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Hydraulic design of a regenerative flow pump for an artificial heart pump

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Abstract: The present study is focused on the design of a regenerative flow pump for artificial heart pump application and the experimental testing and results. It is based on the improved momentum exchange theory proposed by Yoo *et al.* [1]. Salient feature of the present design procedure is that it does not require any re-adjustment of the input data. Using the design procedure, a regenerative flow type blood pump has been designed and manufactured to confirm its validity.

Keywords: regenerative blood pump, momentum exchange theory, design and performance analysis

1 INTRODUCTION

As the regenerative pump is suitable to develop a high head at a low flowrate with a single rotor, it has found many applications in lubrication, control, filtering, and boosting fluid pressure level. Several theories have appeared in the literature concerning the operational principle of the regenerative turbomachine. Among them, a widely used theory is the momentum exchange theory. On the basis of this theory, Yoo et al. [1] suggested a quite complete analysis model that includes the effect of geometric shape on the performance in detail and that introduces the circulatory pivot and effectiveness of circulatory flow. Also, they proposed a slip factor for regenerative pumps and formulated circulatory loss models using available experimental data in references [2] to [4].

The present study aims at designing a regenerative flow pump for artificial heart pump for left ventricular assist device. Note that the regenerative-type pump has not been attempted for use as an artificial heart pump, yet. For use as a blood pump, both hydraulic performance and hemodynamic performance must be satisfactory. Hemodynamic performance is closely related to the damage of the blood hemoglobin. In general, such damage is known to be mostly affected by shear stress as well as the turbulence intensity in the circulating blood through the flow passage in the pump. Regenerative pumps have a spring-like flow pattern, so the blood damage may be considerable. However, as the rotational speed is relatively low, shear stress level is not much higher than the conventional type blood pump. For example, with comparable impeller diameter of 30 mm, axial blood pumps are operated in the range of 8000-12 000 r/min and radial blood pumps are operated about 3000 r/min. However, the regenerative blood pump can be designed <3000 r/min. In addition, regenerative pumps have better cavitation characteristics. And as it has a linear head curve versus the flowrate, it has better control characteristics compared with the other type machines.

In the present design, a regenerative flow pump for artificial heart pump is investigated in view of hydraulic performance only. Its hemodynamic performance will be investigated by calculating appropriate flow variables that is closed related to the damage of the blood hemoglobin in our next study.

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2 DESIGN FORMULAE AND PROCEDURE

The schematic of a rotating impeller vane (shaded part) and the cross-section of the open channel are shown in Fig. 1. The present design procedure is intended to design the channel geometry where the radius of curvatures, $R_{\rm v}$ of the impeller vane and $R_{\rm o}$ of the open channel, are equal to the widths, b and d, of the impeller vane and open channel, respectively, and the vane width is equal to half the vane height, h.

2.1 Design formulae

2.1.1 Number of vanes

The number of vanes is one of the critical variables determining the performance of turbomahcines. When the number of vanes is too small, the fluid is poorly guided and the energy transfer from the vanes to the fluid is decreased. In contrast, when too large number of vanes are used, the increases of skin friction and flow blockage deteriorate the machine performance. Therefore, the appropriate compromise between these two aspects must be made during the design process. Like radial turbomachines [5], the ratio of vane pitch to vane height is given by

$$K = \frac{2\pi r_{\rm g,v}/Z}{h/\cos\beta_{\rm m}} \tag{1}$$

where

$$\beta_{\rm m} = \frac{\left(\beta_{\rm 1b} + \beta_{\rm 2b}\right)}{2}$$

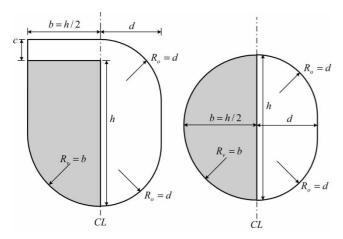


Fig. 1 Designed channel shape

Usual value of K for radial turbomachines is between 0.35 and 0.45. Investigating many regenerative pumps and fans in use at various industries, it has been found that K also falls in the similar range. Within this range, a number of values of K have been tested numerically during the present design process and K = 0.33 was found to yield the maximum efficiency.

Substituting the radius of the vane's centroid (as in Fig. 1) into equation (1) and then arranging it, the number of vanes are as follows

$$Z = \frac{1}{K} \frac{4\pi}{\pi + 4} \left[2C_1 \left(\frac{h}{r_2} \right)^{-1} - C_2 \right] \cos \beta_{\rm m}$$
 (2)

where

$$C_1 = C_2 = \frac{\pi}{2}$$
: one-sided semicircular vane

$$C_1 = 1 + \frac{\pi}{4}$$
, $C_2 = \frac{10 + 3\pi}{12}$: radial vanes

2.1.2 Vane thickness

The volume reduction due to the presence of number of vanes affects the location of circulatory pivot; therefore, it influences the effectiveness of the circulatory flow. Excessively thick vane causes large loss due to its blockage effect and moves the circulatory pivot to the inside of the open channel. In contrast, excessively thin vane moves the circulatory pivot to the inside of the vane, and, hence, it decreases the effectiveness of the circulatory flow. Therefore, it is important to find an optimum vane thickness. In the present study, the vane thickness is determined by considering the effectiveness of circulatory flow.

In order, for all circulatory flow developed inside the channel to pass through the impeller, the circulatory pivot must be located on the line CL in Fig. 1, i.e. the left and the right wetted parts' volumes, divided by the line CL in Fig. 1, of the circulatory flow channel must be equal. This condition yields the following formula to determine the vane thickness

$$\frac{tZ}{2\pi r_2} = \frac{4}{C_1} \left[\frac{A_0 r_{\rm g,o}}{h^3} - \frac{c(r_2 + c/2)}{h^2} \right] \frac{h}{r_2} + \frac{1}{2} \frac{C_2}{C_1} \frac{h}{r_2}$$
(3)

$$BL = \frac{r_{g,v} \cos \beta_{1b}}{r_{g,v} \cos \beta_{1b} - tZ/2\pi}$$
 (4)

Equation (4) is the blockage factor and denotes the area ratio of the frontal area to the actual flow area of an impeller.

2.1.3 Vane height

In order to guarantee that the circulatory flow enters into the vane space smoothly and to maximize the hydraulic diameter of the vane inlet region, the vane height is determined by the following equation

$$h = \left(r_2 - \frac{tZ}{2\pi}\right) \frac{2\sqrt{\pi}}{\sqrt{Z} + \sqrt{\pi}} + (r_2 - r_c) \frac{\sqrt{Z} - \sqrt{\pi}}{\sqrt{Z} + \sqrt{\pi}}$$
 (5)

2.1.4 Stripper angle

The stripper is an isolating wall that prevents the pressurized fluid in the outlet region from flowing into the inlet region at low pressure. When the stripper angle is small, more work is done on the fluid due to the increased pumping region, but the leakage flow through the stripper gap from the outlet region to the inlet region is increased. If the number of vanes in the stripper region is denoted by $Z_{\rm st}$, the stripper angle is given as follows

$$2\pi$$
: $\theta_{\text{st}} = 2\pi r_2$: $\frac{2\pi r_2}{Z}(Z_{\text{st}} - 1) + t$ (6a)

$$\theta_{\rm st} = \frac{2\pi(Z_{\rm st} - 1)}{Z} + \frac{t}{r_2}$$
 (6b)

A recommended value of $Z_{\rm st}$ is three in practice.

2.1.5 Stripper clearance

Usually, the leakage through a clearance affects the overall performance of turbomachines. Especially, when the specific speed is very low with small impeller size, the influence of the leakage flow on the machine performance is noticeable. The clearance in a regenerative pump is usually taken to be <0.5 per cent of the vane height. Assuming that the flow through the stripper clearance is driven partly by the pressure differential acting on a series of orifices and partly by the drag of the impeller, Wilson $et\ al.\ [2]$ obtained the following relation

$$\phi_{1} = \frac{Q_{1}}{Q_{s}}$$

$$= \frac{(h+b)\delta}{A_{o}} \left[\frac{r_{2}}{r_{g,o}} + 2C_{D} \sqrt{\frac{2\theta_{p}}{Z_{st}}} \frac{d\psi}{d\theta} \right] - \frac{1}{2} \frac{h^{2}\delta}{A_{o}r_{g,o}}$$
(7)

Here, the orifice coefficient C_D is assumed to be 0.85.

2.2 Design procedure

In designing the conventional turbomachines, the input variables are design flowrate, design head rise, and rotational speed. For axial and radial turbomachines, the impeller diameter is usually decided by the relation of specific speed and specific diameter in the Cordier diagram. However, such guiding information is not available for the present regenerative turbomachines, and usually the impeller diameter is determined from the given design conditions. Therefore, in the present design procedure, the impeller diameter is treated as an input variable. Therefore, the inputs are flowrate, head rise, and impeller diameter, and the design variables are tip gap, c, and channel's aspect ratio, AR. Through iterative calculations by adjusting the design variables, other geometric variables are found to produce the design flowrate and head at its maximum efficiency. The flow chart of the present design procedure is shown in Fig. 2.

3 PERFORMANCE ANALYSIS

After all design variables have been determined through the present design procedure, a performance analysis may be conducted to predict the offdesign performance of the machine with the designed configuration. The procedure of the performance analysis is presented as follows.

3.1 Performance variables

The basic performance variables are head, power, and efficiency. The head rise from a regenerative pump is computed from the following equation

$$\psi = \frac{gH}{U_{\rm g,o}^2} = \psi_{\rm c} - \Delta\psi_{\theta} \tag{8}$$

where

$$\begin{split} \psi_{\rm c} &= \frac{gH_{\rm c}}{U_{\rm g,o}^2} = \frac{\vartheta Q_{\rm c}}{Q_{\rm s}} \Bigg[\sigma \bigg(\frac{r_{\rm e}}{r_{\rm g,o}} \bigg)^2 - \alpha \bigg(\frac{r_{\rm i}}{r_{\rm g,o}} \bigg)^2 \bigg] \\ \Delta \psi_{\theta} &= \frac{\Delta gH_{\theta}}{U_{\rm g,o}^2} \end{split}$$

In equation (8), $\psi_{\rm c}$ denotes the head rise due to the increase of angular momentum of the circulatory flow in the pumping region. ϑ is the effectiveness of the circulatory flowrate and denotes the substantial circulatory flowrate passing through the impeller.

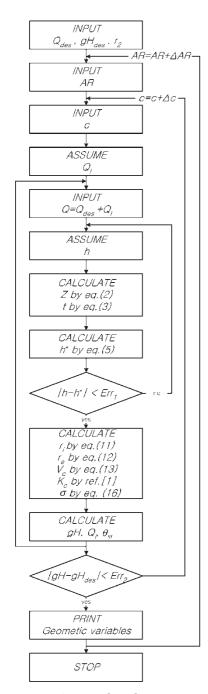


Fig. 2 Flowchart

In the present design, the geometric channel shape is determined to locate the circulatory pivot on the line CL in Fig. 1 and, hence, $\vartheta=1$. $\Delta\psi_{\theta}$ denotes the head loss due to the tangential flow passing through from the inlet to outlet ports. It is evaluated by the classical pipe friction loss formula [6].

Hydraulic power input to a regenerative pump is found from

$$\tau_{\rm h} = \frac{P_{\rm h}}{\rho Q_{\rm s} U_{\rm g,o}^2} = \frac{\rho Q_{\rm s} g H_{\rm c}}{\rho Q_{\rm s} U_{\rm g,o}^2} = \psi_{\rm c} \tag{9}$$

Then, the hydraulic efficiency is evaluated as follows

$$\eta_{\rm h} = \frac{\rho Q g H}{P_{\rm h}} = \frac{\rho Q \psi U_{\rm g,o}^2}{\rho Q_{\rm s} \psi_{\rm c} U_{\rm g,o}^2} = \phi \frac{\psi}{\tau_{\rm h}} = \phi \left[1 - \frac{\Delta \psi_{\theta}}{\psi_{\rm c}} \right] \quad (10)$$

In equation (10), ϕ denotes non-dimensional flowrate. It is an independent variable evaluated by the design flowrate. Therefore, when the increase of angular momentum of circulatory flow, ψ_c , is calculated, all of the performance variables can be evaluated.

3.2 Angular momentum variation

Circulatory flow plays a role to impart angular momentum supplied by the impeller to the tangential flow moving in the open channel that consists of the tip gap channel and the side peripheral channel. Therefore, circulatory flow is the essential feature for an analysis of regenerative pumps.

It is assumed that the circulatory pivot is located at the centroid of the volume containing the fluid in a channel and the circulatory velocity is distributed linearly [1]. From the circulatory pivot and assumed linear velocity distribution, the mean radii of the impeller inlet and outlet can be found from the following equations

$$r_{\rm i} = \sqrt{\frac{3}{10} \frac{4r_0^3 + 3r_0^2 r_{\rm c} + 2r_0 r_{\rm c}^2 + r_{\rm c}^3}{2r_0 + r_{\rm c}}}$$
(11)

$$r_{\rm e} = \sqrt{\frac{3r_{\rm c}^5 + 10r_{\rm 2}^2r_{\rm 3}^2(2r_{\rm 3} - 3r_{\rm c}) + 15r_{\rm 2}^4r_{\rm c} - 8r_{\rm 2}^5}{10(r_{\rm 3} - r_{\rm c})^2(2r_{\rm 3} + r_{\rm c})}}$$
(12)

If the vane is semicircular, the mean radius of the impeller outlet can be calculated by equation (11) with r_0 replaced by r_3 .

To estimate the circulatory flowrate, it is necessary to find an appropriate flow area and an average circulatory flow velocity. From the assumption of the linear distribution of circulatory flow, the circulatory flow area is evaluated by dividing the wetted channel volume of the circulatory flow by the mean circulatory path length which is about two-third of the outer perimeter of the circulatory flow channel.

The average circulatory velocity can be calculated by applying the angular momentum and steady flow energy equations to the impeller and open channel control volumes. Equations (13) and (14) present the circulatory velocities averaged over entire pumping region and in the linear region, respectively. Note that these are obtained for zero inlet vane angle

$$\overline{V_{\rm c}} = \left[1 - \frac{2}{C_3} \ln\left(\frac{2}{1 + {\rm e}^{-C_3}}\right)\right] V_{\rm c,L}$$
 (13)

where

$$C_{3} = \frac{K_{c}A_{c}\theta_{p}V_{c,L}}{Q + Q_{v}}$$

$$V_{c,L} = \sqrt{\frac{2(1 - \phi)(\sigma U_{e}^{2} - \alpha U_{i}^{2}) - (1 - \alpha)^{2}U_{i}^{2}}{K_{c}^{2}}}$$
(14)

Here K_c is the circulatory loss coefficient. The circulatory flow accompanies small secondary eddies generated from the impeller vane corners. This flow phenomenon is similar to the secondary flow in a bend. Yoo *et al.* [1] evaluated the circulatory flow coefficient by dividing the circulating channel into four right angular sections and summing all losses at the four corners with a bend combination factor, I, as follows

$$K_{c} = J(K_{c,I} + K_{c,III} + K_{c,III} + K_{c,IV})$$
 (15)

In equation (14), α is the incidence factor defined by the ratio between the average absolute tangential velocity of the fluid entering the impeller and the vane frame speed at inlet to the impeller. In the present study, the incidence factor is calculated by assuming that the entering velocity profile is linearly decreased through the open channel.

 σ in equation (14) is the slip factor defined as the ratio of the average absolute peripheral velocity of the fluid leaving the impeller to the blade frame speed at the outlet of the impeller. Slip phenomenon in regenerative turbomachines is enhanced considerably by the peripheral adverse pressure gradient. The adverse gradient pressure force pushes the outlet flow backward so that the absolute peripheral velocity of the outlet flow becomes slower than the rotational blade speed. Several models on slip factor have been suggested for radial type machines where there is essentially no peripheral pressure gradient. Hence, these models cannot be directly applicable to regenerative turbomachines. Yoo et al. [1] suggested the following slip factor model for regenerative pumps

$$\sigma = \frac{\sigma^* + (1 - \sigma^*)(1 + (Q_v/Q_s))(r_{g,o}/L_c)\phi}{1 + (1 - \sigma^*)(1 + (Q_v/Q_s))(r_e/L_c)}$$
(16)

where σ^* is a slip factor for radial turbomachines.

4 DESIGN RESULT AND EXPERIMENT

4.1 Design of a prototype regenerative blood pump

As an example of design problem, a blood pump for the left ventricular assist device was designed, a prototype manufactured, and its performance tested to confirm the appropriateness of the proposed design procedure. The required performance of blood pumps is such that the total pressure is in a range of 0-200~mmHg for the flow range of 0-10~l/min and the input power should be <10~W. On the basis of these requirements, the design head of 1300 mmHg at the design flowrate of 5 l/min was selected for the present design study. The performance analysis has been done only on the hydraulic performance, of the pump. The hemodynamic performance, such as hemolysis and thrombus formation, was not considered here.

4.2 Design result

In the present design procedure, a parametric analysis according to the channel aspect ratio is conducted. The design flowrate, efficiency, and the efficiency gradient at design point are plotted versus the channel aspect ratio, as shown in Fig. 3. The channel aspect ratio around 0.8 is shown to have the highest design efficiency in Fig. 3.

Usually, the efficiency of a regenerative pump has its maximum at a certain flowrate, and it is desirable that such a maximum efficiency flowrate coincides with the design flowrate. To find the aspect ratio that yields the design head at the maximum efficiency, an efficiency gradient curve as a function of the aspect ratio is created in Fig. 3, which shows that the aspect ratio of 0.8 exhibits the lowest efficiency gradient. Therefore, in the present study, the computed design variables with AR = 0.8 are chosen as the final ones. As a reference, the effects of the aspect ratio on the blockage factor and the stripper angle are shown in Fig. 4. The results reveal that the stripper

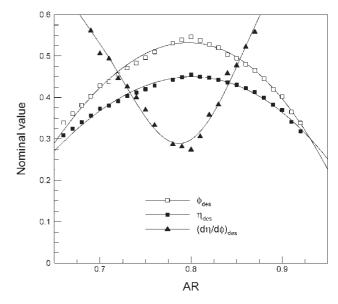


Fig. 3 Dependency of design point on AR

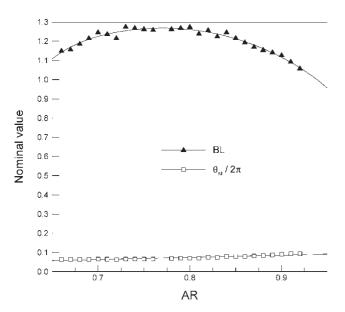


Fig. 4 Dependency of stripper angle and blockage factor on AR

angle is insensitive to the aspect ratio, whereas the blockage factor has a relatively flat plateau at its highest level near 1.2 for the range of AR = 0.73-0.83. It is comparable to 1.2-1.25 for small radial pumps and 1.1-1.5 for large radial pumps. Final design values are presented in Table 1.

The impeller rotor developed through this study is shown in Fig. 5. Tip radius is 15 mm, hub radius is 7.9 mm, and the rotor has 31 vanes. To visualize the flow pattern in the open channel, the casings were made of acryl. Figure 6 shows the assembled set of the blood pump. It has inlet and outlet ports and a couple of pressure taps were installed at each port for measurement of the static pressure rise through the pump.

4.3 Comparison of performance between predictions and measurements

To measure the static pressure difference, a differential pressure transducer (GP:50 216D) was used. Its measurement range is $\pm 2 \text{ mmH}_2\text{O}$. Hydraulic input torque was measured with a rotary type

Table 1 Design result

Design parameter	Value	Unit
Tip radius (r_2)	15	mm
Tip clearance (c)	1.4	mm
Vane height (h)	7.1	mm
Channel aspect ratio (AR)	0.8	_
Vane thickness (t)	0.51	mm
Number of vane (Z)	31	_
Stripper clearance (δ)	0.2	mm
Stripper angle (θ_{st})	25.2	deg.
Rotational speed (<i>N</i>)	2,400	r/min
Hydraulic efficiency (η_h)	34.5	%

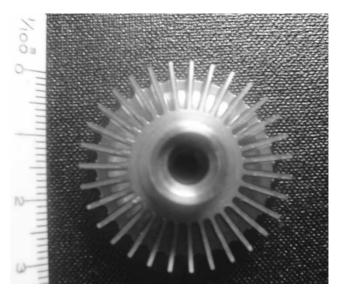


Fig. 5 Designed impeller for blood pump

torque sensor (LeBow 1103). Its measurement accuracy is ± 0.05 per cent of full scale. The volumetric flowrate was measured with TRIMEC multipulse positive displacement flowmeter whose accuracy is ± 0.5 per cent of full scale.

The predicted overall head curves for the designed blood pump are compared with experimental measurements at 2000, 2400, and 2650 r/min, as shown in Fig. 7. It can be seen that the predicted results for three different rotational speeds agree very well with the experiments. Finally, Fig. 8 shows the predicted non-dimensional head and efficiency curves in comparison with the performance

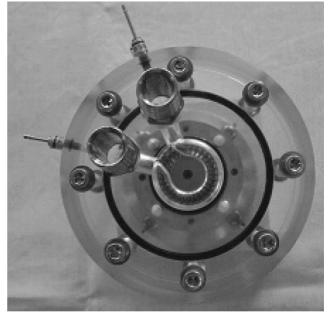


Fig. 6 Front view of blood pump

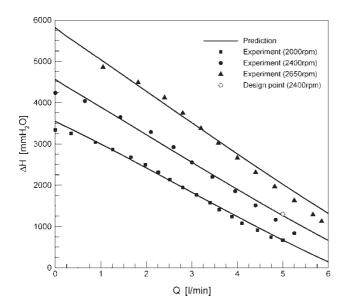


Fig. 7 Dimensional head curve

measurements. Dotted lines and the solid lines represent the non-dimensional performances with the leakage flowrate through the stripper neglected and included, respectively. It can be shown that the effect of leakage is very significant on the overall performance. The present stripper clearance of $\delta=0.2$ mm lowers the efficiency by about 10 per cent. Smaller clearance should have been adopted for better performance.

5 CONCLUSION

The present study designs a regenerative flow pump for artificial heart pump application. The present design procedure leads to find the geometric

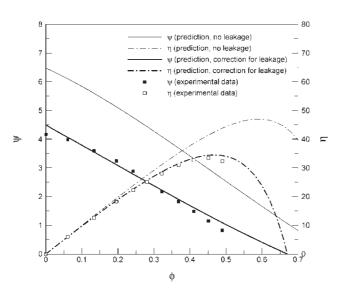


Fig. 8 Non-dimensional performance curve

configurations of the rotor and open channel that produce the desired head at the design flowrate with maximum efficiency. Using the design procedure, a prototype regenerative blood pump has been designed and manufactured to confirm its validity. The predicted overall head and efficiency curves are compared with experimental measurements. It can be seen that the calculated results agree very well with the experimental performance curve, and the prototype pump does produce the desired head at its maximum efficiency. This result confirms the appropriateness of the present analytic model and design procedure.

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APPENDIX

Notation

a	bend width (m)
A	cross-sectional area (m ²)
AR	channel aspect ratio $(=(b+d)/(h+c))$
b	vane width (m)
BL	blockage factor
c	tip gap (m)
C	constant
C_{D}	orifice coefficient
d	open channel width (m)
g	gravitational acceleration (m ² /s)
g h	vane height (m)
H	head (m)
J	bend combination factor
K	vane aspect ratio
$K_{\rm c}$	circulatory loss coefficient
L°	averaged length of circulatory flow path (m)
N	rotational speed (r/min)
P	hydraulic power (J/s)
0	volumetric flowrate (m ³ /s)

$Q_{\rm s}$	solid body rotational flowrate in an open	au	non-dimensional torque coefficient
	channel ($=A_{\rm o}{\rm r}_{\rm g,o}\omega$)	Y	angular speed of relative flow (rad/s)
$Q_{ m v}$	solid body rotational flowrate in a vane	ϕ	non-dimensioanl flow coefficient (= Q/Q_s)
	$(=A_{\rm v}{\rm r_{\rm g,v}}\omega)$	ψ	non-dimensional head coefficient
r_0	radial distance to the impeller hub (m)	$ar{\omega}$	angular speed of absolute flow (rad/s)
r_2	radial distance to the impeller tip (m)	ω	angular speed (rad/s)
r_3	radial distance to the channel tip (m)		
$r_{ m g}$	radial distance to centroid (m)		
R	radius of curvature (m)	Subscripts	
Rc	circulatory Reynolds number	Subscripts	
t	vane thickness (m)	c	circulatory flow
V	velocity (m/s)	des	design point
w	bend height (m)	e	vane exit
Z	number of vanes	h	hydraulic
		L	linear region
α	incidence factor	1	leakage
$eta_{ m 1b}$	vane inlet angle measured from the vane-	m	arithmetic mean
	to-vane plane (rad)	0	open channel
eta_{2b}	vane exit angle measured from the vane-to-	p	pumping region
	vane plane (rad)	st	stripper region
δ	stripper clearance (m)	T	total loss due to circulatory flow
$\Delta \psi$	non-dimensioanl head loss	V	vane
η	efficiency	θ	peripheral direction
θ	angle (rad)		
ϑ	effectiveness of circulatory flow		
Λ	correction factor	Superscripts	
σ	slip factor		
σ^*	slip factor for radial machine		averaged