

## LETTER

# A Goal Programming Approach for Resource Allocation Considering Client Demands in a Multiuser OFDMA Downlink System\*

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**SUMMARY** This study investigates subcarrier and power allocation schemes in an OFDMA downlink system. To consider client demands, a goal programming approach is proposed. The proposed algorithm minimizes the weighted sum of each client's dissatisfaction index. Simulations show that the sum of dissatisfaction indices can be reduced significantly.

**key words:** OFDMA, downlink, subcarrier and power allocation, goal programming

## 1. Introduction

As demand for broadband mobile communication increases, mobile communication systems are expected to support various types of services simultaneously. These systems must support reliable and high-rate data transmission. For these reasons, the orthogonal frequency division multiple access (OFDMA) scheme, based on orthogonal frequency division multiplexing, has emerged as one of the prime multiple access schemes for broadband wireless networks.

In wideband transmissions over multi-path fading channels, inter-symbol interference (ISI) is a major problem, as it inhibits high-rate data communications. However, in OFDMA systems, each subcarrier is assigned exclusively to a single client, and intra-cell interference is eliminated by using an orthogonal frequency. This aspect of the OFDMA can solve the ISI problem. This advantage makes the OFDMA a promising system for future wireless communication systems.

In the OFDMA, resource allocation is an important issue. Depending on resource allocation schemes, the system can maximize throughput or enforce fairness flexibly. Allocating subcarriers and power in the OFDMA downlink system has been widely investigated [1]–[3]. In [1], an optimal

solution of throughput maximization was achieved by assigning a specific subcarrier to the client with the best channel condition and by allocating power using a water-filling algorithm over the subcarriers. In [2], a rate adaptive dynamic resource allocation was suggested. They included the fairness problem in terms of rate criteria. The proportional rate fairness problem was studied in [3], in which an algorithm based on the criteria of proportional rate fairness was proposed.

However, these studies did not include actual client demand. As a result, unfair and inefficient assignments can result. Some clients may not be able to obtain their desired throughput while others may obtain throughput far beyond their needs. Even if client demands are formulated as system constraints, infeasible cases can result due to the limitations of wireless resource. Furthermore, according to the various types of services, clients have mutually exclusive demands. In OFDMA systems, the demands of one client can conflict with the demands of another client. In this situation, it is necessary to develop an efficient algorithm to minimize conflicting client demands while not excluding individual client demands. A goal programming [4] technique is appropriate with these types of problems, as goal programming sets a goal for each client. As the goal is not a strict constraint that must be satisfied, the system can increase a client's throughput while flexibly decreasing another client's throughput. Moreover, due to the property of the goal, the GP technique can generate a feasible solution consistently without the limitations inherent in wireless resources. For these reasons, the GP technique has been used in areas such as project selection and production management. In addition, the GP technique has been used for general downlink transmission systems [5]. However, the OFDMA system and a scheme for power allocation were not included in [5].

In this paper, we propose a goal programming approach for OFDMA downlink resource allocation. In the proposed approach, the dissatisfaction index is introduced, which includes client demands and serves as an index of the dissatisfaction of each client. The objective here is to minimize the weighted sum of client dissatisfaction while satisfying the power constraint. Using the proposed algorithm, it is possible to obtain an efficient solution for real-time applications while considering the demands of each client.

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## 2. System Model & Formulation

In this paper, an OFDMA downlink system in a cell with the power constraint of the base station is considered. Assuming that there are  $K$  clients and  $N$  subcarriers in the system, subcarriers are assumed to be orthogonal to each other. Assuming that each subcarrier has a narrow bandwidth, each subcarrier experiences a flat fading within its bandwidth. In addition, it is assumed that the channel state of each client's information is transmitted by an appropriate control channel so that the base station can obtain perfect knowledge of the channel state information. The scheduler in this system assigns subcarriers and power repeatedly. The channel gain of each subcarrier is further assumed to be fixed within one frame.

In an OFDMA system, it is optimal to allocate a subcarrier to only one client [6]. Due to frequency selective fading, each client experiences different channel gains for different subcarriers. Assuming AWGN noise with variance  $N_0$ , it can be claimed that each client experiences equal noise power over all subcarriers. For client  $i$  and subcarrier  $j$ , the signal-to-noise-ratio (SNR) is as follows:

$$g_{ij} = \frac{|H_{ij}^2|}{B/N \cdot N_0},$$

where  $H_{ij}$  is the channel gain of client  $i$  for subcarrier  $j$  and  $B$  is a total bandwidth used in the OFDMA system.

Based on the Shannon capacity formula for the Gaussian channel, the throughput of a single client can be written as follows:

$$T_i = \frac{B}{N} \sum_{j=1}^N w_{ij} \log(1 + p_{ij} g_{ij}),$$

where  $w_{ij} \in \{0, 1\}$  is the channel allocation index such that  $w_{ij} = 1$  if subcarrier  $j$  is assigned to client  $i$  and equals 0 otherwise.  $p_{ij}$  is the power allocated to subcarrier  $j$  when it is allocated to client  $i$ .

In this paper, we focus on minimizing the amount of dissatisfaction of each client using the goal programming approach [7]. For client  $i$ , it is assumed that he has his own demand  $G_i$  for his throughput. In the goal programming approach,  $G_i$  is actually his goal of satisfaction. He is 100% satisfied when his throughput is greater than or equal to  $G_i$ . Once he has been supplied with  $G_i$ , his satisfaction level does not increase with additional throughput.

If the system has an adequate number of subcarriers to satisfy the goals of all clients, this goal programming approach does not provide any meaningful conclusions. In this case, all clients are 100% satisfied.

The main interest here is a case with subcarrier deficiency. How to allocate scarce subcarrier resources to minimize the dissatisfaction level of clients is the focus of this paper. To model this issue, a dissatisfaction index  $d_i^-$  is introduced, which denotes the shortage of client  $i$ 's throughput

from his goal. It is defined as follows:

$$d_i^- = \max(0, G_i - T_i),$$

where  $T_i$  is the throughput of client  $i$ . The following goal programming model is then suggested:

$$\begin{aligned} \text{Min} \quad & \sum_i^K \alpha_i d_i^- \\ \text{s.t.} \quad & \sum_{i=1}^K w_{ij} \leq 1, \quad \forall j, \end{aligned} \quad (1)$$

$$\sum_{i=1}^K \sum_{j=1}^N p_{ij} \leq \bar{P}, \quad (2)$$

$$T_i = \frac{B}{N} \sum_{j=1}^N w_{ij} \log(1 + p_{ij} g_{ij}),$$

$$d_i^- = \max(0, G_i - T_i), \quad \forall i,$$

$$p_{ij} \geq 0, \quad w_{ij} \in \{0, 1\}, \quad d_i^- \geq 0, \quad \forall i, j,$$

where  $\bar{P}$  is the available total transmission power of the base station. The weighting factor  $\alpha_i$  represents priority factor of user  $i$ , which is introduced to consider different priorities among users. In this model, the objective is the minimization of the weighted sum of dissatisfaction indices of all clients. Constraints (1) ensure that at most one client can be allocated to each subcarrier. Constraint (2) is the total transmission power constraint of the base station.

## 3. Proposed Algorithm

The model in Sect. 2 should be solved within one frame of the OFDMA system. This model is a combinatorial mixed integer programming problem that is difficult to solve. In this section, a heuristic real-time algorithm, the Largest Satisfaction First (LSF) algorithm, is suggested to provide a sub-optimal solution of this model in real time. The LSF algorithm assigns subcarriers such that the aggregated dissatisfaction level is minimized assuming equal power allocation for all subcarriers. It then allocates optimal power to each subcarrier using a well-known optimal power allocation algorithm, the water-filling algorithm [6].

### Largest Satisfaction First

This algorithm is an iterative process. Assuming that in the middle of the iteration process of the proposed algorithm,  $T_i$  is the amount of throughput provided to client  $i$ . If  $T_i > G_i$ , no more subcarriers are allocated to the client  $i$ . At this point, it is assumed that  $T_i < G_i$ . Consider a subcarrier  $j$  that is not yet allocated to any client. If the subcarrier  $j$  is allocated to client  $i$ , his amount of throughput will then be  $T_i + \frac{B}{N} \cdot \log(1 + p_{ij} \cdot g_{ij})$ . If this throughput level is less than  $G_i$ , the incremental amount of satisfaction from the additional allocation of subcarrier  $j$  to client  $i$  is  $\frac{B}{N} \cdot \log(1 + p_{ij} \cdot g_{ij})$ . However, if  $T_i + \frac{B}{N} \cdot \log(1 + p_{ij} \cdot g_{ij})$  is greater than  $G_i$ , the incremental amount of satisfaction is only  $G_i - T_i$  as it is

assumed that the amount of traffic assigned to a client in addition to his goal does not provide any additional satisfaction in the GP approach. Hence, the incremental amount of satisfaction of client  $i$  by the additional assignment of subcarrier  $j$ ,  $S_{ij}$ , is

$$S_{ij} = \begin{cases} \frac{B}{N} \cdot \log(1 + p_{ij} \cdot g_{ij}), & \text{if } A_i \leq G_i \\ G_i - T_i, & \text{otherwise,} \end{cases} \quad (3)$$

where  $A_i = T_i + \frac{B}{N} \cdot \log(1 + p_{ij} \cdot g_{ij})$ .

The allocation process of subcarriers to clients and the power allocation process are interlinked. Once the allocation of all subcarriers has been completed, it is possible to allocate power optimally to each subcarrier by the water-filling algorithm. However, to allocate subcarriers optimally, the information about power levels  $\{p_{ij}\}$  of the subcarriers are required, which can be estimated only after the subcarrier allocation is completed.

In the proposed simple heuristic algorithm, subcarriers are allocated to clients based on the assumption that all subcarriers share the same level of power. After subcarriers are allocated based on this assumption, the optimal power level of each subcarrier can be calculated using the water-filling algorithm. Hence,  $S_{ij}$  is calculated by (3) while setting  $p_{ij} = \bar{p}$  for every  $i$  and  $j$ , where  $\bar{p} = \bar{P}/N$ .

Let  $C$  be an available subcarrier set,  $U$  be a client set to be considered. The LSF algorithm then proceeds as follows:

Step 1. Initialization

$$T_i = 0 \text{ for all } i \text{ and } S_{ij} = 0 \text{ for all } i, j. \\ C = \{1, 2, \dots, N\}, U = \{1, 2, \dots, K\}.$$

Step 2. Subcarrier Allocation

While  $C \neq \emptyset, U \neq \emptyset$ ,

- Calculate  $S_{ij}$  for  $\forall i \in U, \forall j \in C$ :  
if  $G_i < T_i + \frac{B}{N} \cdot \log(1 + \bar{p} \cdot g_{ij})$ ,  $S_{ij} = G_i - T_i$ .  
else,  $S_{ij} = \frac{B}{N} \cdot \log(1 + \bar{p} \cdot g_{ij})$ .
- Let  $(i^*, j^*) = \text{Arg max}_{i,j} \alpha_i S_{ij}$ . Assign subcarrier  $j^*$  to user  $i^*$ .
- Calculate  $T_{i^*} = \frac{B}{N} \cdot \sum_{\substack{\text{all } j \\ \text{assigned to } i^*}} \log(1 + \bar{p} \cdot g_{i^*j})$ .  
 $C = C - \{j^*\}$ .
- If  $T_{i^*} > G_{i^*}$ ,  $U = U - \{i^*\}$ .

Step 3. Power Allocation

Allocate power to each subcarrier using the water-filling algorithm.

In Step 2, the pair  $(i^*, j^*)$  with the largest value of  $\alpha_i S_{ij}$  is chosen and subcarrier  $j^*$  is assigned to client  $i^*$ . Once the subcarrier assignment is complete, power is allocated to each subcarrier using the water-filling algorithm in Step 3.

#### 4. Simulation

In this section, the performance of the proposed algorithm is

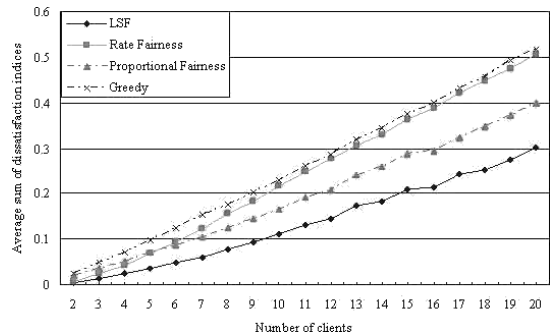


Fig. 1 Average sum of dissatisfaction indices when  $N=512$ ,  $G=0.03$ .

analyzed in computer simulations. The performance of the proposed algorithm was compared with other algorithms, including the rate-fairness algorithm [2] (referred to as the Rate Fairness), the proportional fairness assignment [3] (referred to as the Proportional Fairness), and the greedy assignment [1] (referred to as the Greedy) with varying levels of client demands. All of the utilized algorithms are very simple and their computational times are sufficiently small to be used in real-time applications.

A 5 MHz bandwidth channel divided into 512 OFDM subcarriers is considered. The system is assumed to have 20 clients distributed uniformly in a single cell with a radius of 0.5 km. In addition, the total available transmit power at the base station is 1 Watt. The path loss model is a modified Hataurban propagation model [8], as follows:

$$\text{Path Loss} = \begin{cases} 122 + 38 \log_{10}(d) & \text{if } d \geq 0.05 \text{ km} \\ 122 + 38 \log_{10}(0.05) & \text{if } d < 0.05 \text{ km} \end{cases}$$

where  $d$  (in km) is the distance between a mobile and the base station. In addition to the path loss, for each client, static log-normal shadow fading with a standard deviation of 8 dB is assumed. For multipath fading, pedestrian channel B traffic with a velocity of 3 Km/h is considered. The noise power is  $-100$  dBW in this simulation.

For the simplicity of the performance evaluation and presentation, only the case with  $\alpha_i=1$  for all  $i$  was considered. This simulation can be extended to a case with different values of  $\alpha_i$  easily. However, the most meaningful conclusions could be derived even in the simplified simulation results.

Figure 1 shows the average sum of dissatisfaction indices versus the number of clients when the number of subcarriers is 512 and their goal is  $G_i = 0.03$  for all  $i$ . In order to obtain the average sum of the dissatisfaction indices, 100 iterations for each algorithm were conducted. Due to the limitations of wireless resources, the average sum of dissatisfaction indices increases when the number of clients increases. However, the average sum of dissatisfaction indices of the LSF algorithm increases slowly compared with the other algorithms. It is evident that the proposed algorithm shows much better performance compared to other algorithms in terms of the dissatisfaction level of clients, especially with a large number of clients.

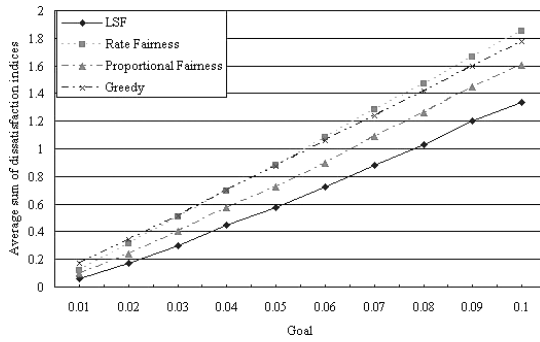


Fig. 2 Average sum of dissatisfaction indices when  $K=20$ ,  $N=512$ .

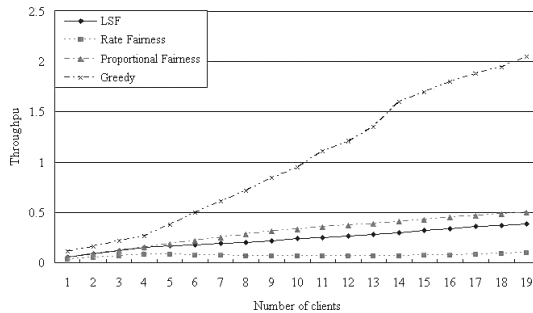


Fig. 3 Total system throughput when  $N=512$ ,  $G=0.03$ .

Figure 2 shows the average sum of dissatisfaction indices for each algorithm while varying the values of the client goal when the number of subcarriers is 512 and the number of clients is 20. As the goals of clients increase, the dissatisfaction level increases due to the limited resources. However, the dissatisfaction level of the LSF algorithm increases much slower compared to the other algorithms. Among four algorithms, the Rate Fairness algorithm shows the lowest performance, as this algorithm adjusts the throughput of each client equally without considering the goals of the clients.

Figure 3 shows the total throughput for each algorithm versus the number of clients. In this figure, Greedy algorithm shows the best performance, and the PF algorithm

shows slightly better throughput than the LSF algorithm. However, these algorithms are not able to reduce the amount of dissatisfaction in client demands as much as the LSF algorithm.

## 5. Conclusion

In this paper, a subcarrier and power allocation scheme that minimizes the weighted sum of dissatisfaction indices of OFDMA downlink system was formulated. As service types are diverse in wireless communication systems, wireless resource allocations should consider the demands of each client according to the type of service. A goal programming model for the subcarrier and power allocation problem was proposed to consider each client's demands as a goal. A simulation showed that the sum of dissatisfaction indices can be reduced significantly by applying the LSF algorithm. It was also found that the LSF algorithm is superior relative to the other algorithms, especially with a large number of clients.

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