \ldots, (q^{-1} - 1) U(k - n_2), -Y(k), \ldots, -Y(k - nr)] \] (13c)
and
\[ \phi_2(k)^T = [C(q^{-1}) Y(k), \ldots, C(q^{-1}) Y(k - nh), C(q^{-1}) U(k - 1), \ldots, C(q^{-1}) U(k - nk), (C(q^{-1}) - 1 + q^{-1}) U(k)] \] (13d)
and
\[ P_1^T = [s_{1}, \ldots, s_{na}, t_{0}, \ldots, r_{nr}] \] (13e)
and
\[ P_2^T = [h_{1}', \ldots, h_{nh}', k_{1}', \ldots, k_{nk}'] \] (13f)

Now, a linear regression form is obtained for the parameter vector \( P \). Note that (12) can be regarded as another representation of the transfer function (2) relating \( Y(k) \) and \( U(k) \), and could be a minimal realization provided \( nc \leq 1 \).

Then let polynomials \( H(q^{-1}) \) and \( K(q^{-1}) \) be of the following form:
\[ C(q^{-1}) H(q^{-1}) = A(q^{-1}) + k_0 R(q^{-1}) \] (14)
\[ C(q^{-1}) K(q^{-1}) = k_0 S(q^{-1}) - q^{-d} B(q^{-1}) \] (15)
In this case the minimal degrees for \( S(q^{-1}) \) and \( R(q^{-1}) \) are \( nb + d \) and \( na \), respectively. These equations can be combined as
\[ S(q^{-1}) H(q^{-1}) - R(q^{-1}) K(q^{-1}) = 1 \] (16)
Equation (16) can be used to tune \( H(q^{-1}) \) and \( K(q^{-1}) \) using \( S(q^{-1}) \) and \( R(q^{-1}) \) which are to be estimated. We now investigate the solvability of (16). If \( S(q^{-1}) \) and \( R(q^{-1}) \) have the forms of (14) and (15), respectively, they have coprime such that (16) is always solvable (see the Appendix). Then by assumption of the coprimeness of \( A(q^{-1}) \) and \( B(q^{-1}) \) the coefficient \( k_0 \) cannot be zero [see (14) and (15)] such that there exist real values of \( h_i' \) and \( k_{1}' \) in vector \( P_2 \).

Observe that the resultant closed-loop system becomes as (7). As the above identity shows, the integrator in \( B \). Adaptive Control Structure

Let us introduce the following recursive least-square algorithm:
\[ \hat{P}(k) = \hat{P}(k - 1) + L(k)[Y(k) - \hat{Y}(k)] \hat{P}(k - 1) \] (17)
\[ F(k) = I - L(k) \phi_1(k)^T F(k - 1) \] (18)
\[ L(k) = \frac{F(k - 1) \phi_1(k)}{1 + \phi_1(k)^T F(k - 1) \phi_1(k)} \] (19)
and
\[ \hat{Y}(k) = Y(k) - \phi(k)^T \hat{P}(k - 1). \] (20)

In (16), the symbols have the following meaning:
\[ \hat{P}(k)^T = [s_{1}(k), \ldots, s_{na}(k), r_{0}(k), \ldots, r_{nr}(k)] \] (21)
\[ S(k, q^{-1}) = (1 - q^{-1})(1 + \hat{h}_i(k) q^{-1} + \cdots + \hat{h}_{na}(k) q^{-na}) \] (22)
\[ \hat{R}(k, q^{-1}) = 1 + \hat{r}_0(k) + \hat{h}_i(k) q^{-1} + \cdots + \hat{r}_{nr}(k) q^{-nr} \] (23)
where \( na = nb + d - 1 \) and \( nr = na \). Then \( \hat{H}(k, q^{-1}) \) and \( \hat{K}(k, q^{-1}) \) are solved from the following identity:
\[ S(k, q^{-1}) \hat{H}(k, q^{-1}) - \hat{R}(k, q^{-1}) \hat{K}(k, q^{-1}) = 1 \] (24)
with
\[ \hat{H}(k, q^{-1}) = \hat{h}_0(k) + \hat{h}_i(k) q^{-1} + \cdots + \hat{h}_{na}(k) q^{-na} \] (25)
\[ \hat{R}(k, q^{-1}) = \hat{k}_0(k) + \hat{k}_i(k) q^{-1} + \cdots + \hat{k}_{nk}(k) q^{-nk} \] (26)
where \( nh = na - 1 \) and \( nb = d - 1 \).
Dividing (25) and (26) by \( \hat{k}_0(k) \), we can obtain the auxiliary parameter vector \( \hat{P}_2(k) \).
\[ \hat{P}_2(k)^T = [\hat{h}_2(k), \ldots, \hat{h}_{na}(k), \hat{k}_1(k), \ldots, \hat{k}_{nk}(k), 1] \] (27)
where
\[ \hat{k}_j(k) = \frac{\hat{h}_j(k)}{\hat{k}_0(k)}, 0 \leq i \leq nh : \hat{k}_j(k) = \frac{\hat{k}_j(k)}{\hat{k}_0(k)}, 1 \leq j \leq nk \] (27a)
And the input \( U(k) \) is also determined as follows.
\[ S(k, q^{-1}) U(k) = \hat{R}(k, q^{-1}) e(k) \] (28)

Lemma 2.1: If the controller polynomials have the forms of (14) and (15), \( \hat{P}_1(0) \) is within the region
\[ \| \hat{P}_1(0) - P_1 \| < \mu, \mu > 0 \] (29)
where \( \mu \) is a radius of the region in the parameter space centered on \( P_1 \), then (24) is solvable for all \( k \) and \( \hat{P}_2(k) \) has bounded parameters.
Proof: It follows from the properties of the least-squares algorithm that if (29) is satisfied for some \( \mu > 0 \), then
\[ \| \hat{P}_1(k) - P_1 \| < \mu, \text{ for all } k \geq 1 \] (30)
Now, for \( \hat{P}_1(k) = P_1(k) \), (24) has a unique finite solution since the Appendix guarantees that the matrix of (24) has a nonzero determinant at \( P_1 \). Then by Assumption A.2 the coefficient \( k_0 \) cannot be zero such that there exists real values of auxiliary parameters. The existence of a \( \mu > 0 \) in (29) such that (24) is solvable for all \( k \geq 1 \) now follows from (30) and the fact that the determinant of a matrix is a continuous function of the matrix elements.

With this adaptive control algorithm, the following convergence may be proved (see [6]).

Theorem 2.1: Consider the system in (2) subject to Assumptions A.1 and A.2, and Lemma 2.1. Then the adaptive control algorithm (17) to (28) leads to:
\[ i) \{ U(k), \{ Y(k) \} \text{ bounded for all time} \]
\[ ii) \text{the closed-loop response satisfies:} \]
\[ \lim_{k \to \infty} \| C(q^{-1}) Y(k) - G(k - 1, q^{-1}) U(m) \| = 0 \] (31)
where
\[ G(k - 1, q^{-1}) = q^{-d} \hat{B}(k - 1, q^{-1}) \hat{R}(k - 1, q^{-1}) \] (31a)
and \( \hat{B}(k - 1, q^{-1}) \) is the time-varying polynomial of \( B(q^{-1}) \).

Remark 2.1: The local nature of Theorem 2.1 arises by way of the solvability requirement. In cases when the determinant of (24) is zero and the cofactor of the element \( \hat{k}_0(k) \) is also zero at only a finite number of time instants, then any control can be used at these instants and global convergence will follow since (24) is solvable for all \( k \) greater than some finite \( N \). And in the specific case when (24) does not have a solution but
the cofactor of the element $K_d(k)$ is not equal to zero, the auxiliary parameters in (27) can assume any arbitrary values. This may be thought to alleviate the solvability problem of (24).

Remark 2.2: Note that nothing has been said about the richness condition on the control input $U(k)$. In this direct pole placement scheme, no persistent excitation condition is needed. This scheme only requires the solvability condition of (24). And compared to the indirect pole placement scheme, an advantage is that this scheme reduces the computational effort at each step because the determinant of (24) needs not to be calculated.

III. COMPUTER SIMULATIONS

The following example illustrates some features of the algorithm. UDUT factorization method [8] was used throughout for the estimation of the controller parameters.

Consider the following system [6]:

$$A(q^{-1})=1.0-1.2q^{-1}$$

$$q^{-d}B(q^{-1})=q^{-4}(1.0-3.1q^{-1}+2.2q^{-2}).$$

The following conditions were used:

$$C(q^{-1})=1.0$$

$$Um(k)=1.0$$

Initial condition of $P_1$ was taken as $[152.9 -293.1 -152.1 159.9]^T$, and initial condition of $P_2$ was given from (24).

IV. CONCLUSIONS

In this note we have presented a direct scheme for adaptively controlling linear time-invariant discrete-time single-input single-output nonminimum phase systems. In this scheme, a polynomial identity has been derived from the pole placement equation and the Bezout identity, and an integrator has been introduced into the controller to satisfy a condition of the unique solution for the polynomial identity. An example illustrating the performance of the algorithm has also been given.

APPENDIX

COPRIMENESS OF $S(q^{-1})$ AND $R(q^{-1})$ IN (16)

The proof proceeds by contradiction. Suppose that $S(q^{-1})$ and $R(q^{-1})$ have the greatest common factor $M(q^{-1})$, then

$$S(q^{-1})=M(q^{-1})S_0(q^{-1})$$

$$R(q^{-1})=M(q^{-1})R_0(q^{-1}).$$

Substituting in (5), we arrive at

$$M(q^{-1})[A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})]=C(q^{-1}).$$

Since $nc \leq 1$, we will consider here only two cases.

i) If $nc = 0$, then $M(q^{-1}) = 1$, which means $S(q^{-1})$ and $R(q^{-1})$ are coprime, and

ii) if $nc = 1$, we have

$$M(q^{-1})=C(q^{-1})$$

and

$$A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})=1.$$
Continuous-Time Model Reference Adaptive Control Based on Weighting Functions

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Abstract—This note presents a model reference adaptive control for a plant with unknown system degree. A continuous-time observer based on the weighting function is introduced, instead of a finite-dimensional state variable filter which needs an upper bound of the plant degree. The state is reconstructed by a sum of convolution integrals in a finite span of time.

I. INTRODUCTION

The continuous-time MRAC (model reference adaptive control) for the single-input single-output finite-dimensional linear time-invariant plant has been developed theoretically. The global stability of the control system was proved by Narendra et al. [1] and Morse et al. [2]. The control system is designed and analyzed under the following assumptions on the plant.

1) The numerator polynomial of the transfer function is stable.
2) An upper bound of the plant degree is known.
3) The relative degree is known.
4) The sign of the high-frequency gain is known.
5) The plant is controllable and observable.

Recently, relaxation of these assumptions has been intensively studied using two approaches. One is an extension of the control system where elimination of the assumption 3) or 4) is considered [3]-[5]. The other is an introduction of the robust control, which can cope with the noise and parasitics of the plant [6]-[9]. It concerns the assumption 2). However, the robust control needs an upper bound of the degree of the plant dominant mode and the parasitics are restricted to the finite-dimensional system. It also deteriorates the control quality to some extent.

To overcome these disadvantages, we propose an MRAC which does not require the upper bound of the plant degree. We use a continuous-time observer whose transfer functions belong to the finite Laplace transform [10], since the observer used in the usual control system requires the upper bound [11],[2]. The observed state is constructed by a sum of convolution integrals in a finite span of the time. As the adaptive controller estimates these weighting functions, it does not require the upper bound.

The case where parameters are known is considered first in Section II and the adaptive control is designed in Section III. Numerical simulations are also presented in Section IV in order to show the effectiveness of the proposed control system.

II. MODEL FOLLOWING CONTROL

A. System Representation

It is assumed that the known single-input single-output linear plant is described by the state equations

\[
dx(t)/dt = Ax(t) + bu(t),
\]

\[
y(t) = c^T x(t),
\]

where \(x(t) \in \mathbb{R}^n\), \(u(t) \in \mathbb{R}\), and \(y(t) \in \mathbb{R}\) are the state, the input, and the output of the plant, respectively, and \(n\) is the plant degree.

The transfer function becomes

\[
g(s) = c^T (sI - A)^{-1} b = k_0 \cdot \alpha(s)/\beta(s)
\]

where \(k_0\) is a constant, \(\alpha(s)\) is a monic stable polynomial, \(\beta(s)\) is a monic polynomial with degree \(n\). The \(s^m\) means a time differential operator.

The relative degree \(m\), which is the difference between the degree of \(\alpha(s)\) and that of \(\beta(s)\), is expressed as follows:

\[
m = \min\{1, c^T A^{-1} b \neq 0\}.
\]

Recursive differentiations of (1.b) and substitutions of (1.a) lead to

\[
sy'(t) = c^T x(t), \quad c^T \sim m - 1
\]

\[
x^m y(t) = c^T x^m(t) + c^T A^{m-1} bu(t),
\]

Let \(q(s)\) be a monic stable polynomial

\[
q(s) = s^m + q_{m-1}s^{m-1} + \cdots + q_0.
\]

We get

\[
g(q(s)y(t)) = f^T x(t) + k_0 u(t),
\]

where

\[
f^T = c^T q(A), \quad k_0 = c^T A^{-1} b.
\]

If the state \(x(t)\) is available, the inverse system is given as follows:

\[
u(t) = k \{ - f^T x(t) + q(s)y(t) \}
\]

where \(k = k_0^{-1}\).

The reference model to be followed is defined by

\[
y_m(t) = q^{-1}(s)r_m(t),
\]

where \(r_m(t)\) and \(y_m(t)\) are the reference input and the output, respectively. Replacing \(y(t)\) by \(y_m(t)\) in (8), the control input for the model following control becomes

\[
u(t) = k \{ - f^T x(t) + q(s)y_m(t) \}
\]

\[
= k \{ - f^T x(t) + r_m(t) \}.
\]

B. Observer Proposed

The state \(x(t)\) is not generally available, therefore we introduce the continuous-time observer which reconstructs \(x(t)\) by using convolution integrals in a finite span of the time. The observer is designed as follows.

First, the differential equation (1) can be solved as

\[
x(t) - x(t-h) = \exp \{-Ah\} x(t) + \int_t^{t-h} \exp \{A(t-s-a)\} bu(s) \, da.
\]